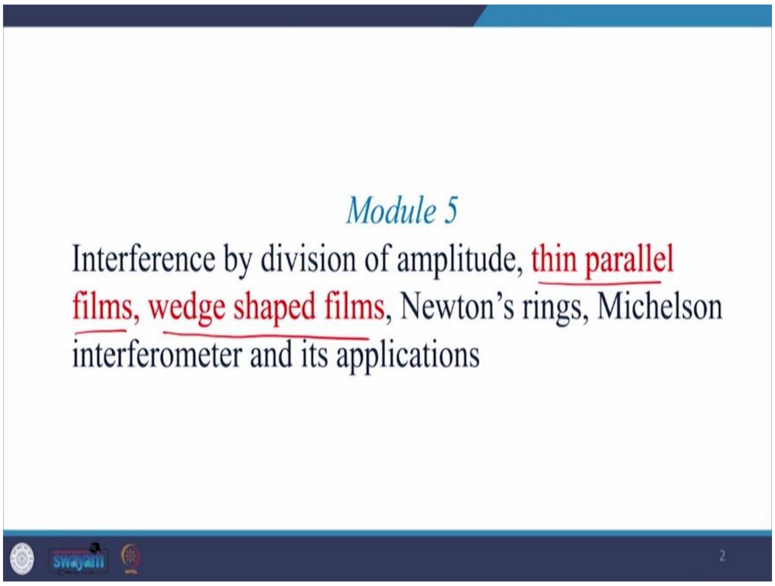
Applied Optics
Professor Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology Roorkee
Lecture: 22
Thin Parallel Films, Wedge Shaped Films

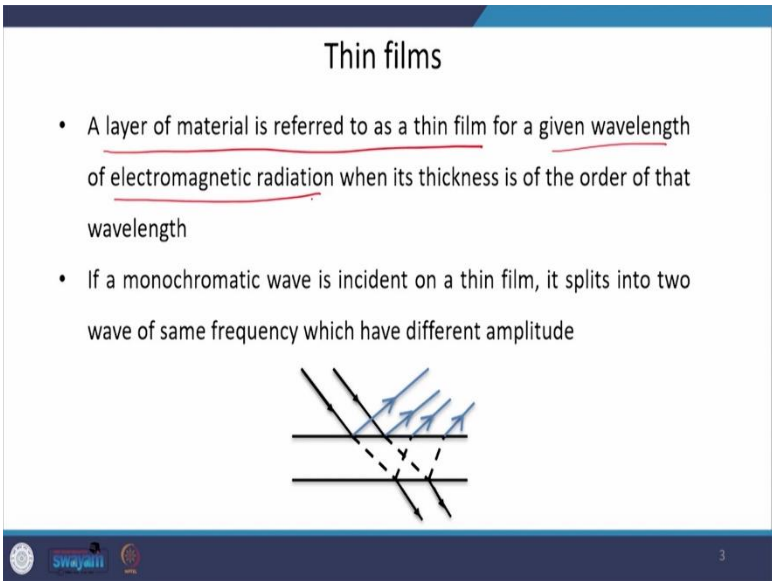
Hello everyone, welcome back to my class and we were in module 5. And in the last lecture, we were introduced to our interferometer, which relies on division of amplitude, wherein we talked about interference between 2 reflected rays and then we studied about the condition of maxima and minima both in reflected arm as well as in transmitted arm. Today we will generalize this concept and we will go to multiple reflections.

(Refer Slide Time: 1:01)



Therefore, today we will talk about thin parallel films or generalized case and we will again then move to wedge shaped films, we will talk about interference pattern produced by a wedge-shaped film also apart from the usual parallel films.

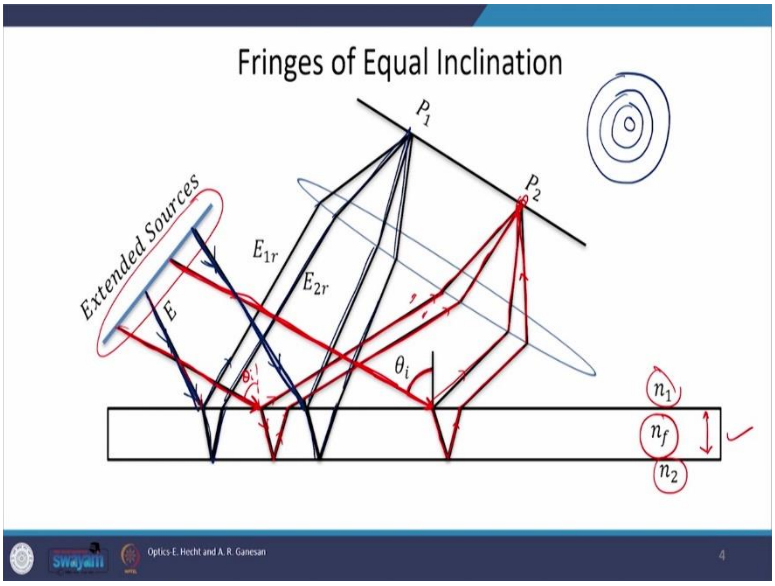
(Refer Slide Time: 1:15)



Now, just to revive your concept, a layer of material is referred to as thin film for a given wavelength of electromagnetic radiation when its thickness is of the order of that wavelength, if the film thickness is the order of the wavelength then only we call the film as thin film, apart from this here, we also consider that the light which is falling on the film is coming from a monochromatic source. And what does the film do is that it splits the light into different reflected and transmitted light or transmitted and reflected beams.

Now, since the same light is being splitted or the same light is being transmitted multiple time and reflected multiple times therefore, all these transmitted and reflected components will have same frequency, but of course, they will all have different amplitudes.

(Refer Slide Time: 2:29)



Now, let us consider that we have an extended source here now, we shifted to extended source from point monochromatic source. Now, if the source is extended, then the rays from these extended sources will fall at different angles on the first interface of the thin film, the thin film is again here and it has a certain thickness and refractive indexes above and below the film is n_1 and n_2 respectively. Now, the ray which starts from here will fall here and the ray which is start from this point of this extended source will fall here.

Now, suppose a parallel beam of light is starting from the extended source which are represented by this red line here, these are the two rays or waves which are parallel and they are falling on this film, the first interface of the film therefore, after reflection this ray will go here and then there is a converging lens which will tilt the direction and it will fall at point P_2 on the screen.

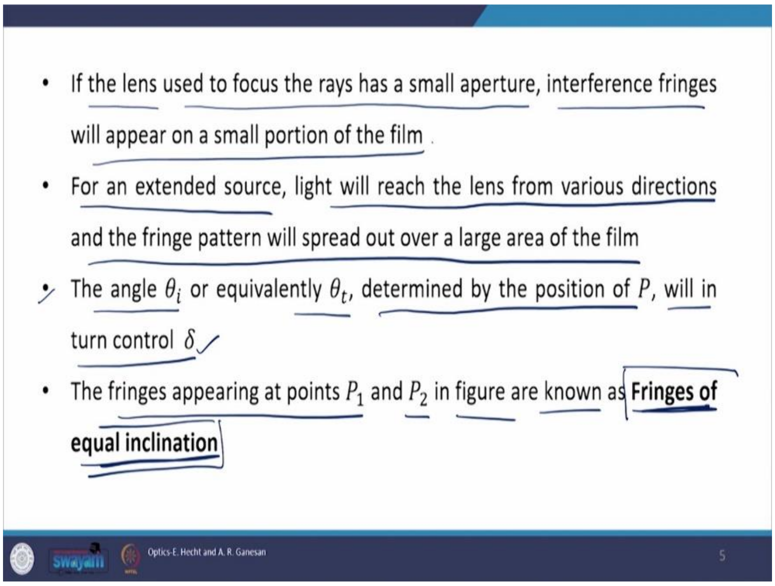
But apart from this, part of the ray will get transmitted, it will suffer reflection from the lower interface of the film and here again after transmission, it will become parallel to the first reflected ray, these two rays would be parallel and then they will again since they are the parallel rays, it will again converge to P_2 point on the screen. Similarly, this ray which is parallel to the first incoming ray, it will also suffer reflection and transmission and ultimately it will also get converse to point P on the screen.

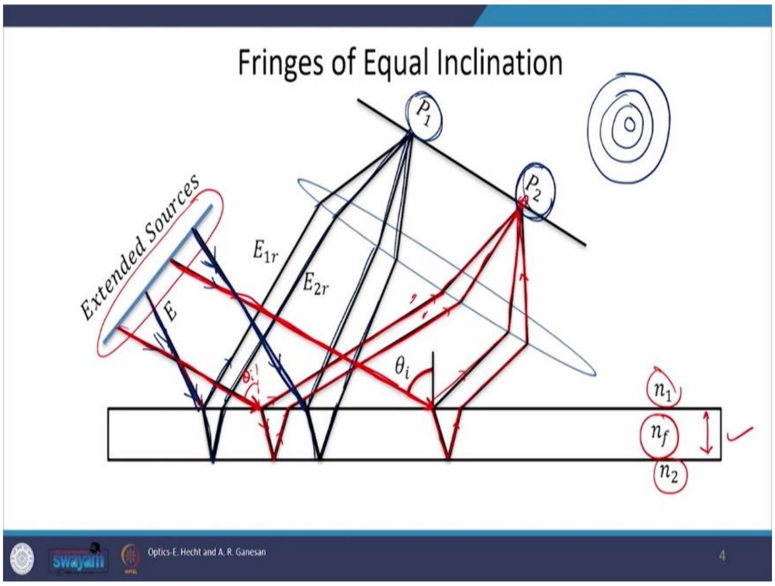
It means the rays which are parallel they will fall at the same angle at the first interface. What I mean to say is that this angle would be equal to this angle, the θ_i would be same here in both the cases. The rays which are parallel will fall at the same angle of incidence at the first interface of the film. And after its reflection all these parallel rays will go to the same point, but this is 2d, it concept in two dimensional.

Similarly, the rays which are again parallel, but inclined at different angle this blue line represents the second set of ray, this is again a parallel beam of light, but with a different inclination with respect to the first interface of the thin film, they will again suffers through the reflection and transmission and then they will converge to point P_1 here, it means the angle of incidence, the sides where all these rays will go. The rays which are falling at the same angular getting converge to the fixed point on the screen.

Now, let us go move in a three dimensional plane or three dimensional space. Now in 3D, the source is extended and the film is also a 3D film now, it has both length and width and the depth is very small which is of the order of the wavelength. Now, in that particular case this common point P_1 and P_2, they will trace out a circle and therefore, the fringe pattern which we will see in this particular case where the source is extended, it will be of the form of concentric circles, now this will be the fringe form, the fringes would be in the form of concentric circles.

(Refer Slide Time: 7:02)





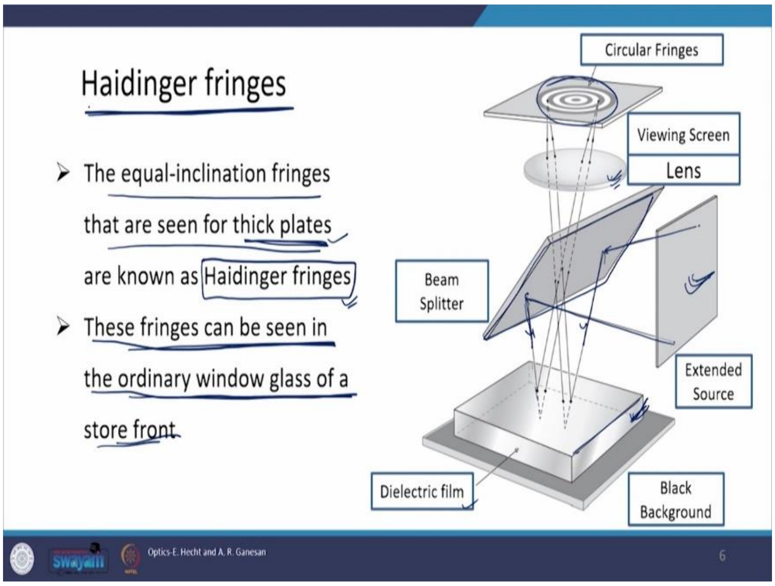
Now, there are few points which must be noted before moving ahead, if the lens used to focus the rays has a small aperture then the interference fringes will appear only on a small portion of the film. Here the lens is very small, the aperture is very small then only the fringes will appear only on a very small portion of the film.

For an extended source light will reach the lens from various directions as shown in the previous figure and the fringe pattern will spread out over a large area of the film. For the point source, there was a small area where the fringes were visible, for extended source, ray will fall on the film from different direction and then the fringes will also be will be spreaded and therefore, the they will appear on a large area of the film.

Now, the third point the angle of incidence θ_i or equivalently the angle of transmission θ_t because they are directly related, determined by the position of P, will in turn control the phase difference there. The angle of incidence or the angle of transmission therefore, will control the phase difference which is very much obvious, because if you change the angle of incidence the path difference will change and therefore, the phase difference will also change and therefore, they will decides whether at a particular position on their screen whether there is a maximum or minimum.

The fringes appearing at point P_1 and P_2 in figure are known as fringes of equal inclination, which is very much obvious, why? Because the rays which subtend the same angle with respect to the normal to the point of incidence, they are converging to the same point. Let us go back to the figure you see this the red beam is going to point P_2, while the blue ones are going to point P_1, the red ones are subtending the same angle therefore, they are getting converge to the same point which is P_2 and similarly for the blue. And therefore, the fringes, which are obtained in this way, are called fringes of equal inclination, the rays which inclined by same angle will fall on the same fringe and therefore, fringes of equal inclination.

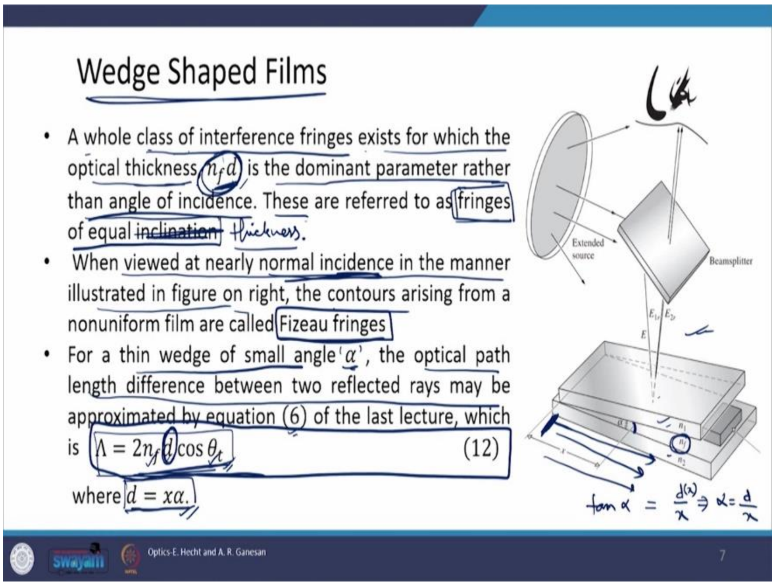
(Refer Slide Time: 9:52)

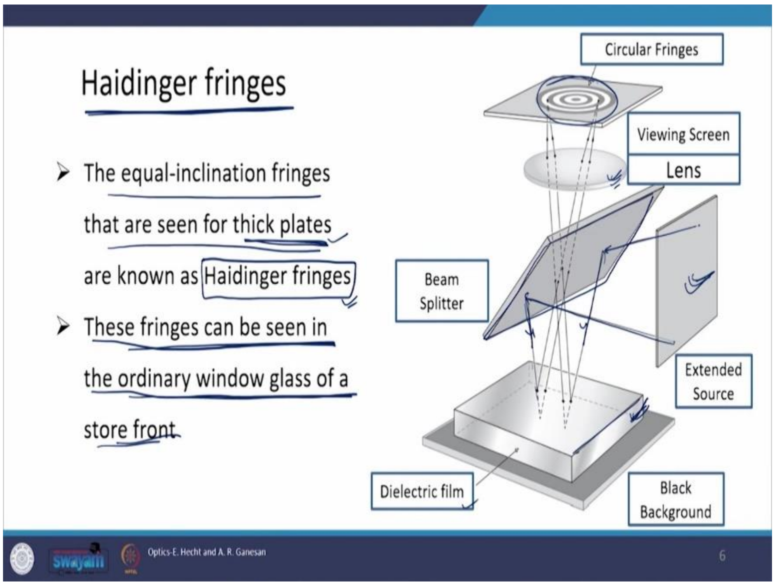


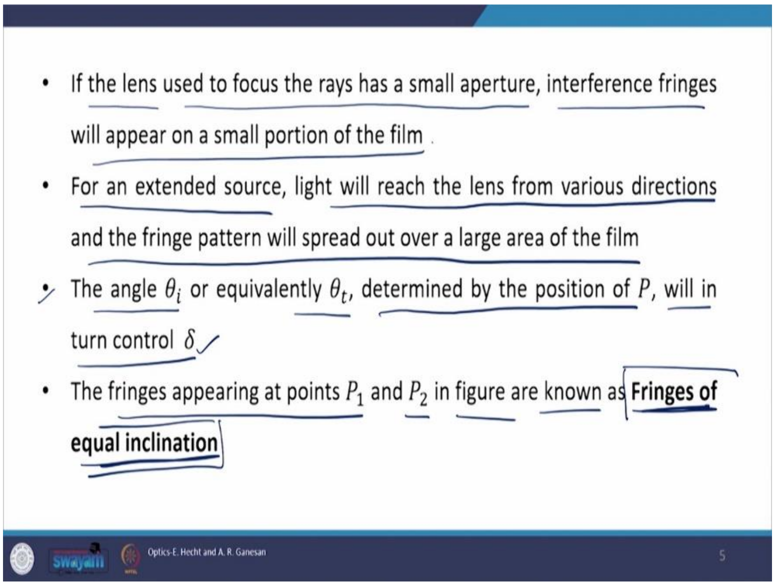
Now, this is the setup for the fringe formation. You see, this is the extended source and from the source the beams are going to this semi silvered glass plate or semi reflecting glass plate and this glass plate direct this rays to the thin film which is this one, this is the thin film or thin dielectric film and then the rest of are multiple reflections here and then through this lens, they are converged to the screen and we see the formation of circular fringes.

The equal inclination fringes that are seen for thick plates now, if the plate is thick are known as Haidinger fringes, there is a particular property of these fringes and what is this property or what are the prerequisites for the formation of these fringes? For the Haidinger fringes, the angle of incidence must be either equal to or very close to 0, or these fringes must fall on the glass plate almost normally. And these fringes can be seen in ordinary glass window also or the glasses which we which are installed on a store front, these are called Haidinger fringes, just remember normal incidence and for the thick glass plate.

(Refer Slide Time: 11:25)







Now, apart from these fringes of equal inclination, there is another class of fringes and what is this new class of fringes? This new whole class of interference fringes exists where the dominant term is the thickness, let me reframe it now, apart from this fringes of equal inclination there exists a separate class of fringes, which is called fringes of equal thickness, fringes of equal thickness means this path difference which is given by Λ here you see that there are terms which depends upon n_f and there are term which depends upon θ_t or θ_i.

If the path difference is angle dependent then we get fringes of equal inclination. It is also possible that we have a film in which d is changing, d is variable and in that particular case fringes become d dependent, they become dependent on the thickness of the film. And therefore, the dominant parameter now becomes $n_f d$, which is optical path length in a medium of refractive index n_f.

And therefore, a whole new class of interference fringes exist for which the optical thickness which is $n_f d$ is the dominant parameter rather than the angle of incidence or angle of transmission which is shown here in this equation number 12 and these fringes are referred to as fringes of equal inclination, this is the new type of fringes, fringes of equal inclination. Thus, we are introduced to two types of fringes, first is fringes of equal thickness and the second is fringes of equal inclination.

Now, here we talked about fringes of equal inclination, there is a mistake. These fringes are referred to as fringes of equal thickness, not inclination, is wrongly written, because $n_f d$ is now the variable and the fringes is now dependent on $n_f d$ instead of θ_i *or* θ_t therefore, such kind of fringes are called fringes of equal thickness.

Therefore, we studied two types of fringes, first is called fringes of equal inclination which we study in case of parallel film and the second type of fringes is called fringes of equal thickness and this type of fringes appears where the thickness varies and the best example is wedge shaped film. Therefore, under this topic we will talk about this type of fringes, which are called fringes of equal thickness.

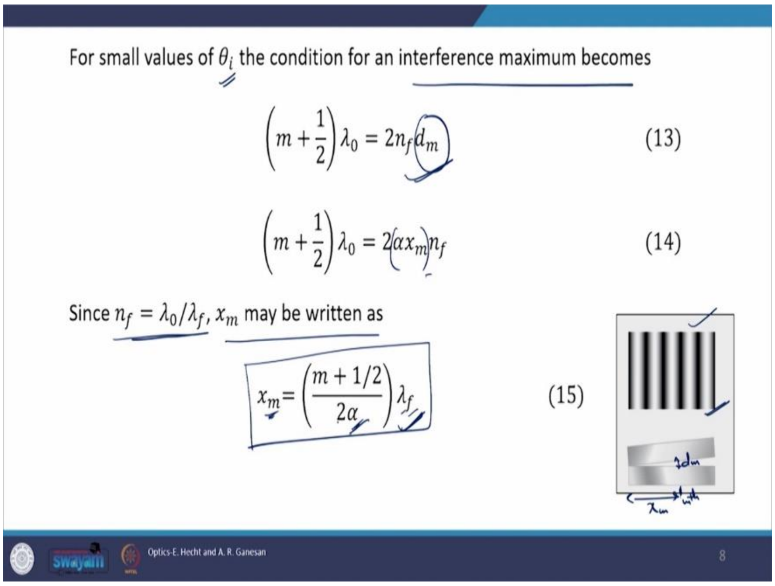
Now, you see that there is a medium of reflective plates of refractive index n_1 and n_2 and they are inclined at some angle α, this is angle of inclination is α. Then between these two plates, there is a film of refractive index n_f, whose thickness is increasing as we go away from this point. We move in this direction, in plus x direction, you see that its thickness is increasing. This film of refracting index n_x, this shape of the film is called wedge shape, this is the shape of the film and this film thickness is increasing as we move away from the point of our line of contact between the two plates then we see that thickness of the film is increasing slowly and this angle is kept very small.

Now, if we may incident a beam almost normally to the first interface of this wedge shape field then we again get some type of interference pattern due to again multiple reflection and transmission both in the transmitted arm and reflected arm we will see a fringe formation. Now, for normal incidence the fringe pattern which arises is called Fizeau fringes, we talked about Haidinger fringes, now this is Fizeau fringes, Haidinger fringes appear in case when the fringes are of equal inclination, but for equal thickness case, we have Fizeau fences.

Now, consider we have a thin wedge and the angle of the wedge is very small which is represented by α here then the optical path length difference between the two reflected rays may be approximated by question number 6 which we covered in last lecture. Now, this equation says that optical path length difference is equal to $2n_f d cos\theta_t$, n_f is the film refractive index, but what is d here, d is the thickness but thickness is varying as you move away from the line of contact.

Therefore, d is expressed by this relation $d = x\alpha$, where α is very small angle therefore, from this figure you can see that $tan\alpha = d/x$, here d is a function of x of course, and when α is very small then we can write $\alpha = d/x$ or $d = x\alpha$, d the thickness is function of x. Here x is measured from the line of contact, we assume that the x is 0 there and as we move away from that line of contact x increases in positive direction.

(Refer Slide Time: 17:48)

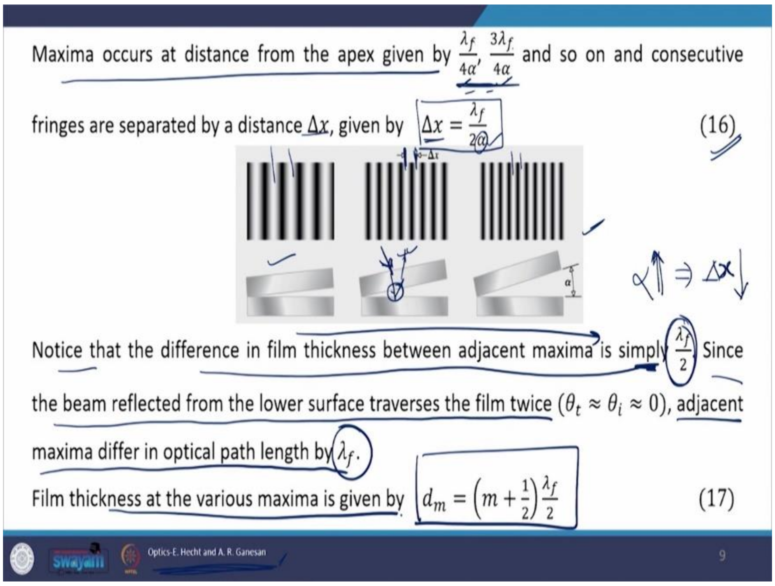


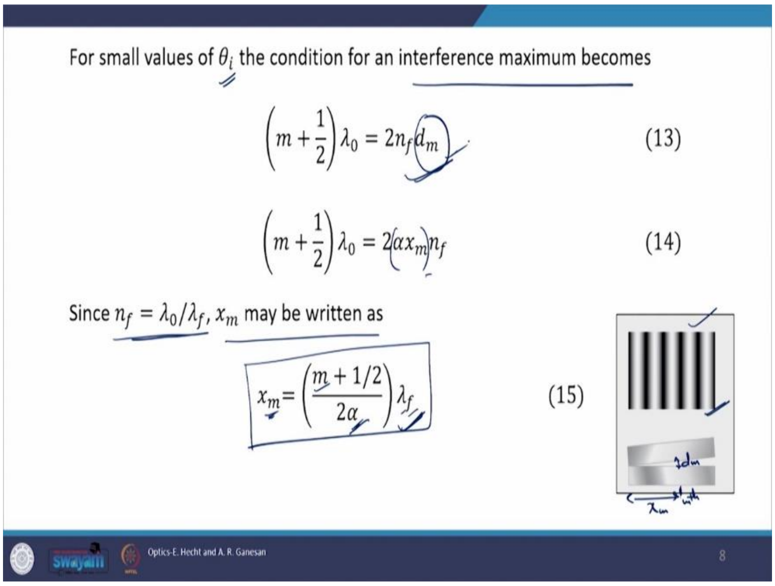
Now, under this condition for small values of angle of incidence the condition for interference maximum will be given by equation number 13, we are just following the previous, the last, class lecture. And here the d_m is the thickness of the film where m^{th} maxima is observed and as we know $d = x\alpha$ therefore, we write $d_m = \alpha x_m$, x_m represent the position of the m^{th} maxima from the line of contact of course here. If the maxima is appearing here then if m^{th} maxima is appearing here then this is x_m and the corresponding thickness is d_m.

Now, we have already defined that $n_f = \lambda_0/\lambda_f$ in the last class therefore, x_m from the equation number 14 can be expressed by this relation which is equation number 15. Now, you see that if you plot these things then you can easily visualize that fringes would be straight line fringes and the fringes switches from dark to bright and bright to dark as we move away from the line of contact, as you increase the thickness of the wedge. If you vary the thickness of this wedge, the fringes alters here it goes from bright to dark and dark to bright and so on and this is shown here.

But you also see that this position of the fringe, it depends upon the angular α also. So, the m^{th} position of m^{th} bright fringe, it depends upon α it depends upon n_f, it depends upon λ. Therefore, if you change α or λ, the position of this bright fringe will change accordingly.

(Refer Slide Time: 19:56)





Now, from the previous relation the first two consecutive maxima occur at x which is given by $\lambda/4\alpha$ and $3\lambda_f/4\alpha$, how? Here if you substitute m is equal to 0, then you get $\lambda_f/4\alpha$. Second, the next interference maxima will occur when m is equal to 1 and which if you simplify will give you $3\lambda_f/4\alpha$. And from these two relations as we did in Young's double slit experiment, we can calculate the fringe width, the separation between the two constitutive maxima which is Δx and if you subtract these two you will get this relation, this is fringe width as shown here in this figure you see, the separation between two maxima is Δx .

Now, this separation is inversely proportional to α, the angle here and therefore, if you vary α if you increase α the fringe width decreases and this what exactly is shown here in this figure here. Here if you go from left to right, α is increasing, α is increasing if you go from left to right and therefore, the fringe width since fringe width is inversely proportional to α, it will decrease and this is what exactly happened here, the fringe width is large here, it is has got reduced and here it is very small relatively it is slowly decreasing as α is getting increased as we increase α the thickness of the fringes are reducing, which is very much clear from equation number 16 also.

Now, this all figures are taken from this reference, I always mentioned the references here down, all this material is either from optics by E-Hecht and A.R. Ganesan or from optics by Ghatak, all this figure, all these contents are from there. Now, notice that the difference in the film thickness between adjacent maxima is equal to $\lambda_f/2$, the d, if we move from one maxima to another maxima, the d varies by only $\lambda_f/2$. Now, once you can calculate d from equation number 30.

Now, since the beam reflected from the lower surface traverses the film twice, because when you fall light here, when you incident light here, then it goes into the film and then it gets reflected and then comes out, it means the reflected light travels twice the length of the film. Therefore, the adjacent maxima differ in optical path length by λ_f and the film thickness at various maxima is given by this relation.

And similarly, you can calculate the expression from various minima and this is all for fringes formed in wedge shape film. And this is and this is all for today and we will meet in the next lecture and then read further, we will talk about the next topic in this module 5 which are newtons ring, very important topic. And I end my lecture with this and thank you all for listening me, see you in next class.