Applied Optics

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Lecture: 25

Michelson Interferometer and its Applications - II

Hello everyone, welcome back to my class and today is the last lecture of module 5. Now, as you remember, in the last class, we started with Michelson interferometer.

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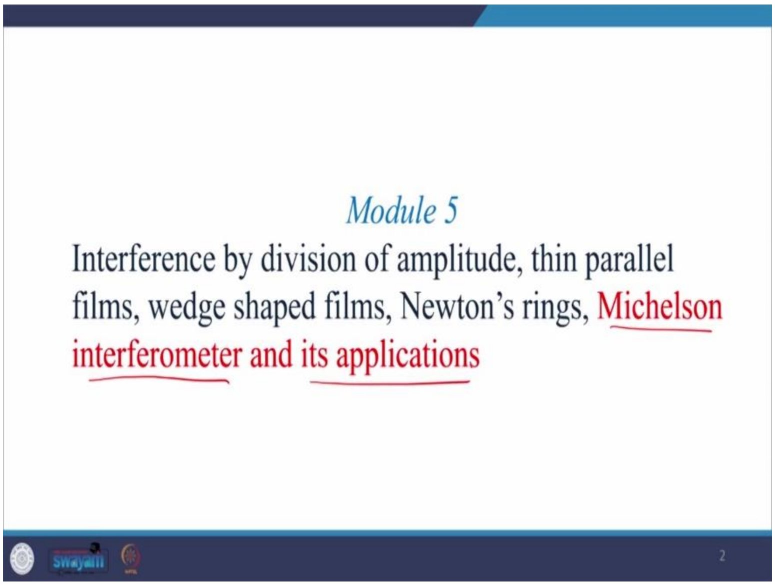

Today we will cover one more topic in Michelson interferometer and then we will start talking about its application.

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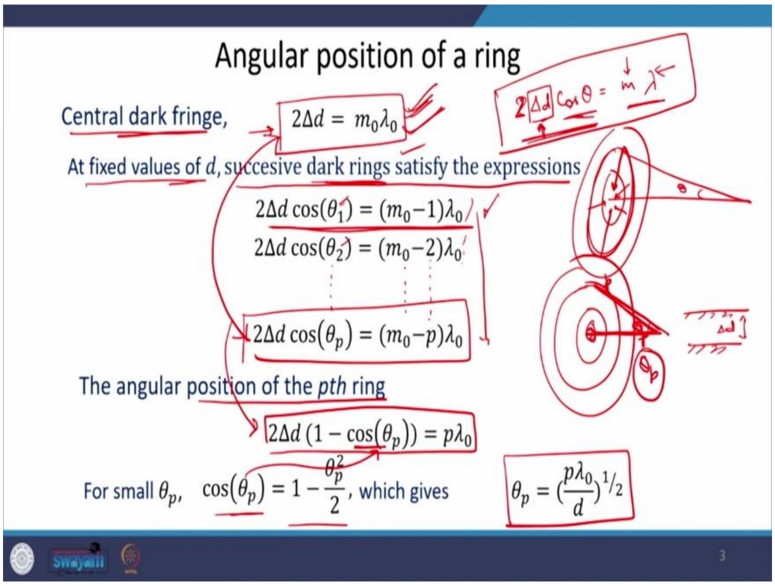

Now, in last class we derived an expression for maxima, the condition of maxima. Now, we have also talked about the case when the separation between the two mirrors which is Δd if it is reduced then we saw that the angular position of different fringes it should also be reduced. This can be understood through this relation which we derived in the last class $2\Delta d cos\theta = m\lambda$ this was the relation which we talked about in the last class.

Now, in this relation what I am saying is that if Δd which is the separation between the two mirrors or the thickness of the field, if this is reduced, then what will happen is that, since on the right hand side, we have λ which is fixed and we have m which is an integer which is again fixed therefore, right hand side is fixed.

Now, since we are varying Δd and we want the left hand side to be fixed, then we must play with $cos\theta$ term. Now, since Δd is getting reduced then θ should also be decreased so as to keep the left hand side of this equation constant. Now, if we reduce θ then what will happen now, these are the fringes which we observe and what is θ, θ is the angular position of the fringes this is the θ.

Now, since we are reducing θ then what will happen is that the fringes will move towards the center, they will shrink and with shrinking they will come at the center and then they will collapse, they will disappear from the field of view. Therefore, with a reduction in the fringe the mirror separation, with reduction in Δd, the fringes will disappear at the center of the fringe pattern and slowly if we keep reducing Δd, we keep reducing the separation between the two mirrors, the number of fringes in our field of view will reduce and a situation will come when there will be only few number of fringes.

Now, if we again reduce Δd to 0 or if we overlap the two mirrors, so, that the Δd is equal to 0 then what will happen, all the fringes will collapse at the center and there would be a uniform darkness in our field of view because the air gap is 0 and since air gap is 0 the condition of minima will satisfy here because we are also taking into account the extra phase difference of π due to internal and external reflection which happens at beam splitter.

Therefore, a uniform darkness will prevail when the airfield thickness or the separation between the two mirrors is 0. Now, in this situation, suppose the angle of incidence is almost equal to 0 are if we are normally launching the light into the system then $cos\theta$ would be 0 in this equation, we will be having this equation here $2\Delta d = m_0\lambda_0$, m_0 is again integer. Therefore, for central dark fringe, we will have this relation.

Now, we were having two mirrors we slowly reduced the separation and then merge them. Then what is the other possibility we can keep rotating the micrometer screw which is attached to the mirror M_2, then the image of M_1 will now travel on opposite side and the air gap between the mirror will again now open but in the opposite direction and slowly Δd will now increase and if Δd is increasing the new fringes from the center will start to appear, this is what we discussed in the last class.

Now, suppose the separation between the two mirrors is fixed. For fixed Δd as is written here, the successive dark rings satisfy the following expression here. Now, what we are doing is that we have fixed the separation between the two mirrors here, these are the two mirrors and the separation which is Δd is now fixed, due to this separation there is a formation of some ring fringe pattern and then we can just assign some angular positions to the different rings.

Now, Δd is kept fixed now, if we vary θ if we focus our attention to different rings then different dark rings starting from the center sorry starting from the center the success of dark rings will satisfy these criteria. This condition is from the central ring and then this condition is for the next order dark ring and for this we will have to write $m_0 - 1$. For next order it would be $m_0 - 2$ and the corresponding θ will vary here, for the first one $\theta = \theta_1$, for the second one $\theta = \theta_2$. Similarly, for p^{th} ring, $\theta = \theta_p$ and this number will be equal to p now, similarly, we can write this relation for all the dark rings which are appearing in our field of view.

Now, if you want to know the angular position of p^{th} ring, suppose this is our p^{th} ring and we want to calculate this θ_p if we want to calculate this θ_p then you will have to calculate the angular orientation of the central fringe and calculate the angular orientation of the θ_p and then subtract them and this will give the angular orientation or angular position of the pth ring and this is what exactly is done here, in this relation what we did, we subtracted this, which is the condition for central dark fringe with this relation these two expressions when they are subtracted this gives us this relation which is nothing but the expression which gives us the position of p^{th} dark ring.

Now, in this relation, θ_p is coming with cos term and for small θ the cost term can be expanded, just consider the first two terminal expansion of $cos\theta$ and from here if we substitute this expression of $cos\theta$ back into the previous expression then we get the value of θ_p the angular position of pth ring. And from here we can also get the angular position of $(p-1)^{th}$ ring or angular position of $(p-1)^{th}$ ring and from there we can calculate the angular width. Once angular width is there, we can also calculate the usual width, the width have the particular ring as we did in Young's double slit experiment, as we did in Newton's ring experiment, the similarly we can do it here also.

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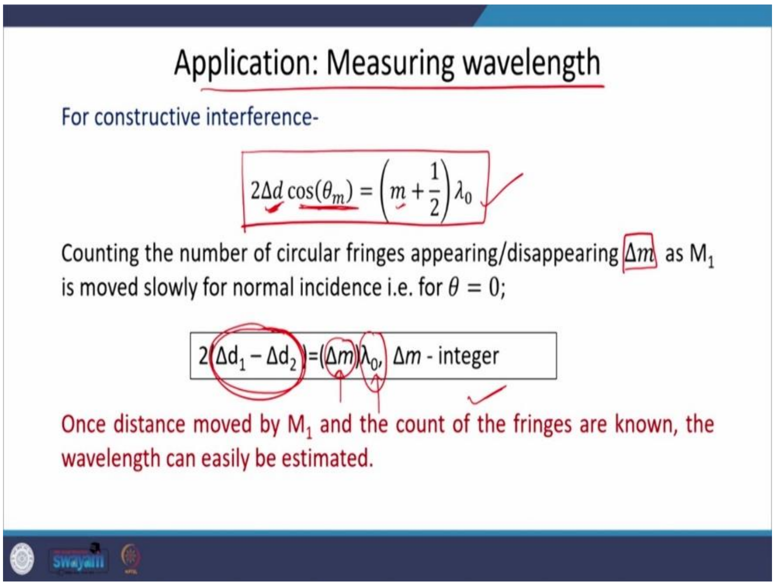

Now, in this slide, we will talk about applications of Michelson interferometer and the first application is measuring wavelength we can measure the wavelength using Young's double slit Newton's ring and as well as Michelson interferometer, how to do it, we know that for constructive interference, this condition holds $2\Delta d cos\theta_m = (m + 1/2)\lambda_0$, where m is the order of the bright ring and θ_m is the corresponding angular position.

Now, we know that in our setup, we have two mirrors, and attached to one of the mirror is our micrometer, if we play with the micrometer, or if you play with the separation between the two mirror, the fringes in our field of view either appear or disappear. Now, to measure the wavelength of the light what we do is that, we start rotating the micrometer attached with the mirror. This rotation will gives either appear or disappear certain number of fringes, let us say that the change in the number of fringes or the fringes which disappeared from our field of views Δm and for this disappearance or appearance, we rotate the micrometer by a certain distance.

Now, what would be this distance, this distance would be the separation between the two mirrors before the rotation of the micro meter and the separation between the two mirrors after the rotation of micrometer, if we subtract these two separations, this will give the readings of the micrometer, this is what exactly the micrometer is doing. Therefore, we have this relation then we change Δd and therefore, m changes say the change in m is Δm and change is Δd is $\Delta d_1 - \Delta d_2$ everything is being calculated for the normal incidence therefore, we have neglected here the $cos\theta$ term.

Now, once it is done, once a micrometer is rotated by a certain distance which can be easily read on the micrometer scale and the fringes appeared or disappeared are counted then using this relation, we can calculate λ because Δm is known, we know how many fringes appeared or disappeared and this distance is known this is nothing but micrometer reading and from here, λ_0 the wavelength of the source can be calculated.

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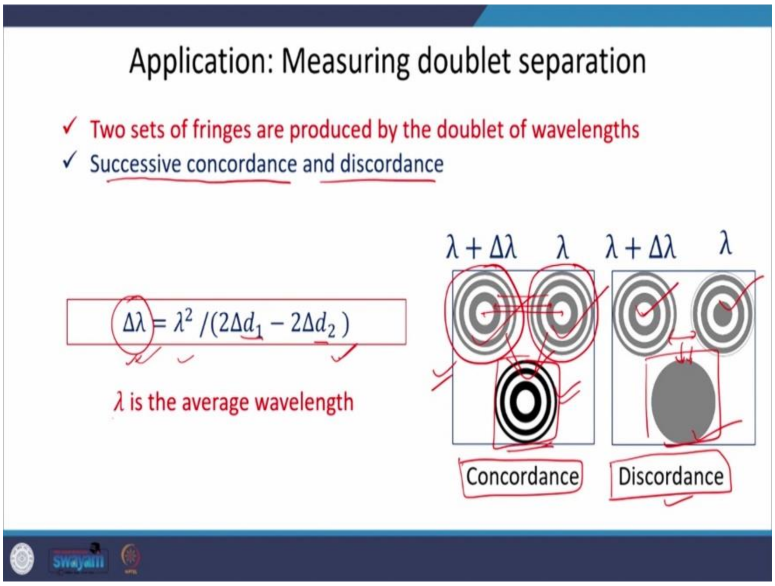

Now, next application is in measuring the wavelength difference in doublet. If we use sodium lamp in Michelson interferometer, then we know that in sodium lamp there are two lines 5890 nanometer and 5896 nanometer, they are very closely placed, the separation between the two wavelengths is very small. In this situation, when we are having a source which produces doublet or we are having a source which have two wavelength which are very closely spaced, in that particular case, we can calculate the wavelength separation of the doublet and how to do, this can be done by observing successive concordances and discordances then what is concordances and discordances.

Now, since there are two wavelengths in our source, the first wavelength say λ_1 it will produce its own interference fringe pattern, which of course would be concentric circular ring pattern. Similarly, the second wavelength will also produce its own concentric circular ring pattern. Now, it may so happen that both the wavelength simultaneously produce its own concentric ring patterns and the dark of one falls on top of the dark of other, similarly, the bright of one falls on top of the bright of other, in this situation the dark fringe will become darker and the bright fringe will become brighter and this is what exactly is shown here in this figure.

Now, you see the here for the first wavelength, this is the circular ring pattern and for the second wavelength this is the circular in pattern. And the separation between the two mirrors is such that or the micrometer is rotated in such a way that the center falls on the center, the first ring falls on the first, the second falls on the second and so on.

And in this particular case, the dark that the result and this is the resultant of the overlap, when this fringe overlaps with the this fringe pattern that this results and we know that dark is more dark here, dark is darker and bright is brighter, because dark fringes overlapping the dark fringe of one pattern is overlapping with the dark fringe of another and similarly bright is overlapping with the bright pattern for the second wavelength and therefore, we get better contrast now and this is called concordance.

Similarly, now if the dark of one fall on the bright of other. If the dark rings for wavelength one falls on the bright ring of wavelength two then we will get uniform illumination, this is the pattern of first wavelength, this is the pattern of second wavelength if they both combine we get this and since dark is being compensated by bright and bright is being compensated by dark, we get uniform illumination in our field of view and this phenomena is called discordance, better contrast, better visibility, darker fringe dark, darker dark, brighter bright, this means concordance and if uniform intensity distribution, uniform field of view than discordance.

Now, by setting the system on successive concordance or on successive discordance we can calculate the separation between the two wavelength, we can set the system such that the concordance appear in that particular case we can write the either maxima or minima condition for the two wavelengths, and then we rotate the micrometer and then we fix again for next concordance.

The next concordance means, if it is concordance number one, the successive concordance will give the next condition for maxima or minima. And then we write the condition of maxima or minima for this new concordance and from this two relations, we can get the wave length separation which is $\Delta\lambda$ here, and Δd_1 and Δd_2 is the separation between the mirrors, Δd_1 is the separation between the mirrors when there is the first concordance and Δd_2 due to the separation between the mirror when there is the second concordance and these two concordances must be successful, they must be coming one after that.

And once you do this, we can measure the wavelength separation. Now, apart from the circular fringe pattern, the Michelson interferometer can also produce another kind of fringes, how to do this.

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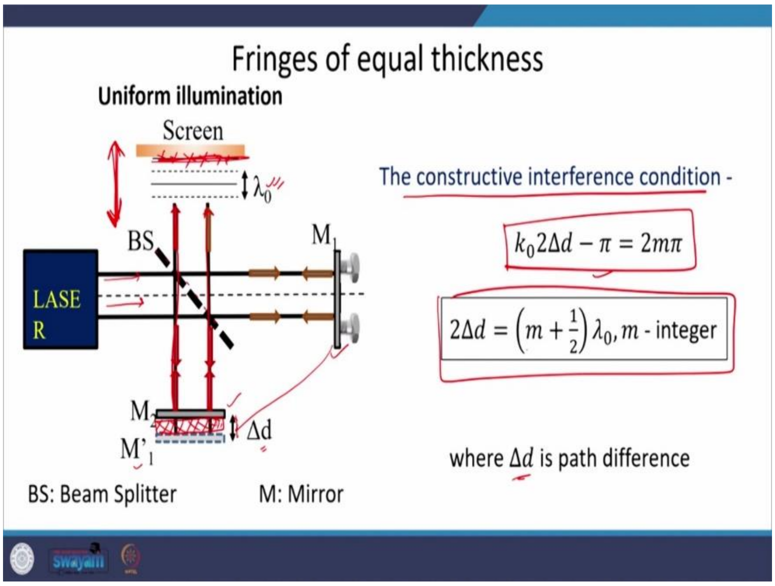

Now, here instead of using point source, a parallel beam of light is launch, in points of source what happens is that beam diverges, but here a parallel beam of light is being launched on this Michelson setup, here you see mirror M_1, mirror M_2, M'_1 is the virtual image of mirror M_1 and the separation between the two mirrors is Δd, this is our beam splitter.

Now, since parallel beam of light is coming and the film thickness which is the air film which is getting formed between the two mirror is uniform, the film is being formed here, see, this is the film which is of uniform thickness, then the rays which are getting reflected from the top interface of their film and from the bottom interface of the air film, they will interfere to produce some fringe better.

But, from the picture itself it is very much clear that these two rays, they will have a certain phase difference which would be constant throughout the field of view, since the source is launching parallel beam of light, the path difference between the rays which are coming to the screen, they would be constant throughout the field of view, the path difference here would be constant to the path difference at this point, this would be equal to the path difference at this point, this would be equal to the path difference at this point, therefore all points on the screen.

We will see same type of interference, either constructive or destructive, or something in between but it would be of same intensity and therefore, uniform illumination would be there on the screen if you launch parallel beam of light and if the mirrors are parallel, there is no misalignment. If they are perfectly parallel than uniform illumination on the screen.

And here to find constructive interference this is the condition, since the incidence is normal theta is equal to 0 and $cos\theta$ term is gone and from here we can see that $2\Delta d = (m + 1/2)\lambda$ and where Δd is the path difference. Therefore, we see that fringes are far which are a straight line fringes, but in the vertical direction but if you put a screen there you will see a uniform illumination on the screen.

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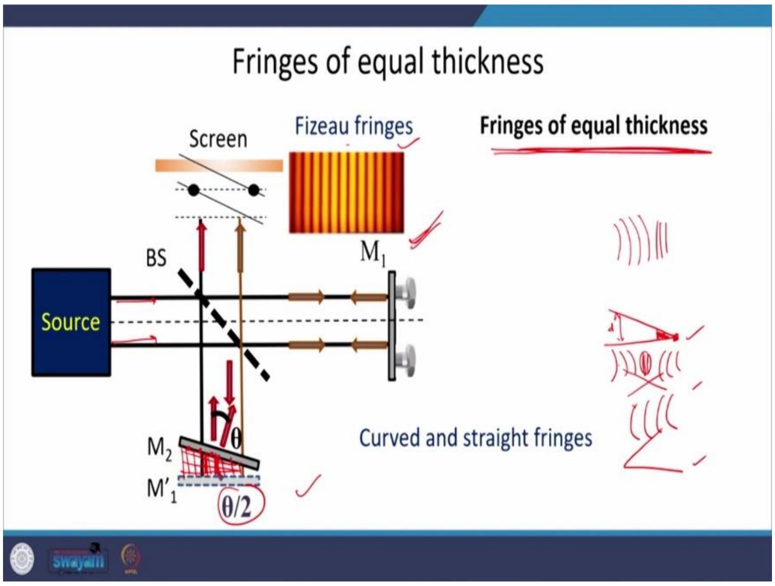

Now, next case is shown here, in this setup not one of the mirror, there are two mirrors and in this setup one of the mirror is tilted, with tilt one of the mirror. Now, again a parallel beam of light is being launched, but we deliberately tilt one of the mirror and therefore, the air film which is formed between the two mirrors is of wedge shape. Now, in this particular figure, the mirrors are tilted by $\theta/2$ and therefore, the reflected ray, the angle between the vertical and the reflected ray would be θ.

But in any case, we clearly understand that the air film is now wedge shaped. Since the air film is now wedge shape and we have already studied what is the fringe pattern in case of wedge shape and the fringe pattern there we know, it would be a straight line fringes and these fringes are called Fizeau fringes or fringes of equal thickness.

Wherever the thickness satisfies the criteria of maxima or minima, we get bright and dark fringes and these fringes are called fringes of equal thickness because for all the dark fringe we can pick certain points in the wedge which satisfies the condition of minima for the dark fringe and if we pick all the bright fringe the corresponding points on the wedge they will satisfy the condition of maxima.

Now, apart from this straight line fringes, we can also get a curve fringe here. Now, suppose this is how the mirrors are arranged. Now, the fringes at the center, like as long as we are looking in this area, the corresponding fringes they will look like a straight, but if we go for larger d, if we increase d, then we see that the fringes start to become like this.

The convex surface always points towards the meeting point of the mirror, the point on which the two mirrors are joining here. Now, if you put mirror like this, then what will happen is that here we will see for this region we will see straight line fringes, and here you will see this type of fringes on the other side you will see this type of fringes and if the mirrors are like this, then you will see this type of fringes, we see that just by tilting the mirrors, we can generate different type of fringes. The widely studied is this straight line Fizeau fringes.

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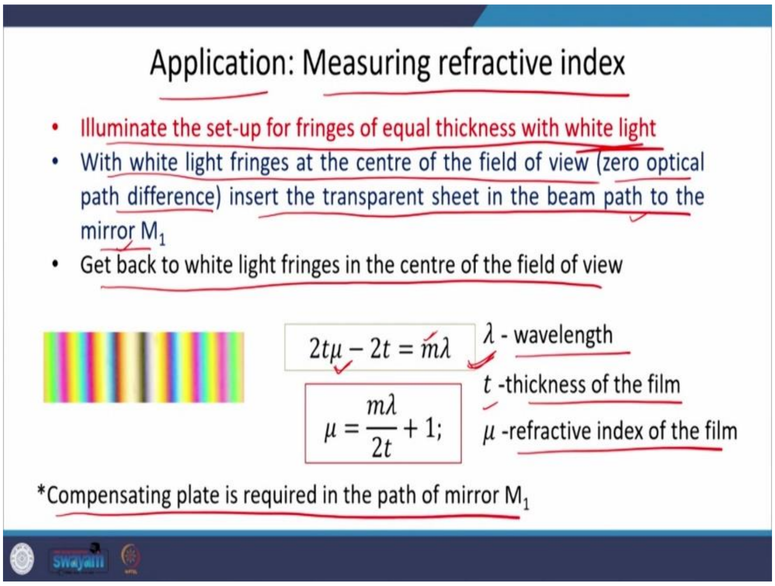

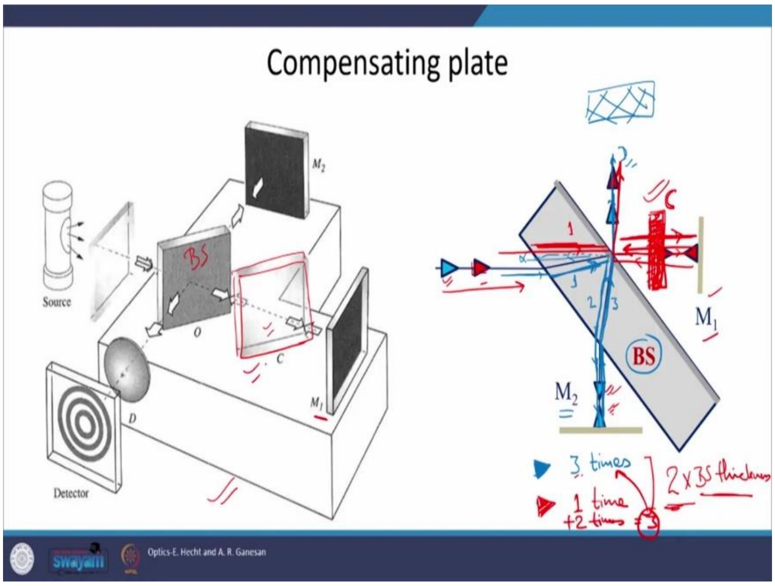

Now, let us again move to the next application which is measuring refractive index, how to measure the refractive index, we perform the same experimenting in Young's double slit and there we covered one of the slit with a thin transparent film and then we saw that there is a shift in the maxima and by measuring the shift in the central maximum, we calculated the refractive index of the film.

Here what we do here and one more thing and while doing that, I also mentioned that we must use white light interference pattern too, because the fringes in Young's double slit experiments are indistinguishable. And since they cannot be distinguished from each other, we require a reference and that reference is provided by white light interference fringes and the shift in the white light interference fringes can easily be measured and detected, detected and measured.

Now, to perform the experiment for here, we first illuminate the setup for fringes of equal thickness with white light because they act as a reference, now with the white light fringes at the center of the field of view for 0 optical path difference, insert the transparent sheet in the beam path to the mirror M_1 in one arm, it just means that in one arm beams out the thin film, we have two arm M_1 and M_2, in either of the arm, you can insert the thin film, here I am particularly focusing on inserting it in the arm of mirror M_1. Now, if you insert the thin film, the fringes will be shifted.

But if there is no white light interference pattern, we will not be able to measure the shift because the fringes are identical and therefore, while inserting the right after inserting the thin film, we switch on again white light and then see how much is the shift in the center of the field of view of white light fringes, measuring this shift, we can measure the refractive index of the film using this formula and here μ the refractive index of the film, t is the thickness of the film and λ is the wavelength and m is the order.

Now, one point, compensating plate is required in the path of mirror M_1, why do we require compensating plate? If you remember in the first class here, I saw you this figure and what we saw is that this is our beam splitter and this is our compensating plate which is inserted in path half mirror M_1 why it is required, to understand this let us see in this diagram, its a simplified version of the setup. You will see that the light is launched in this direction and the blue light or the blue arrow is going to mirror M_1 and the red arrow is going to mirror sorry; the blue one is going to mirror M_2 while the red one is going to mirror M_1.

Now, let us trace the path of blue arrow, I will pick blue color for this, the blue arrow is going in this direction and then it goes here and then it sees that the back phase of the beam splitter is painted and therefore a painted means it is coated with silver or some reflecting material and then after getting reflected it goes in this direction and then it follows this path and goes to mirror M_1. At mirror M_2, this blue ray goes to mirror M_2 and at mirror M_2 it suffers reflection, after reflecting it goes back then it follows this path here and then it goes in this direction where detector is kept.

A part of the light also get reflected in this direction, but we are neglecting it for a while because this is not of our concern, because our detector is here, the fringe pattern is getting formed here therefore, we are only concentrating here.

Now, this blue ray, before reaching to detector it crossed the thickness of the beam splitter, this is our beam splitter, it crossed the thickness of beam splitter thrice, how, from here it crossed first here this is the first crossing and then when then it travels to mirror M_2 this is second crossing and after getting reflected from mirror M_2 it again crosses the thickness of mirror sorry this beam splitter this is third crossing, it means it is passing through the thickness of the beam splitter thrice and then ultimately it is going to the screen or the detector therefore, blue arrow this is crossing our detector, traversing the thickness of the detector three times.

Now, let us do the same for the other color, for the red one. The red one is going in this direction and the red one is the part of the ray which is getting transmitted here we said that beam splitter, on the back of the beam splitter there is a reflecting surface, which partially reflect means part of the light is transmitted and part of the light is only reflected therefore, the transmitting part is represented by red arrow here. This red arrow is transmitting and going to mirror M_1 and here it is a first reflection it comes here and again get reflected towards the detector.

Now let us count how many times it crossed the width of the detector. We see that here it cross just once, therefore, this red arrow it crossed the detector width once, one time. It means the blue arrow is traveling through the thickness of the beams splitter thrice, it means the difference is 2, 2 into beam splitter thickness. This we did not take into account in our calculation while calculating minima and maxima, we assume that the two beams are traveling the same path length.

But here we see that the beam splitter thickness itself is incorporating the path length difference, because blue color is traveling thrice and red color is traveling once only. Since the blue arrow or the blue line or blue beam is crossing thickness three times and the red beam is crossing the thickness one time then therefore, there is an additional path length difference which is equivalent to two times the thickness of the beam splitter, to compensate for this additional path length difference or duplicate copy of transparent beam splitter now, now this duplicate copy does not reflect it only transmit here or almost perfectly transmitting glass plate is placed in the path of red beam.

Now, if we put a glass plate whose thickness is equal to the thickness of the beam splitter, then what will happen, the red ray will now pass through this compensating plate once while going to mirror M_1 and the when it comes back after reflecting from mirror M_1 then it goes again through the thickness of this glass plate, it means it is covering this thickness twice and therefore, total times becomes equal to three which is equal to that of the blue beam because of insertion of this glass plate which we name as compensating plate, the path length between the two beams becomes equal provided the micrometer is adjusted in such a way that the distance of mirror M_1 and M_2 from the beam splitter is same, this is the importance of the compensating plate.

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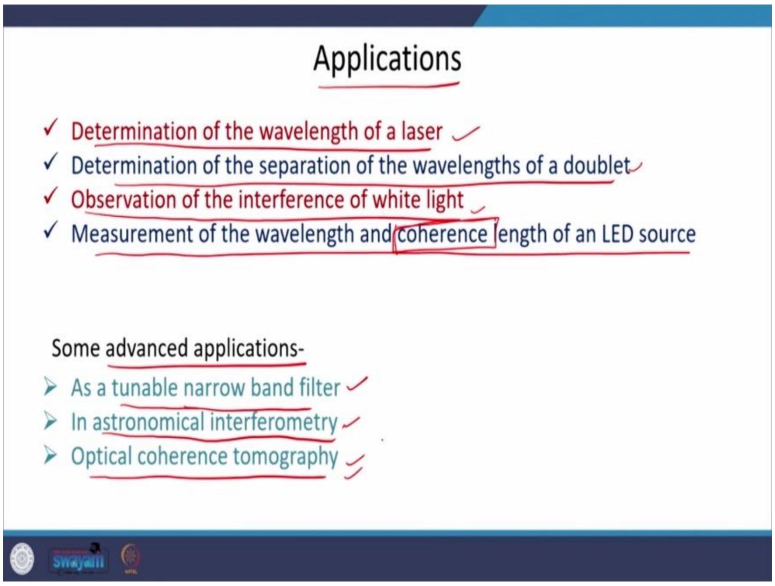

Now, these are the applications, I have listed the applications of the Michelson interferometer here in this slide. We can determine the wavelength of laser light which we saw in our previous slides how it is done, we can determine the separation of the wavelengths of a doublet, we saw how it is done, we can observe the interference of white light we know how it is done and measurement of wavelength and coherence length of an LED sources.

I will talk about coherence in my next module in detail, there I will talk more about, what a coherence length is, but just for the clarification, when the light propagate, then it remains coherent for only a finite duration of time and during this time the length cover is called coherence length.

As long as the optical path length is equal to or below coherence length, we can observe interference fringes. And if the optical path length is larger than the coherence length still the waves will interfere but we will not be able to observe it because the coherence is compromised and the fringes would not be sustainable anymore. Having said this, I would also like to list a few advanced applications of Michelson interferometer. We know LIGO is there, we know it is used in Michelson Morley experiment, but recently, in last few year, people have searched come advanced applications and a few of them is listed here.

The first one is tunable narrow band filter and second is astronomical interferometry, LIGO you know, the third is optical coherence tomography. This is used in medical science here if you want to image biological sample layer by layer, then you also can use this Michelson interferometer. With this I end my lecture, and thank you all, see you all in next lecture. Thank you.