**Applied Optics**
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**Lecture – 26**
**Multiple Beam Interference**

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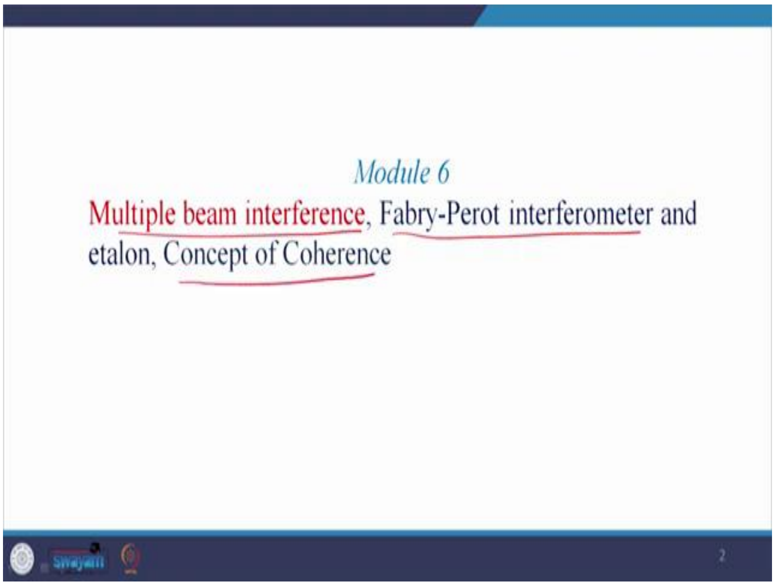


Hello everyone, welcome back to my class, today we start module number- 6. In module-5, we talked about interference. And in module-6, we will again talk about interference, but in this module, we will particularly talk about multiple beam interference. Just to clarify the difference, in module-5 we talked about two beam interference. There we took a thin film and then we considered only first two reflected and first two transmitted rays and there we talked about interference between the two. Here we will take multiple reflected and multiple transmitted rays.

And the interference between reflected rays would be considered as well as the interference among transmitted rays would be considered and there we will talk about the condition of maxima and minima. And thereafter Fabry-Perot interferometer would be discussed and at last I will introduce the concept of coherence.

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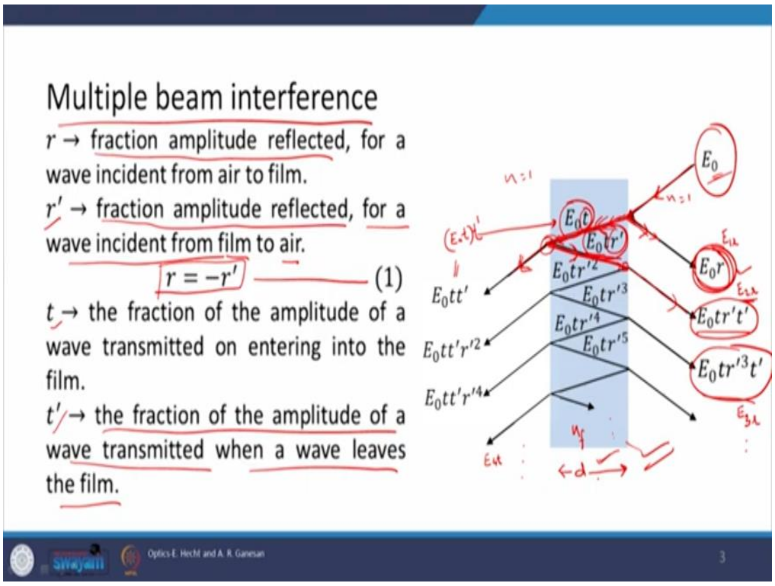


Now, let us start with multiple beam interference. Now, to realize multiple beams, what do we do is that we take a thin glass plate or thin film, which is shown by this shaded color here and light beam is launched of amplitude $E_0$ here at this interface. And as we discussed earlier also, a part of the rays get reflected and a part get transmitted. Here in this figure, this shaded region is glass, which have say some refractive index n while the outside region, it is air, n=1 here in outside medium, this film is kept in air.

Now, the first ray which is being launched on the interface of amplitude $E_0$, this ray suffers reflection at this interface. And here we have assumed that amplitude reflection coefficient is a r and this amplitude reflection coefficients tells us about the fraction amplitude which got reflected at this interface. Therefore, the amplitude of the ray which will be reflected would be equal to $E_0$ which is our initial amplitude into this amplitude reflection coefficient which is r.

In the same way, we have assumed that the amplitude transmission coefficient in the film is t. And therefore, the fraction of the amplitude which is transmitted in this experiment is $E_0t$. Now, the reflection coefficient within the film is assumed to be $r'$, $r'$ is fraction amplitude reflected.

And this is for a wave incident from film to air, r is amplitude reflection coefficient for ray which is made to incident from air to film. But, $r'$ is the amplitude reflection coefficient from film to air, it is opposite. And we have already studied Stokes relations, which relates r with $r'$ and the relation is given by equation number one here, which says that r=$-r'$. There is 180 degree phase difference between these two amplitude reflection coefficients. t is the amplitude transmission coefficient from air into film. Similarly, $r'$ is the again transmission coefficient from film to air.

Therefore, it is fraction of the amplitude of a wave transmitted when the wave leaves the film and this is shown here. When this ray falls at this interface, a part get transmitted and a part get reflected, within the film reflection coefficient is $r'$. Therefore, the total reflected amplitude is incident amplitude into $r'$, and incident amplitude is $E_0t$ and therefore, the total amplitude would be $E_0tr'$.

Similarly, the transmitted amplitude would be $E_0tt'$. Why? Because $E_0t$ is our incident amplitude which is given here and then we will have to multiply it with the transmission coefficient which is $t'$.

This $t'$ is transmission coefficient from film to air and if you multiply this with $t'$, you get this relation. $E_0tt'$ is the amplitude which get the transmitted, this is the first transmitted beam. And then we keep on multiplying the relevant transmission and reflection coefficient and this will give us the various orders of a reflection and transmission amplitudes. And these are given here with this relation, this is the amplitude of the first reflected wave. This is the amplitude of the second reflected wave, this is the amplitude of the third reflected wave.

Therefore, we call this the first reflection as $E_1r$, the second reflection as $E_2r$, the third reflected beam as $E_3r$ and so on. And similarly, on the transmitted part, the first transmitted wave is called $E_1t$, the second is called $E_2t$, the third is called $E_3t$, and fourth is called $E_4t$ and so on and so forth. There will be infinite many such reflections, because at each interface, the beam will suffer partial reflection and partial transmission, and it will keep on going. And here we have also assumed that the film, this thin film, is lossless it is not absorbing any amplitude.

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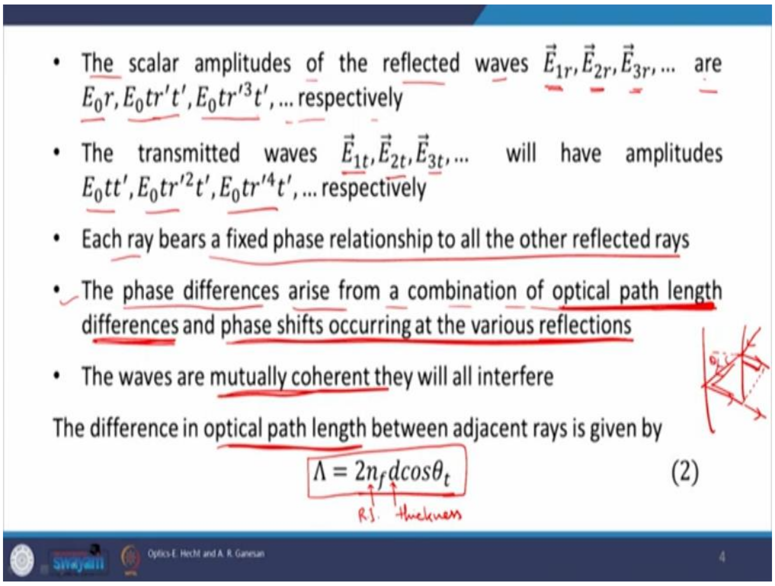


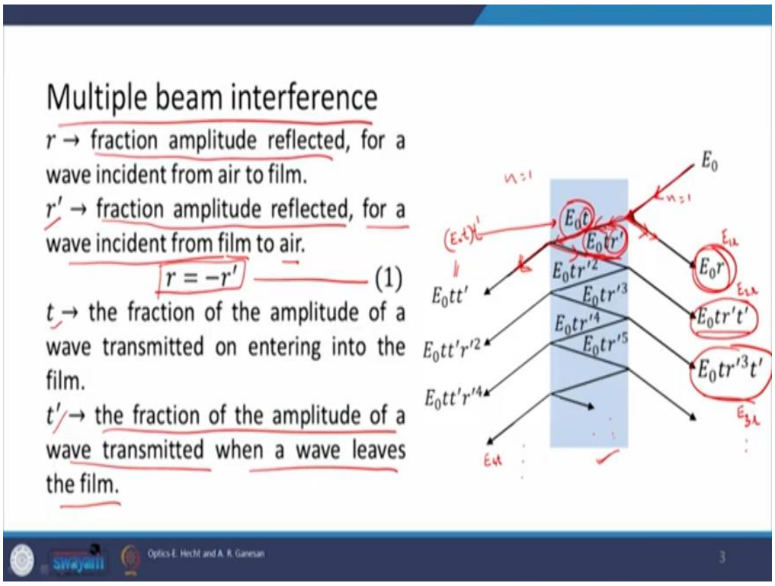


Now, therefore, we will get a number of reflected rays and a number of transmitted wave. And this is what exactly is written here. The scalar amplitudes of the reflected waves which are designated as $E_{1r}$, $E_{2r}$, $E_{3r}$, these are the reflected wave. And their respective magnitudes are $E_0r$, then $E_0tr't'$, then $E_0tr'^3t$, $t'$ and so on and so forth. Similarly, on the transmitted part, if transmitted beams are $E_{1t}$, $E_{2t}$, $E_{3t}$ then their respective magnitude would be given by these three terms respectively.

Now, each ray bears a fixed phase relationship to all other reflected rays. Because each ray you see here in this figure, the first reflection is appearing and this reflected part is not going inside the film; but, while the second reflection which is $E_{2r}$ is traversing the thickness of the film twice.

Therefore, each reflection, each next reflection is traversing the thickness of the film twice. And therefore, there is a certain phase relationship between all these rays, both transmitted and reflected. Now, all these phase differences arises from a combination of optical path length differences and phase shift occurring at various reflection. Now, the phase relationship which reflected rays as well as transmitted rays develops, it has two contribution. The first contribution is optical path length differences, because the different reflected ray, they traverse different thickness of the film.

Similarly, different transmitted waves, they travel different thicknesses of the film. And therefore, they have a certain phase relationship and this thickness of the film appears in path length differences. This is the first contribution in the total phase.

The second contribution in the total phase arises out of reflections. Because we know when a wave travels from a rare medium to denser medium, the reflection there contributes 180 degree phase shift. There are few reflections there in the thin film is of this nature where the wave sees 180 degree phase shift and few reflections are there where the wave does not see this 180 degree phase shift.

And these two phase shift where first due to the path length difference, and second due to the reflection. These two phases get accumulated and this give the resultant phase and this resultant phase is seen among different transmitted and reflected rays. Now, all these transmitted and reflected rays as can be seen in this figure they all are getting generated from the wave of amplitude $E_0$. One single beam is falling and it is generating the different orders of transmission and the reflections.

We see different transmitted beams and different reflected beams and they all have their origin in the first beam which is of magnitude of $E_0$. And since they all are generating from the same source, they all are mutually coherent and therefore, they will interfere and give some sustained interference fringe pattern.

Now, we can calculate the optical path length difference. How to calculate? Take two adjacent rays and then suppose this is our film and the rays falling like this. One ray will go down and then we will get reflected from here and then it will go in this direction, first reflection would happen here.

Now, to calculate the path difference between the two ray, we will have to drop a perpendicular here, then calculate these two path and then subtracted from this path. And this will give the path length difference which we have already calculated.

And this optical path length would be $2n_f dcos\theta$, where d is the thickness, d is the thickness of the film and $n_f$ is assumed to be the refractive index of the film, $n_f$ is the refractive index of the film and d is its thickness. If these two quantities are known and angle of incidence is known, then we can easily calculate the path length difference between the adjacent rays. And this is equal to $2n_f dcos\theta_t$, $\theta_t$ is the angle of transmission, this is $\theta_t$.

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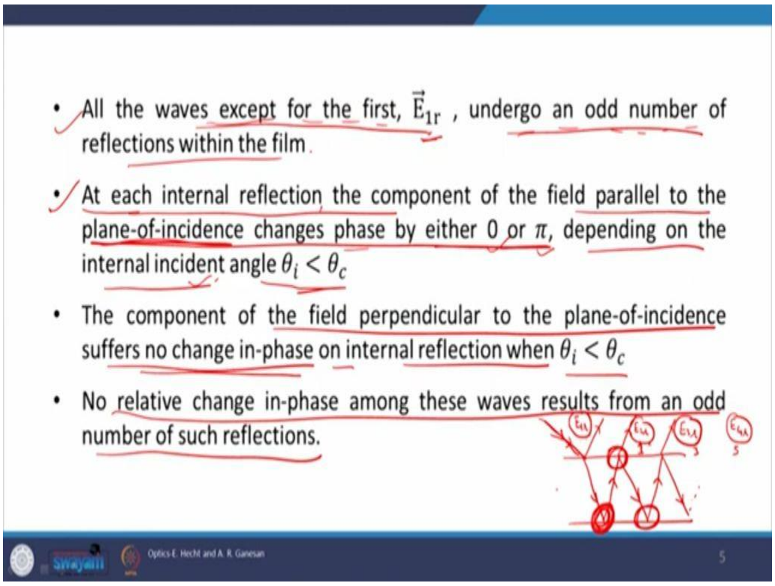


Now, all the waves except for the first which is $E_{1r}$, undergo an odd number of reflection within the film. How to ensure this? Now, you again see the same picture. This is our film and this is the wave which is falling on the film and this is the first reflection and then a partial transmission is there, again reflection, again transmission-reflection, again reflection-transmission, reflection and so on.

Now, this is your first reflected ray, this is your second reflection, this is our third reflection. Now, this statement says all the wave except for the first which is $E_{1r}$, $E_{1r}$ is given here, undergo an odd number of reflections.

Now, let us see, the first reflection for $E_{2r}$, this is the only reflection which is happening, which is 1. For $E_{3r}$, this is the first reflection, and this is the second reflection, and this is the third reflection. It means $E_{2r}$ is undergoing one reflection, $E_{3r}$ is going three reflection and similarly

$E_{4r}$ undergo five reflection. You can draw multiple reflections and realize this. And we see that the apart from $E_1r$, all other waves they undergo odd number of reflection.

Now, at each internal reflection, the component of the field parallel to the plane of incidence changes by either 0 or $\pi$, this is known. If a wave is falling at the interface and if the field is in the plane of the paper, because the plane of the paper will contain the incident ray, the interface, and the point of incident. And such a plane is called plane-of-incidence. I repeat plane-of-incidence contains the incident ray, the point of incidence and normal to the interface.

And if the electric field of the incoming wave is in this plane, then its phase changes either by 0 or $\pi$. And it depends whether the internal incidence angle is less than $\theta_c$ or not, $\theta_c$ is critical angle.

Now, the component of the field perpendicular to the plane-of-incidence suffers no change in-phase on internal reflection when $\theta$ is less than $\theta_c$. Now, we are launching a wave it has certain polarization. And the first statement said that the component of the polarization which is in the plane of incidence, it will change in-phase by 0 or $\pi$ and it depends upon the internal incidence angle. Similarly, the component of the field which is perpendicular to the plane-of-incidence, it will suffer no change in-phase or internal reflection.

Then, from these two statements what we can conclude is that no relative change in phase among these wave results from an odd number of such reflections, let me reframe it. $E_{1r}$ is reflection when the wave fall on this film interface from air. First interface separating the rare medium from the denser medium and the wave is coming from the rarer medium therefore, $E_{1r}$ will suffer a phase shift of $\pi$.

Now, in $E_{2r}$, it suffers only one reflection. Similarly, $E_{3r}$, it suffers three reflection, there are odd number of reflection, then therefore, the $E_{2r}$ will be in phase with $E_{3r}$. Similarly, $E_{3r}$ will be in-phase with $E_{4r}$, but there is no comment on whether $E_{1r}$ is in-phase with the $E_{2r}$. But, till now we have not talked about the phase which the wave accumulate due to the path difference, we are just talking about the phase the wave accumulate due to the reflections.

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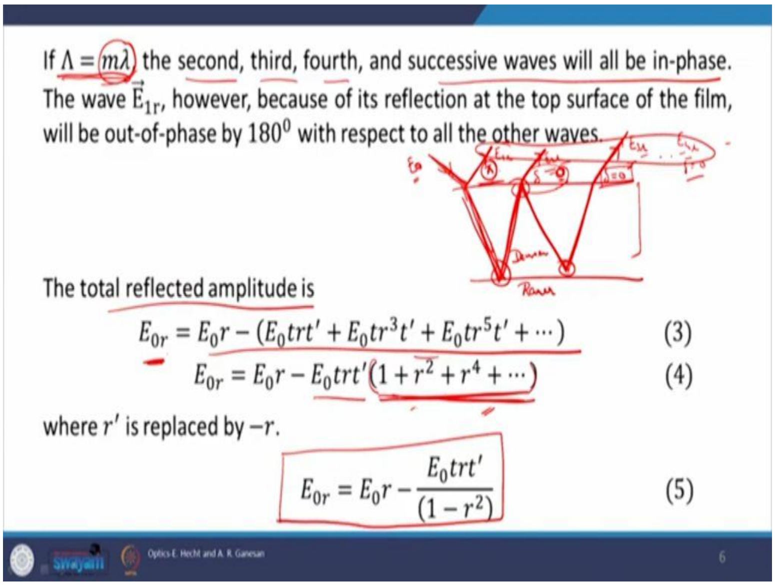


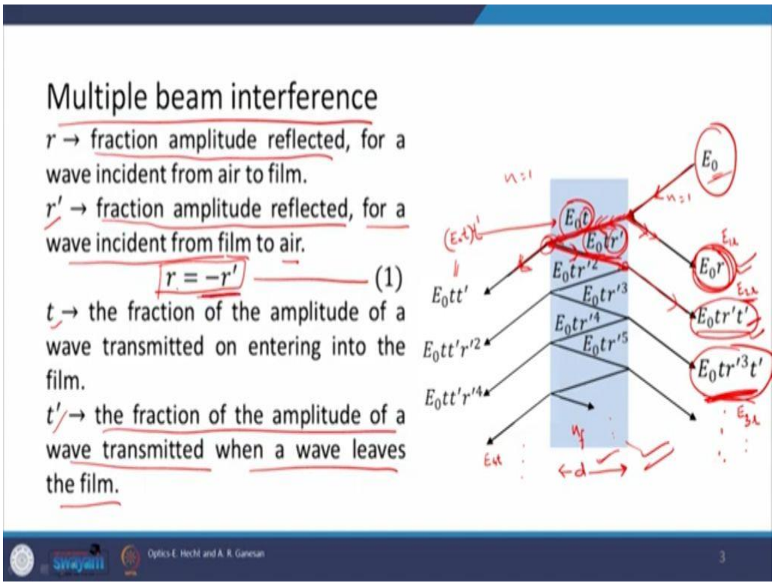


Now, let us move in to next slide. And in this slide, we take into account the phase contribution due to the optical path length differences. Now, before taking the optical path length difference into account, let us divide this case into two part.

Now, this optical path length consideration is divided in two part, the first part, the optical path length difference is assumed to be integral multiple of wavelength. Now, if optical path differences integral multiple of wavelength, then the second, third, fourth and successive waves will all be in-phase. Why do I say so? Now, this is our film. This is the incident wave, this is the first reflected wave, and this is the second reflected wave. Similarly, this one is the third one and so on, we can draw several like this.

Now, if the optical path length differences integral multiple of wavelength, then $E_{2r}$, $E_3 r$, $E_{4r}$ would be in-phase if we only consider the phase contribution from the optical path length difference, this is very clear. Since, optical path length difference is the integral multiple of wavelength or phase differences integral multiple of 2 $\pi$. Therefore, $E_{2r}$, $E_{3r}$, $E_{4r}$, $E_{5r}$ they all would be in-phase. Now, we see that the first wave $E_{1r}$, it falls on the interface and then the first reflection is there and which due to reflection suffer a phase shift of $\pi$.

Now, here it is internal reflection, it is a denser medium and here it is a rarer medium. Therefore, there would be no phase difference and then $E_{2r}$ with respect to the $E_{1r}$, it will have 0 phase difference. Let us represent the phase difference by $\delta$ and $\delta$ is equal to 0 here for $E_{2r}$ with respect to $E_{1r}$, the incident one. Why? Because this path difference is integral multiple of wavelength, no extra phase is accumulated due to the path length difference.

Similarly, $E_{3r}$, the path length difference between $E_{2r}$ and $E_{3r}$ is integral multiple of λ, therefore phase difference between $E_{2r}$ and $E_{3r}$ would again be 0. Because, all the reflections, the ray is starting from $E_0$, then it is going in this direction, in this direction, again in this direction, and in this direction and then going in $E_{3r}$.

Therefore, three reflections happened here and all three reflections are internal. And here the reflection is happening at denser to rarer medium boundary and which does not contribute to any phase here and therefore, the total phase difference would be 0. The phase difference between $E_{1r}$ and $E_{2r}$ is $\pi$, because $E_{1r}$ itself accumulate a phase $\pi$ due to the reflection, $E_{2r}$ it does not accumulate extra phase. $E_{3r}$, it also does not accumulate any extra phase. Similarly, for $E_{4r}$, it also have a phase difference of 0, it means $E_{2r}$, $E_3r$, $E_{4r}$, $E_{5r}$ they all would be in-phase. While, $E_{1r}$ would be out of phase by 180 degree with respect to $E_{2r}$, $E_{3r}$, $E_{4r}$ and so on.

Therefore, due to this $r = -r'$ relation, the Stoke relation, we have this extra phase difference of $\pi$. Now, since all the reflected rays are coming here in this direction on the first part of this thin film, to calculate the overall disturbance, we will add up the total reflection amplitudes. And total reflection amplitude, let us assume it is designated by $E_{0r}$, it would be represented by $E_{0r}$. And then we will have to add up all these contributions we will add $E_0r + E_0tr't'$ then we will add up this. Add them up and then exercise equation number-1, which is $r = -r'$.

If you do this, then we get equation number-3. You see in equation number-3, on the right hand side, there is no $r'$, because $r = -r'$. Once it is done, then we see that $E_0trt'$ is common in all this term. Therefore, it can be taken out and what is left is here in this bracket. Now, you see that this is GP (Geometric Progression) and we know how to add a geometric progression. We used our knowledge of GP here, added them up and we have this final relation for the total reflected amplitude.

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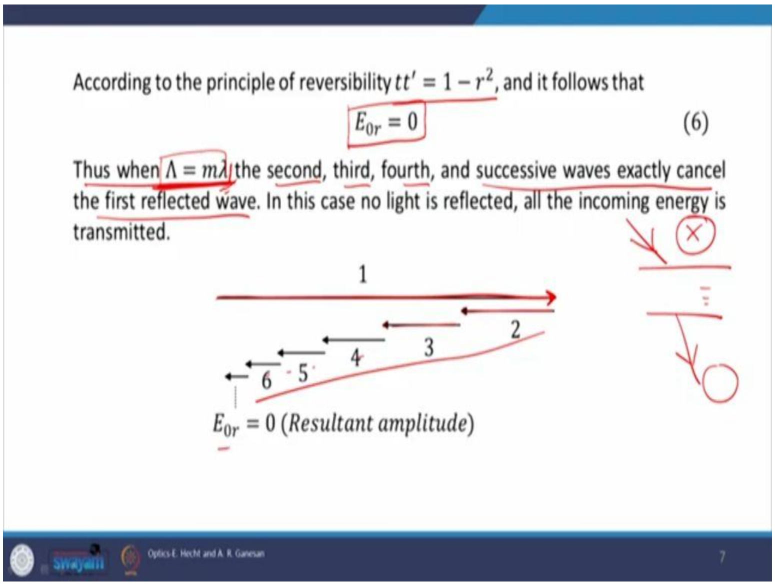


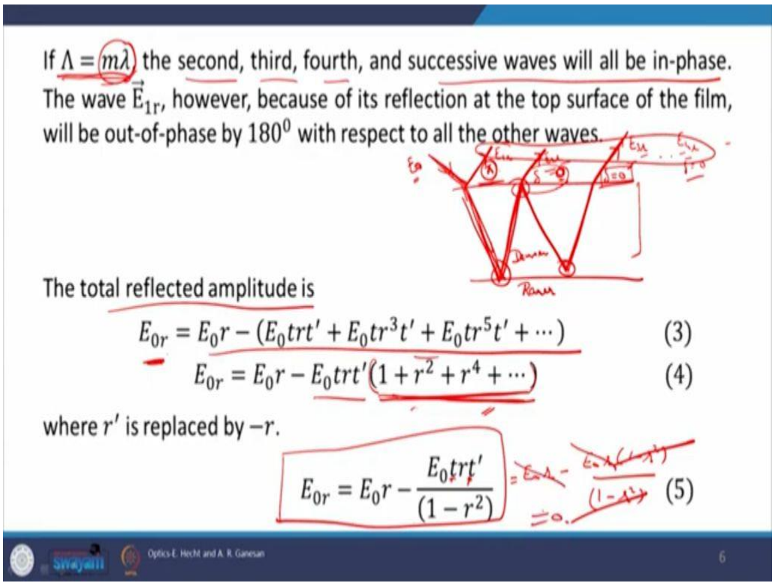


Now, the principle of reversibility also gave another Stokes relation, and which is $tt' = 1 - r^2$. Now, if you put $tt'$ which is here in the numerator, if you replace $tt' = 1 - r^2$, then this is what you get here $E_0 r - E_0 r$, $tt' = 1 - r^2$ and in the denominator too, we have $1-r^2$, this will go away. And this too will go away and we are left with 0 on the right hand side.

And therefore, the total reflected amplitude is equal to 0. I repeat, if the path difference contribution is such that the optical path difference is integral multiple of λ then in multiple beam interference, the resultant electric field distribution amplitude is equal to 0.

Now, thus, when optical path length difference is integral multiple of $\lambda$, the second, third and fourth these waves, successive waves, they exactly cancel the first reflected wave. And therefore, we will not get any light in the reflected arm, all the energies would be transmitted. This is the film, we launched some light and there is nothing in the reflected arm, everything is getting transmitted. Whole energy will appear here, unless there is some absorption within the film.

Vectorially, it can be considered like this, this is the amplitude of the first in wave or first reflection, this is the amplitude of the second reflection, this is the third and so on and so forth. The magnitude is decreasing because on successive reflection, only smaller part goes into the reflected arm and therefore, they all are add up when $\Lambda = m\lambda$ and the resultant is 0.

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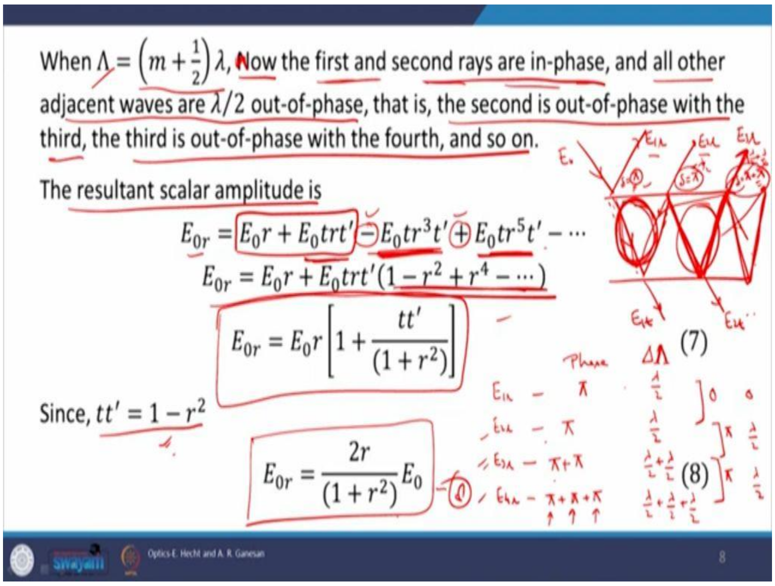


Now, let us consider the second case of optical path length difference, where we assume that optical path length difference is half integral multiple of λ. Now, in this case what will happen? Now, the first and second rays are in phase, how? Let us again draw the picture. This is the wave which is falling on the film and this is the transmitted wave, this is the first reflection, which is $E_{1r}$, again reflection, transmission. If this is our $E_{2r}$ and this is our $E_{3r}$ and similarly here we have $E_{1t}$, $E_{2t}$ and so on.

Now, what we are saying is that the optical path length difference between $E_{1r}$ and $E_{2r}$ now is equal to half integral multiple of wavelength, it means the corresponding phase difference is $\pi$. Now, for $E_{2r}$, $\delta$ is equal to $\pi$ now and for $E_{1r}$, we know $\delta$ is already $\pi$ because of the reflection. It means $E_{1r}$ and $E_{2r}$ are in phase. What about $E_{3r}$? Now for $E_{3r}$, the beam has to travel two time two times more the thickness.

Therefore, one more phase will be added here, $\pi + \pi$. First $\pi$ is due to this optical path length difference, the due to the first traversal, first two traversal within the film thickness and the second $\pi$ is due to the second two traversal between the film in the film. This is the first $\pi$ and this is the second $\pi$. First $\pi$ again, I repeat first $\pi$ is due to the optical path length difference when the beam traverse the film thickness twice for the first time. Second $\pi$ is due to the optical path length difference due to the traversal of the beam between the film thickness twice again.

Now, you see that the $E_{2r}$ is out of phase with $E_{3r}$. Similarly, if the beam travels once more, this whole film's twice then what we see is that $E_{4r}$ would be again out of phase with respect to $E_{3r}$. Why? Because, an extra $\pi$ will again get added there. If you talk in terms of optical path difference, then path difference here is λ/2, in this case λ/2+λ/2.

Let me write it here, phase or path length difference, $\Delta\Lambda$. I will write phase or path length difference for $E_{1r}$, $E_{2r}$, $E_{3r}$, $E_{4r}$, the first four. $E_{1r}$ accumulates a phase difference of $\pi$ due to reflection or path length difference of λ/2 due to reflection. This is $\Delta\Lambda$, which is difference in the optical path length. $E_{2r}$ it travels within the film twice.

And therefore, due to the optical path length difference contribution, it accumulates a phase difference of $\pi$ or path difference of λ/2. There is no contribution from the reflection for $E_{2r}$. Similarly, $E_{3r}$, it before coming out of the film $E_{3r}$ travels the film four times.

Therefore, it would be one $\pi$ for two traversal, one again extra $\pi$ for next two traversal, four traversal is equal to 2 $\pi$ and similarly here λ/2+λ/2. $E_{4r}$, it is traveling six times in the film. Therefore, first two traversal contributes $\pi$, second two again $\pi$ and the last two again $\pi$.

Similarly, in path length difference, we will have 3λ/2. And what do you see is that $E_{3r}$ and $E_{4r}$ are out of phase by $\pi$, r the path length difference is λ/2. Similarly, $E_{2r}$ and $E_{3r}$ they are out of phase by $\pi$, therefore the path length difference is again λ/2. While, first two $E_{1r}$ and $E_{2r}$, the phase difference is 0, path difference is also 0. It means $E_{1r}$ and $E_2r$ would now be in phase while other would not. The other $E_{2r}$ would be out of phase with $E_{3r}$, $E_{3r}$ would be out of phase with $E_{4r}$ and so on and this is what is written here.

Now, the first and second rays are in phase and all other adjacent waves are λ/2 out of phase, that is second is out of phase with the third, third is out of phase with the fourth and so on. Therefore, the resultant scalar amplitude now which is written here, now, you see that these two are in phase therefore, they are added. While, this is out of phase with this, $E_{3r}$ is out of phase with $E_{2r}$; therefore, we have a minus sign.

Similarly, $E_{4r}$ is out of phase with $E_{3r}$, therefore a plus sign here, here we have minus, here plus and so on. Each next term will have different sign. With this we added all these term and perform the geometrical sum. Since the series is in GP, then we added them up and this is the relation which we get ultimately. Again exercise Stokes relations which is $tt' = 1 - r'$ and this modifies the equation number-7 and this is equation number-8. This is the final expression of the resultant scalar amplitude of reflected beam.

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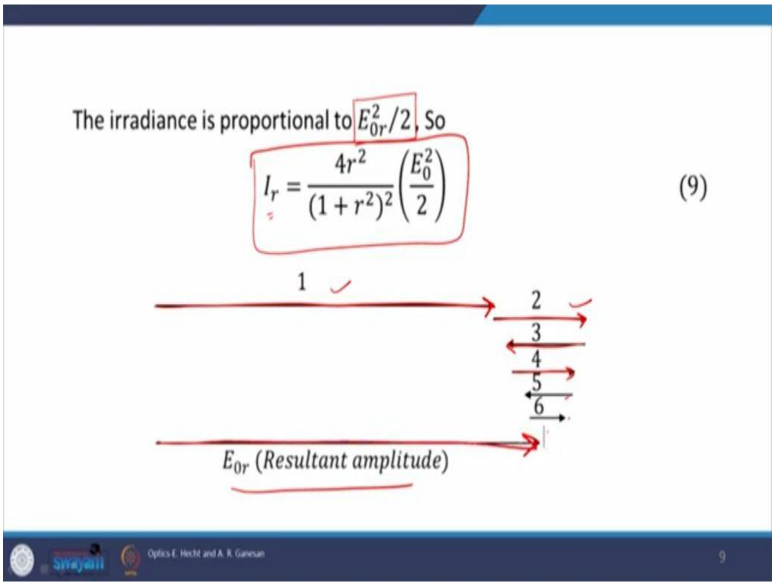


From this relation if you want to calculate the irradiance, then you know irradiance is ${E_{0r}}^2/2$. And from here we got this expression of irradiance or intensity of the reflected light. And in this case, if you want to see what exactly is happening vectorially and then you see that this is the contribution from the first reflected ray. This is the contribution from the second ray. And you see that 1 and 2, they are pointing in the same direction, they are in-phase although their magnitudes are different, and which is supposed to be, which is a very much obvious why the second magnitude would be smaller than the first one.

Similarly, the $E_3$ is also smaller than 2, but 3 is in the opposite direction because it is out of phase with respect to 2 by $\pi$. Similarly, 4 is out of phase with respect to 3 by $\pi$, similarly, 5 and 6 and so on. Now, if you add them up, then we will have this resultant this is a bigger non-zero amplitude.

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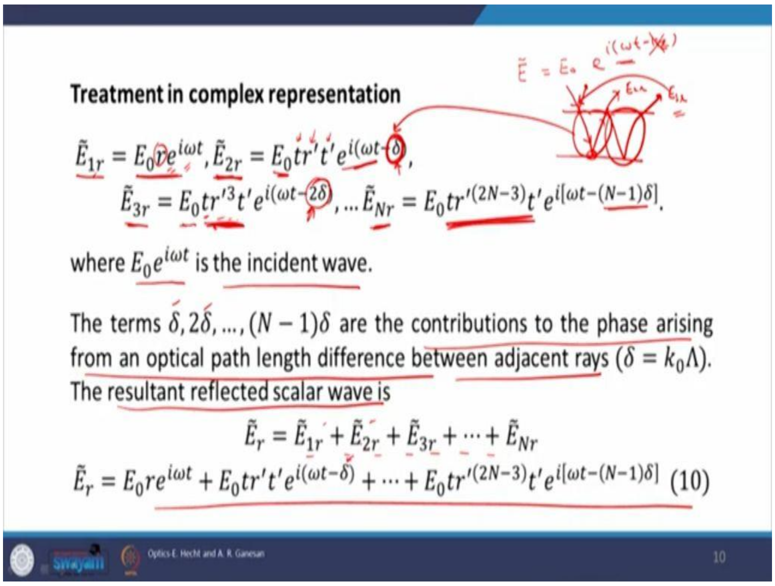


Now, the same thing can be done in a complex representation very easily, how to do these things in the complex representation. We know a wave in the complex representation is written as follows. $E = E_0 e^{i(\omega t - kz)}$, z is the direction of the propagation you see.

Now, here what we assume is that the first reflected wave which is represented by $\tilde{E}_{1r}$ it represented by $E_0 r e^{i\omega t}$, where r is the coefficient of amplitude reflection, amplitude coefficient of reflection. Now, we have assumed that this part is not there for $\tilde{E}_{1r}$, the reference is taken in such a way that the time component is only here in the $\tilde{E}_{1r}$. While, in $\tilde{E}_{2r}$ which is the second reflected beam it traverses through the thickness of the film, it means it will accumulate some phase.

And to take into account of this phase, we have incorporated this phase term $\delta$, which incorporate two traversal into the film. This is the film, the ray is coming in, it going once into the film and then twice into the film. The two traversal are embedded here in this phase $\delta$. Therefore, the second reflected ray, it will have amplitude since it is traversing through the film twice therefore, two transmission coefficient, one reflection is happening here.

Therefore, one reflection coefficient, $e^{i\omega t}$ is there already in the incoming wave, but apart from this we have a phase difference $\delta$, this phase difference is between $\tilde{E}_{2r}$ and $\tilde{E}_{1r}$.

Similarly, for third reflected ray, we have $E_0$, then the extra transmission and reflection coefficients are there which is here in amplitude part. One apart from this we have twice of $\delta$ because the this is $\tilde{E}_{2r}$ and the for the third reflection, the wave again have to travel through the thin film twice, it will again add up the phase $\delta$. Therefore, overall phase of $\tilde{E}_{3r}$ with respect to $\tilde{E}_{1r}$ would be twice of $\delta$. Similarly, for $n^{th}$ reflected wave, the phase would be$(n-1)\delta$ .

And the relevant number of transmission and reflection coefficients are also appear here in the amplitude part and $E_0 e^{i\omega t}$ as I said before it is the incident wave. And this phase $2\delta$, $(N-1)\delta$ are the contribution of the phase arising from the optical path length difference between the adjacent rays.

Now, the resultant scalar wave would be the sum of all these waves let us sum them up and substitute for $\tilde{E}_{1r}$, $\tilde{E}_{2r}$, $\tilde{E}_{3r}$. And this is the final big expression which figured in form of equation-10. This gives the resultant disturbance a resultant amplitude in the reflected part. Now again, remember the $\delta$ is optical path length difference between adjacent ray.

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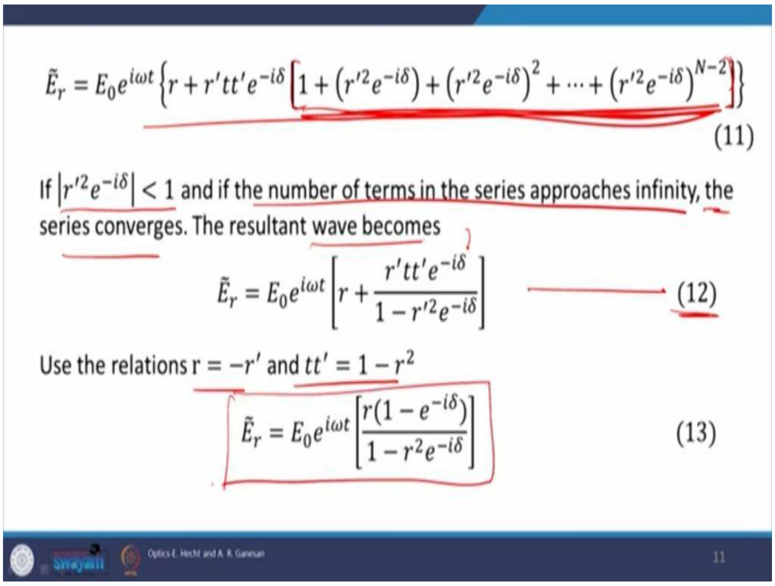


Now, doing some mathematics leads to equation number-11 and we see again that in this parenthesis, these terms are in GP. And if $r'^2 e^{i\delta} < 1$ and if there are infinite many reflected rays, then the series given in this parenthesis, it converges. And the resultant can be written here by can be given by equation number-12, we just added up this geometrical progression.

Now, we will use the Stokes relation, the two Stokes relation $r = -r', tt' = 1 - r^2$ and this slightly modify the equation number-12. And this modified equation is given by equation number-13.

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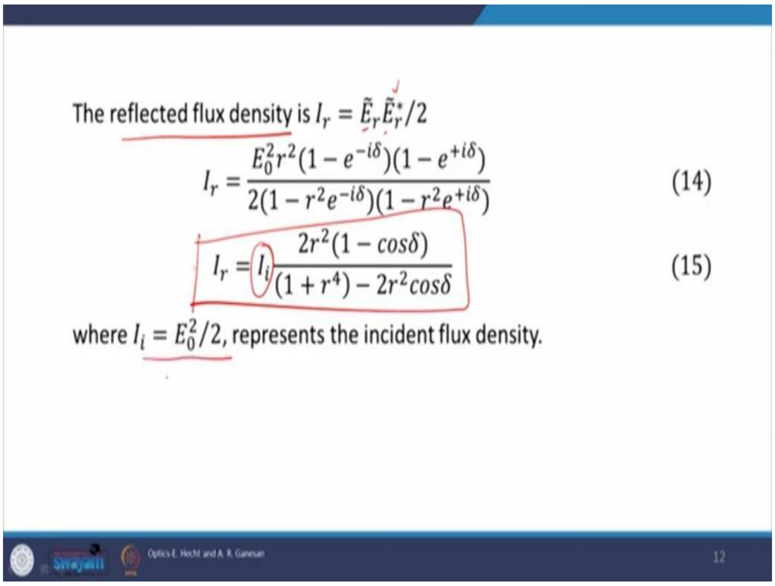


Let us now calculate the intensity or irradiance. The reflected flux density would be given by $\tilde{E}_r\tilde{E}_r^*/2$, star means complex conjugate. If you do this, then slight modification again gives us equation number-15. We are this term, $I_i$ is the incident flux density which is given by $E_0^2/2$.

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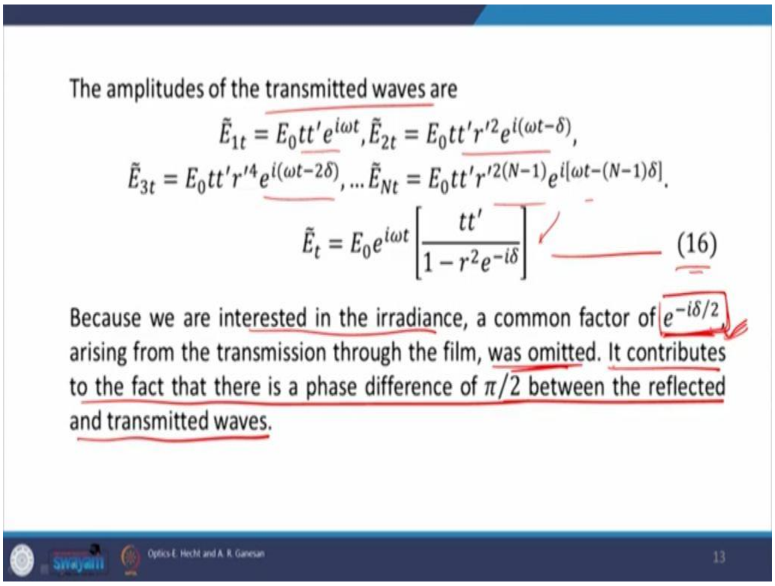


Now, once the reflected amplitudes are calculated, let us again do the same for transmitted amplitude. We will start in the complex representation followed the same thing what we did with the reflected one. Added them up and the total transmitted amplitude in complex representation is given here by equation number-16.

Now, if you add them, then you will see that we get $e^{i\delta/2}$ extra in equation number-16. And since we are interested in irradiance, irradiance means we will multiply $E_t$ with its complex conjugate. Therefore, the phase part will any way go away.

And therefore, this term is neglected here in equation number-16 it will not contribute to the irradiance and it is deliberately omitted. It contributes to the fact that there is a phase difference of $\pi/2$ between the reflected and transmitted wave. The reflected and transmitted wave has a phase difference of $\pi/2$ and there only this phase part appear. But, while considering irradiance or intensity, it has no meaning and therefore, it is deliberately omitted.

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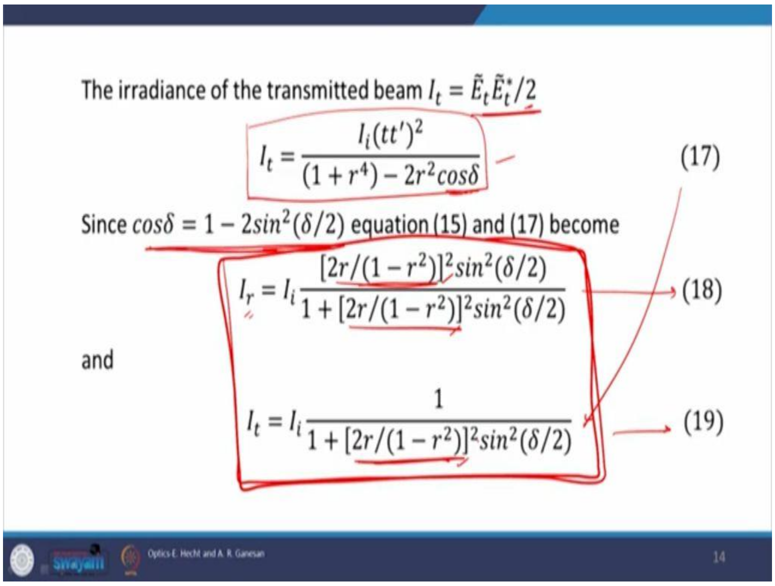


Now in the transmitted term, let us calculate the irradiance which is $\tilde{E}_t\tilde{E}_t^*$, which is of course complex conjugate by 2. And the expression for irradiances is this and to simplify it let us say represent cost in terms of sin. We know that $cos\delta = 1 - 2sin^2\delta/2$. Using this formula, the equation-17 here, while the irradiance expression for reflected ray is given by equation number-18. Wherein, we have just expressed $cos\delta$ in terms of $sin\delta/2$.

Now, in equation 18 and 19, they are important relations. In this expression, what do you see is that they are very bulky. But, there are certain things which are in common. In 18 and 19 what are the common things? In this bracket which is $2r/(1-r^2)$. This bracket is common, both in equation number-18 and equation number-19, these terms are getting repeated. This bracket is there in the numerator of 18 as well as in the denominator of 18.

Similarly, the bracket is also there in the denominator of 19. It means that this has to do something like we can replace this bracket with some other parameter and then probably this relation will be simplified, we will do it later.

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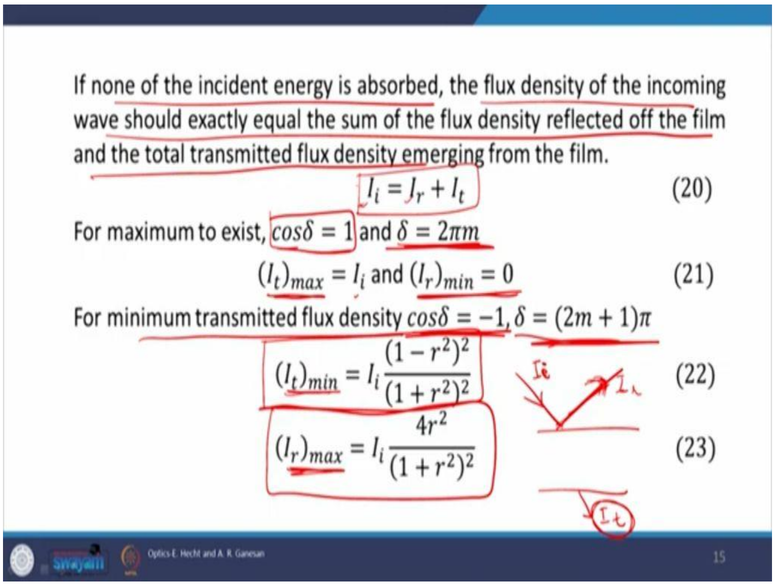


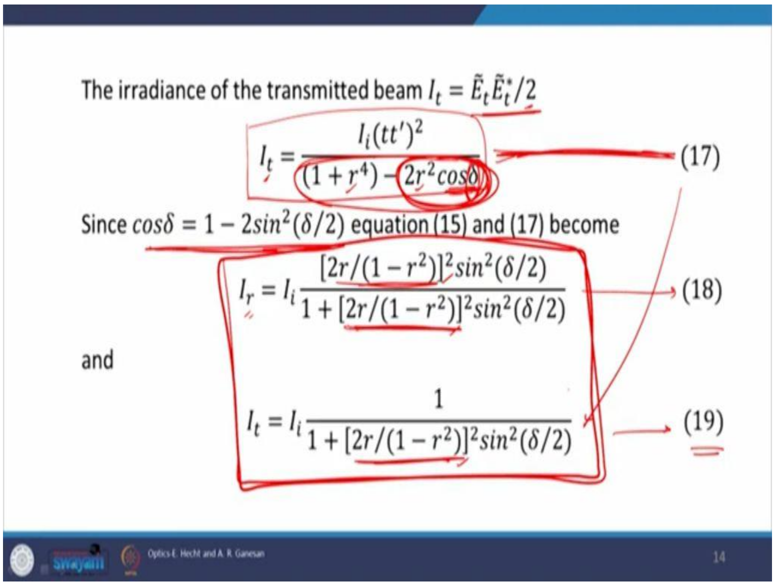


Now, if we assume that none of the incident energy is absorbed, the flux density of the incoming wave should exactly equal to the flux density reflected off the film and the total transmitted flux density. What I mean to say is that, suppose a certain energy is falling in the system, in this thin film and part will get reflected, and a part will get transmitted. And if the film is not absorbing any energy, then the reflected intensity plus transmitted intensity would be equal to the incident intensity. And this relation holds if the film is non absorbing and this is what equation number-20 is saying.

The incident in irradiance would be equal to the reflected irradiance plus transmitted irradiance. In this condition, if we want to observe maxima in transmittance, then let us go to back to the equation number-19 and here we want to observe maxima, or let us go to question number-17.

If we want to observe maxima in 17, then the denominator must be very small. To maximize $I_t$, we must minimize the denominator in right hand side of equation number-17. How to do this? To minimize the denominator in 17, we know that r is fixed, r is amplitude reflection coefficient, we cannot touch it.

The phase is something which is varying, it is variable. If we somehow play with this phase $\delta$ and that leads to minimum denominator, then the $I_t$ would be maximize. And how to minimize then this $\delta$? Now, if we take $\delta = 2\pi n$, then $cos\delta = 1$. And if $cos\delta = 1$, then this term will have its maximum value.

And if this term has maximum value, therefore whole denominator would be minimum and if denominator is minimum, $I_t$ is maximum. And therefore, we can say that if $\delta$ is equal to integral multiple of $2\pi$, $I_t$ maximizes, the transmitted intensity maximizes and it would be equal to $I_i$. And at that time, there would be 0 intensity, there would be 0 irradiance in reflected part.

For minimum transmitted flux density, how to minimize transmittance? To minimize transmittance, we will have to maximize the denominator of equation number-17. To maximize it, of course $cos\delta$ should cos $\delta$ must be equal to -1.

And for this $\delta = (2m + 1)\pi$, where m is an integer, it should be odd integer multiple of $\pi$. And under this circumstances, minimum irradiance would be given by equation number-22. And the corresponding maximum, the corresponding reflectance which would be of course maximum at that time because the sum of transmitted and the reflected irradiance is fixed.

If the transmitted irradiance is minimizing, then the of course this irradiances or intensities are going into the reflected arm. And since this intensity is appearing in the reflected arm, there would be maxima in the reflection. Let us try to understand it schematically. This intensity is falling here, $I_i$ is falling and $I_r$ is getting reflected and $I_t$ is getting transmitted. These are the transmitted, reflected and incident irradiances are intensities.

Now, as soon as $I_t$ decreases, this part of intensity goes into $I_r$ arm and it will keep it will be increased. Because, $I_t + I_r = I_i$, things are conserved. Therefore, when $I_t$ minimizes, $I_r$ maximizes. And therefore, we can write the maxima for a maxima condition for $I_r$ which is $cos\delta = -1$ .

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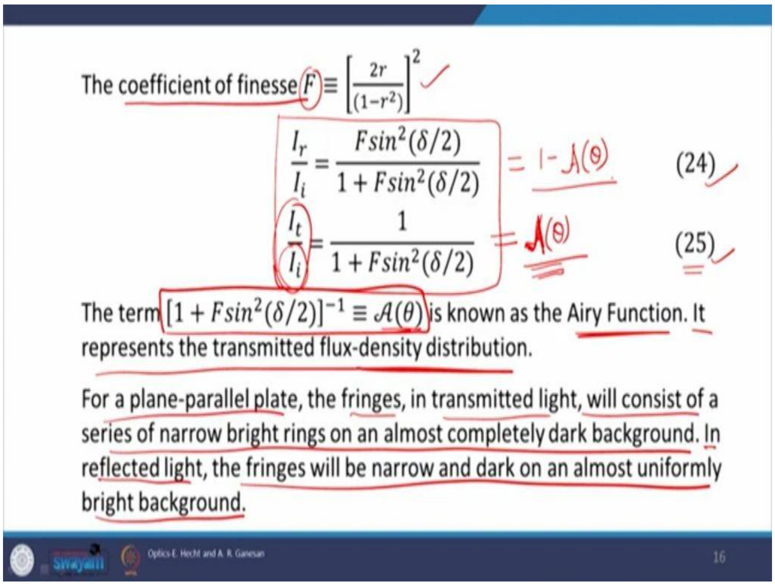

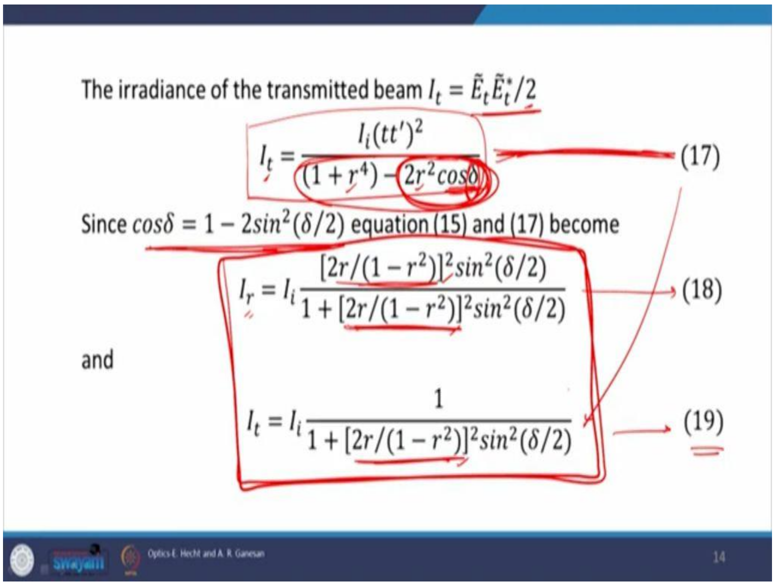


Now, as I said before, in equation number 18 and 19, 1-r by sorry, this bracket term $2r/1-r^2$ is appearing several times. And therefore, we define a term which is called coefficient of finesse, which is represented by F and is equal to $(2r/(1-r^2))^2$ . With this introduction, we again write equation number 18 and 19 which are given here these are this is equation number 18 and this is equation number 19. Let us rewrite equation 18 and 19, then we get better form of these two equations earlier they were looking bulky.

Now, here again you see that the denominator of both equation number 24 and 25, they are same. Therefore, another definition is introduced, $1/(1 + Fsin^2\delta/2)$ is replaced with italic $A$ function of $\theta$ and this is known as Airy function. Italic $A$ is known as Airy function and it represents the transmitted flux density distribution. Why? Because in equation number-25, if you replace $1/(1 + Fsin^2\delta/2) = A(\theta)$. Then, you see that $A(\theta)$ is nothing but the transmitted flux density distribution. And in the equation number-25, on left hand side denominator we have $I_i$, which is the incident irradiance.

Therefore, equation 25 exactly represents the relative transmitted flux density. Now, for a plane-parallel plate, the fringes in the transmitted light will consist of a series of narrow bright rings on an almost completely dark background. In reflected light, the fringes would be narrow and dark on an almost uniformly bright background. How can I comment this? Just plot equation number 24 and 25. How to plot them? Let us first plot equation number-25, which is nothing but Airy function, this is Ai. And what is 24? 24 If you see it closely, then it is $1 - A(\theta)$.

Now, $1/(1 + Fsin^2\theta) = A(\theta)$, then $1 - A(\theta)$ would be equation number-24. It means if I plot $A(\theta)$ and $1 - A(\theta)$, then this will give us the transmitted and reflected irradiance distribution.

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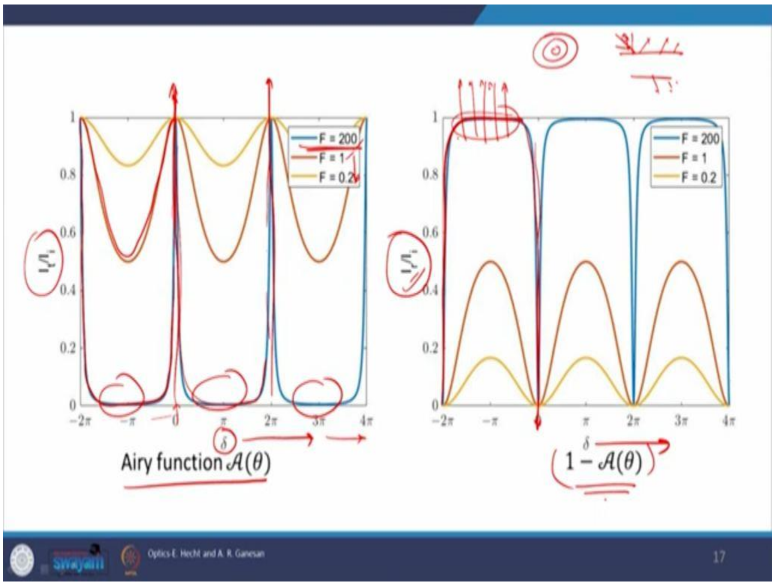

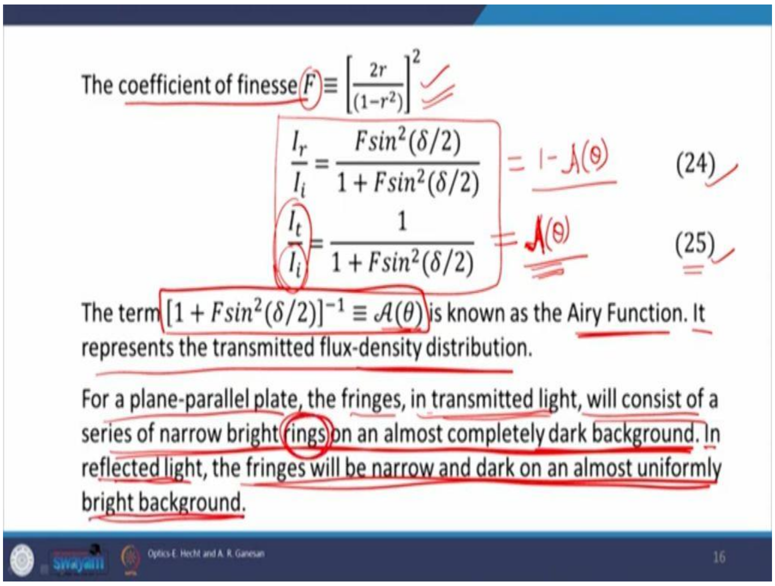


Let us plot them. Here $A(\theta)$ is plotted, $A(\theta) = I_t/I_i$. And $1 - A(\theta)$ is plotted here on the right hand side which means , $I_r/I_i$ is plotted here. And these plots are for three different values of coefficient of finesse.

The coefficient of finesse is defined here and for three values of coefficient of finesse, we can see these three plot. On the horizontal axis, $\delta$ is plotted the phase difference. On the vertical axis, relative transmitted irradiance and relative reflected irradiances are plotted. You see that for large value of coefficient of finesse, the irradiances first sharply dropped down and then it becomes 0, and then it again reaches to maxima.

And then again drops down and then it remains very low and then again reaches to a maxima, a periodic variation is found. These are called fringes of course. And if you decrease the value of F, then you see that the intensity variation or irradiance variation, they are not touching the 0. Fringes would still be there, but the difference between maxima and minima would be small. And as you go down in F, coefficient of finesse, this difference reduces.

Now, in this plot also, we see these peaks represent bright fringe, where this is the bright fringe, and this part represent darkness. The bright fringes appear in the background of darkness while opposite happens for the reflected irradiance. Here what do we see is that for larger value of F, the intensity remains high or irradiance remains high for most of $\delta$ and then it reduces down.

Now, you see that the width of bright fringes very wide while the dark fringe is very thin. So here the dark fringes appear in the background of brightness, in the background of brightness dark fringes appear and this is what this sentence said.

In transmitted light, the fringes will consist of a series of narrow bright rings on almost completely dark background. And similarly, in the reflected light, the fringes will be narrow and dark on an almost uniformly bright background. But, the ring concept is still to be explained, why it is ring. You see that this is the phase on the horizontal axis I am plotting the phase. And this thin film is a three dimensional object and one light is being launched here and multiple reflection and refraction is seen. You can rotate it in 3d. If you rotate this in 3d, then you will see that ring is formed.

If you rotate the angle, the incident ray, like this is the film, and this is the incident ray and these are the reflected and transmitted light. And then you can rotate it with keeping this angle of incident fixed and then you will see a ring type fringe pattern is being formed, concentric ring pattern would be seen. In one case, the width of the bright fringes would be sharp. In other case, white bright fringes would be very wide and coefficient of finesse this define now here the sharpness of these fringes.

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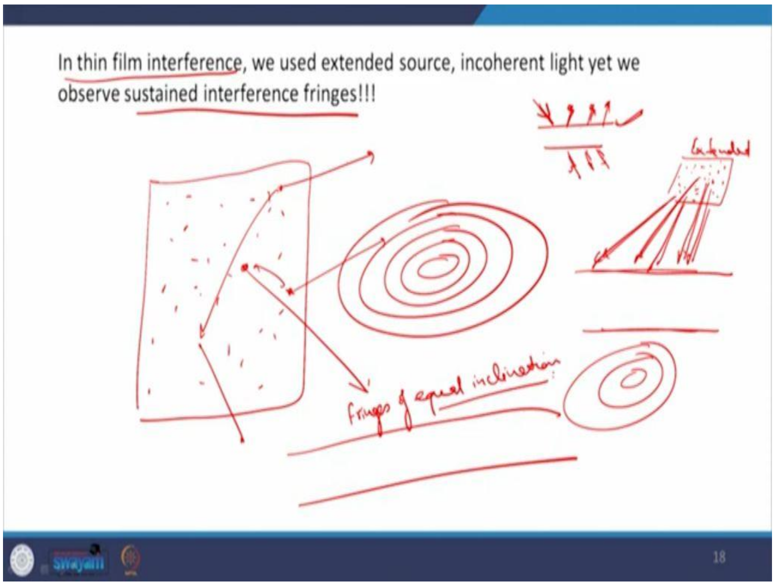


Now, having explained this, a few concepts are still left, there are still few confusion which I see that it surface in the mind of students. Now, in thin film interference experiment, we studied thin film in previous classes too.

Now, if you remember here we used one ray and we saw interference among different reflected rays as well as different transmitted rays. In the previous classes, we exposed illuminated the fringe with an extended source. And then rays from different directions and at different angle fell on the film, and then they produce some interference pattern.

Now, in this case, where we just launched one ray and then multiple coherent reflected and transmitted rays are generated and then they interfere. It is very much understood, because all this reflected and transmitted rays, they are getting generated from the same parent ray. And therefore, they all are coherent and they are supposed to interfere, and give rise to sustained interference fringes. But here what you see? Since the source is extended. Here also we saw that we get circular fringes if you remember.

But, since the source is extended, these rays are coming from different point sources which is constituting our extended source. And since, the rays are coming from an extended source, extended source means very wide broad source. And it has infinite many points sources, which are totally uncorrelated with each other. Therefore, the ray which is coming from this source is totally uncorrelated with the rays coming from this source is totally uncorrelated with the rays coming from the source, they all are mutually incoherent.

Now, the question arises how come an interference among incoherent rays giving rise to sustained interference fringes. It is a big question, because from the very beginning of interference, we demanded or we study it that there are certain guidelines to observe sustained interference fringes. And what are the most important guideline? The most important guideline is that interfering rays they must maintain a constant phase difference. The interference ray, the interfering rays they must be coherent they must maintain a constant phase difference.

And then there are a few other criteria that they must have almost same frequency, same amplitude, but they are subsidiary. The most important part is that they must be coherent or they must maintain the same constant phase difference.

Now, here we see that they are not coherent. These rays are emanating from totally incoherent sources, there is no relation between these two point sources or these two points sources are any two point sources in an extended broad source. And yet, when this light fall on this thin film, we saw that they generate very beautiful concentric ring pattern and we call them fringes of equal inclination. How come we are getting this? Sustained fringes particularly they all comes in the domain of interference where we have mutually coherent sources. But, with the incoherent sources, there getting sustained fringes.

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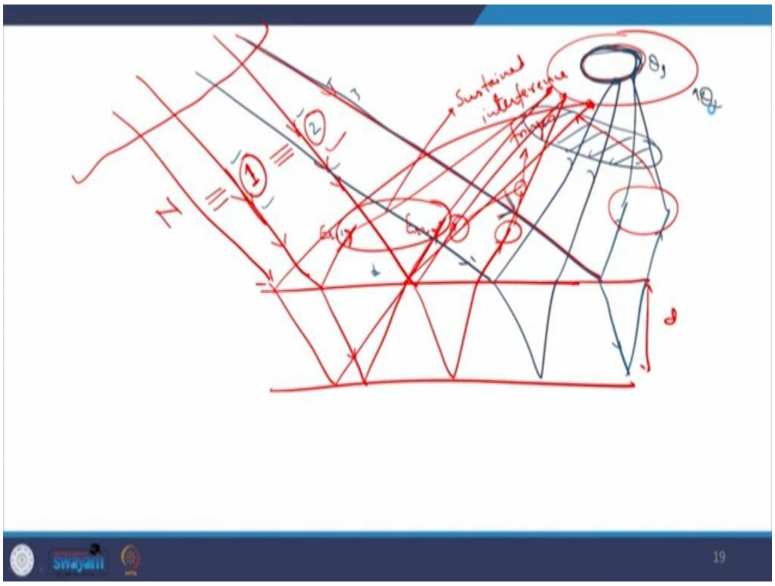

Now, the explanation for this is as follows. This is our film, this is a certain thickness d. Now, here we have extended source from which we are getting rays at different angles. What happens here is that when these rays come, it partially get reflected from here, partially get transmitted and then it get reflected from here too. Similarly, another ray which is parallel to the first one it falls here, it partially gets reflected from here, transmitted and then again reflected.

Now, let us draw the another ray with another color, suppose this is the second ray, which is falling at different angle. Now, these two rays, they fell on the angle of incidence, for these two rays were same here. This ray is coming at different angle, and then this will go inside and then we will have a reflected rays out of it.

Now, what happens here that although all these three rays ray number 1, ray number 2, ray number 3, they are mutually incoherent. But, out of ray number 1, the rays which are getting generated here in case of this reflected first ray, which is $E_{r1}$ and $E_{r2}$ they are coherent. Because, they are getting generated from the same ray, the ray number 1. Their origin is same, therefore they are mutually coherent and they will interfere, and give rise to sustain interference fringes for only these two rays $E_{r1}$ and $E_{r2}$, they will give rise to sustain interference fringes.

Similarly, for ray 2, we are getting this reflected ray, this reflected ray they too will give sustained interference fringes. Similarly, these blue rays, they will also give rise to sustain interference fringes. It means whatever pattern they are forming on the screen, be it dark, be it bright, it would remain dark and bright throughout.

Now, the second concept is that the rays which are parallel among each other, they will fall on the same circle. The rays which are parallel, suppose the final pattern is concentric ray, then first two will fall on this circle. And this ray 2 is since parallel with ray 1, it will also fall somewhere on the same circle. Some other rays, say ray number n it is again parallel to ray number 1 or ray number 2, it will again generate two reflected and refracted ray and after they will again fall on the same circle.

It means, all the rays which are parallel, which are getting generated from this broad source they will fall on the same circle. And at their point of incidence, they will generate some pattern and which will be sustained pattern. At the spot of falling, they will create some intensity distribution and that intensity distribution will be constant throughout time, it would be a sustained interference fringe.

And the ray which is falling at different angle which is given here in blue color, this array in particular, this will fall on different circle. Similarly, a ray which is parallel to this blue ray, this will again create a ray and which after reflection, it will go through some lens, and then they will overlap and they will again fall on the other circle. It means each circle represents a parallel beam of light or alternatively in multiple beam interference, in case of thin film, which is illuminated by a broad source of light, a particular circle represents set of rays which fall at the same angle of incidence.

The blue circle is for angle of incidence $\theta_1$, the red circle is for angle of incidence $\theta_2$. And this is why even we do not have coherent sources, we get sustained interference fringe here, very important concept. Now this is all for today and thank you all for listening me. See you in the next class.