Applied Optics
Professor Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology Roorkee
Lecture: 21
Interference by Division of Amplitude

Hello everyone, welcome again in my class. Today, we will start module number 5. In module 4, we talked about interference and in particular, we talked about interferometers, which give a fringe pattern by splitting its wavefront. Now, from today onwards, we will talk about interferometers which rely on division of amplitude.

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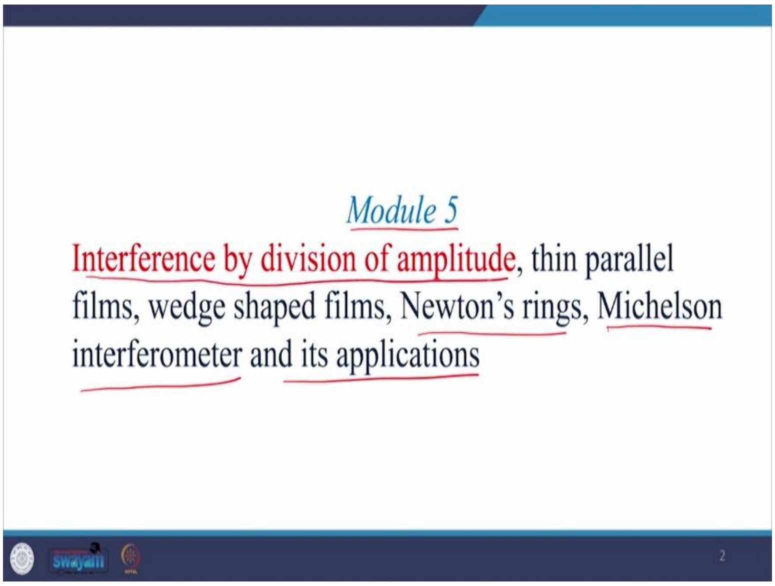

Module 5

Interference by division of amplitude, thin parallel films, wedge shaped films, Newton's rings, Michelson interferometer and its applications

Now, this is the whole syllabus of module 5. In this module, we will talk about interference by division of amplitude wherein we will talk about thin parallel films then wedge-shaped films then the famous Newton's ring experiment and very well-known Michelson interferometer and its applications. For today, we will cover interference by division of amplitude, I will talk about the concept here and then we will go to thin film which wherein we will assume that only there are two reflected ray and then we will talk about its interference and that conditions of maxima and minima here.

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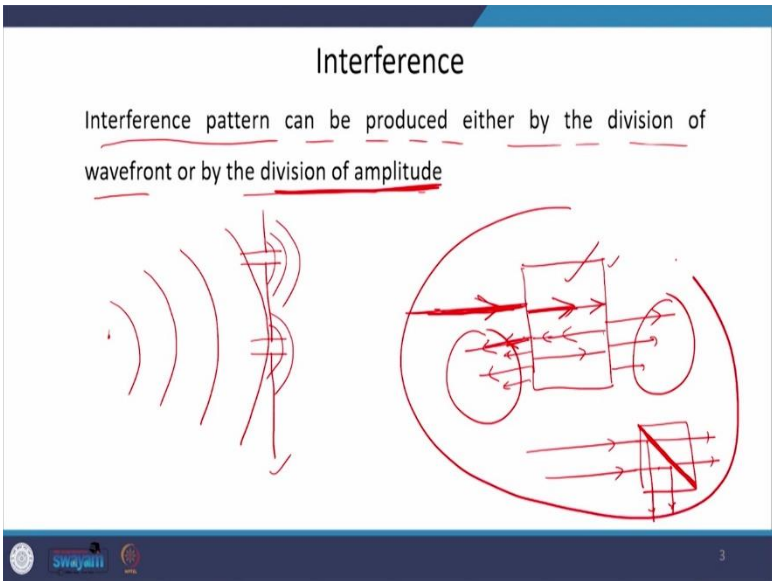

Interference

Interference pattern can be produced either by the division of wavefront or by the division of amplitude

Now, just to revive your concept, the interference pattern can be produced either by division of wavefront or by division of amplitude here, this concept was introduced in prior lectures. Now, whenever we say the division of wavefront it means that suppose we have a point source here and it is emitting spherical wavefront and then we put some slits in front of these wavefront, spherical wave fronts, then what will happen? This wavefront will fall on this slit and a part of this wavefront will pass through first slit another part of it will pass through the second slit and then from these slits new spherical wave fronts would be generated and then they will propagate and interfere to produce interference fringes. And this typical example utilised or seen in Young's double slit experiment.

Now, what is division of amplitude in this particular case, the instrument divides the amplitude of the wave. How does it do so? Now, as an example, we can consider a thin film and then suppose a light is falling here or rays falling on this film and the part of the ray will get reflected and the part will get transmitted, this transmitted part will again fall on the second interface and again a part will be reflected and a part will be transmitted.

Now, this reflected ray will again go back after reflection and a part will be transmitted and therefore, there will be multiple transmitted and multiple reflected wave and these reflected and transmitted wave under suitable circumstances may overlap and then they may give interference fringes. Now, here you can see that a particular amplitude is being launched and this amplitude is being split in here, the part is getting transmitted into part is getting reflected, it means that reflected and transmitted magnitude sums up and then it gives the incident magnitude provided the film is lossless, such a splitting may also be observed in beam splitter.

Now, here we launch a beam of a ray. Now, suppose a beam splitter has a semitransparent hypotenuse which is here then after entering into the beam splitter, the part of the ray will be reflected downward and the part will get transmitted, the beam splitter is also amplitude splitting device similar to thin films, we will learn about this part of interference in module number 5.

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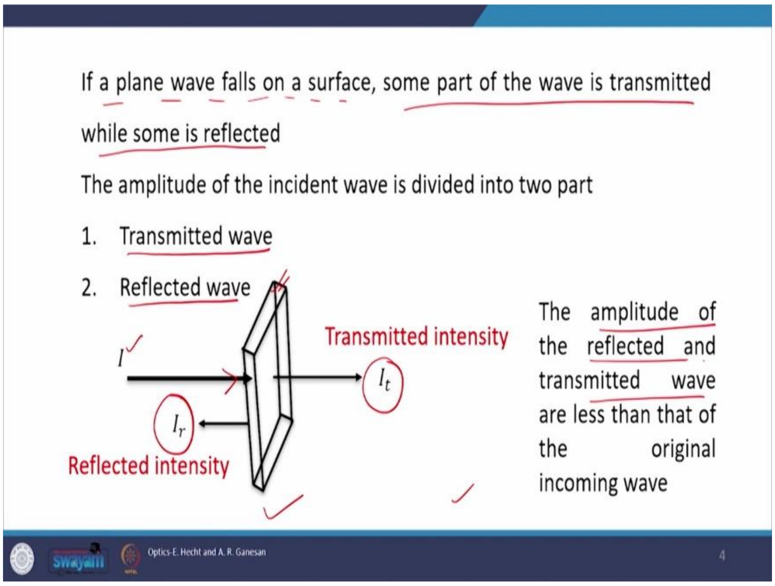

If a plane wave falls on a surface, some part of the wave is transmitted while some is reflected

The amplitude of the incident wave is divided into two part

1. Transmitted wave
2. Reflected wave

The amplitude of the reflected and transmitted wave are less than that of the original incoming wave

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Now, if a plane wave falls on a surface, of course this surface is assumed to be a transparent, we assume that there is a transparent film and the light or the plane wave is falling on the first interface of this transparent film, then some part of the wave is transmitted while some is reflected. Therefore, the incident amplitude is being divided in two parts, the first part is transmitted wave and the second is reflected wave which is shown schematically here in this figure.

The intensity I is launched and the transmitted intensity is represented by I_t and the reflected intensity is represented by I_r, if you sum these two intensities up then we will get the original intensity which is I, provided the medium, this film, is lossless. And of course, the amplitude of the reflected and transmitted wave is less than that of the original incoming wave here this is the very simple concept, but we must understand it very properly.

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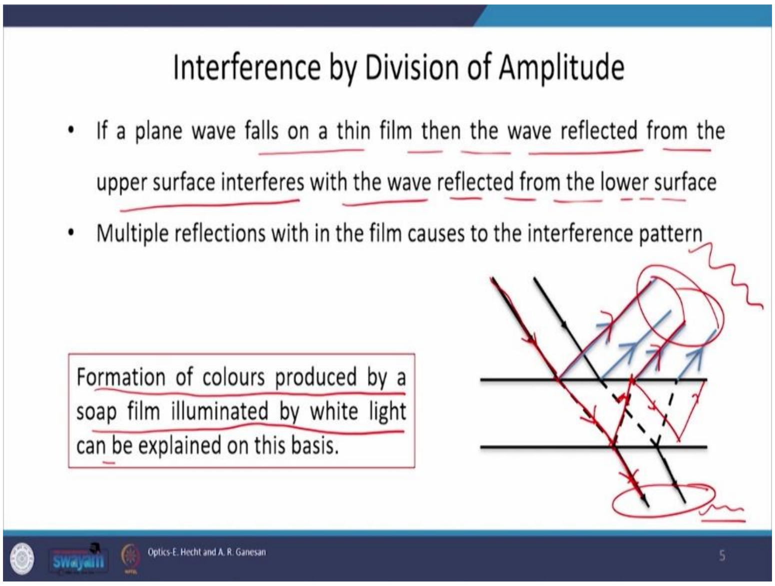

Interference by Division of Amplitude

- If a plane wave falls on a thin film then the wave reflected from the upper surface interferes with the wave reflected from the lower surface
- Multiple reflections with in the film causes to the interference pattern

Formation of colours produced by a soap film illuminated by white light can be explained on this basis.

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Now, if a plane wave falls on a thin film then the wave reflected from the upper surface interferes with the wave reflected from the lower surface and which can be easily understood from this figure here, wave is falling here, a part is getting reflected, a part is getting transmitted and at the lower interface again part is getting transmitted and a part is getting reflected, now, these two parts may interfere.

Similarly, a ray may again get reflected here and then several reflection and transmission may happen and all this reflected ray may interfere and give some fringe pattern in this direction and always transmit transmitted rays may also interfere and give some fringe pattern in this direction and these fringes we often see in our daily life. Where do we see these fringes?

The best example is formation of colors produced by a soap film illuminated by white light, because whenever we use detergent we see that soap bubbles get formed and these soap bubbles are very colorful, and what is the origin of these colors? This interference, different components or different wavelength in white light they interfere in different way or in different directions and the produces their maxima at different angles and therefore, we see different colors in soap bubbles.

Now, here and we will talk about interference between two rays only, two reflected rays or two transmitted rays, why only two? Because we assume that the reflection coefficient or transmission coefficient is such that the waves which beyond second reflection is very weak in intensity and therefore, we assume that these rays can be neglected, the effect of these rays or contribution of these rays can be neglected and of course, under this study we assume that the film is lossless.

This interference is studied for thin film which is assumed to be lossless as I said and the film is assumed to be thin because its thickness is of the order of wavelength. We have already discussed the relevance of word thin, whenever we say thin there is some reference thickness with which we compare it. If the film's thickness is of the order of the wavelength then only we call this film a thin film.

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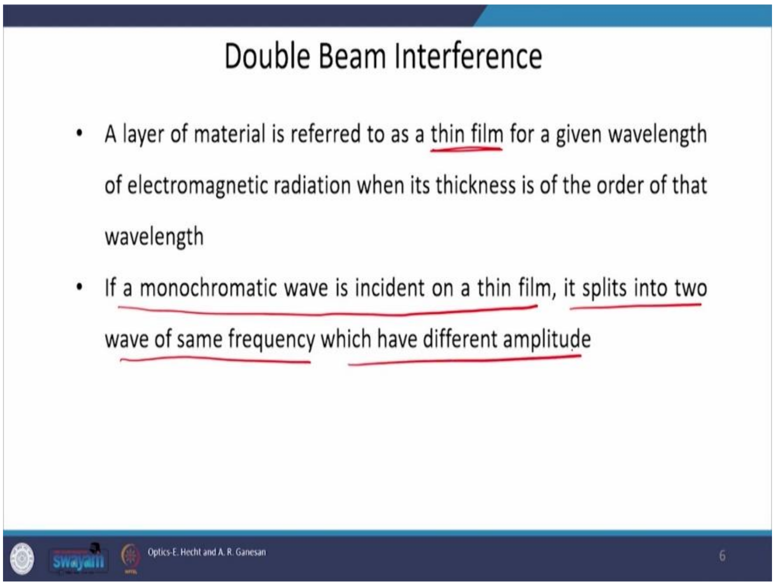
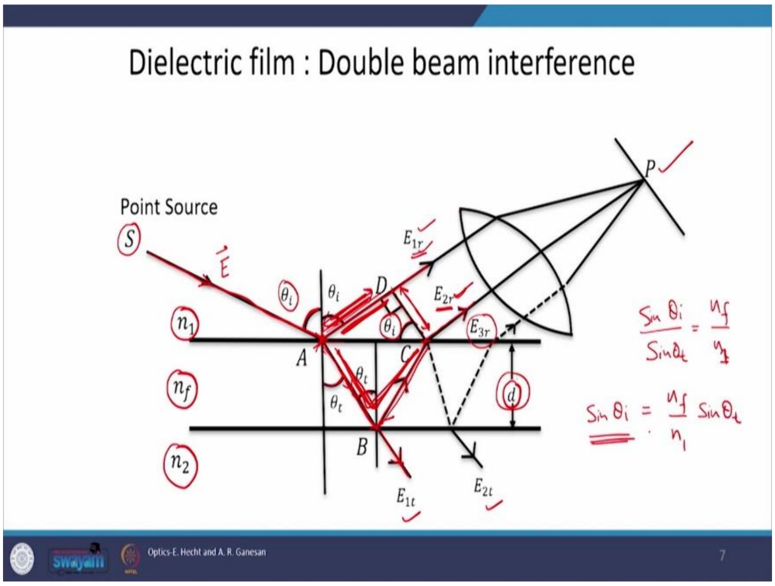


Double Beam Interference

- A layer of material is referred to as a thin film for a given wavelength of electromagnetic radiation when its thickness is of the order of that wavelength
- If a monochromatic wave is incident on a thin film, it splits into two wave of same frequency which have different amplitude



Dielectric film : Double beam interference

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_f}{n_1}$$

$$\sin\theta_i = \frac{n_f}{n_1}\sin\theta_t$$

Now, if a monochromatic wave is incident on such a thin film then it splits into two waves of same frequency which have different amplitudes and these are shown here schematically in this figure, we have a point source which is situated at point S and the ray starting from point source or wave starting from point source falls on the first interface of this thin film and the thin film has thickness d and refractive index n_f, the medium above the thin film has refractive index n_1, while the medium below the thin film has referred active index n_2.

Now, when the wave strikes at point A on the first interface, a part of this wave will be reflected while a part will go inside the film which we call transmitted wave. This transmitted ray, it will again fall on the lower interface or the second interface of the film and there again it suffers reflection or partial reflection and partial transmission, this is the transmitted wave and this is the reflected ray and similarly multiple transmission and reflection will happen.

But here in this particular case, we assume that only first two reflected rays contribute to the fringe deformation because the reflection and transmission coefficients are chosen such that that further reflection and transmissions are so weak that they almost do not contribute to the effective electric field at the point of observation P. Therefore, we assume that E field is being launched and that reflected amplitude is represented by E_{1r}, this is the first reflected amplitude or first reflected field, the second reflected field is represented by E_{2r}.

Similarly, first transmitted field is represented by E_{1t}, while second transmitted field is represented by E_{2t}. We assume that the incident ray falls at angle θ_i at the first interface and the angle of transmission is θ_2. Now, giving these conditions we can now calculate the path difference between E_{1r} and E_{2r}, we will follow the traditional approach here, we will draw a perpendicular from Point C to point D, D is on a ray E_{1r}.

Now this perpendicular will make an angle θ_i with respect to the first interface and the path length after line DC is the same for both of the rays. And here we put a lens and this lens what does it do? It make them overlap, it converges the rays and therefore these rays superimposed at point P and where we are supposed to get interference pattern or fringe formation.

Now, we will see like we will now talk about the path difference between the E_{1r} and E_{2r}, then what is the path difference? Till point A, the path length is same and after line DC the path length is same therefore, whatever is happening whatever difference is being created between the two paths is after A and before DC.

Now, you see that E_{2r}, the second ray is following path A, B, C, while the E_{1r} ray is following path AD therefore, the path difference would be this length minus this length, we will have to calculate the length A, B, C and this is the usual length, but if you want to find the optical path length then we will have to multiply this distance by n_f. Similarly, we will have to multiply AD with n_1 to calculate the optical path length for first ray which is E_{1r} ray.

Now, if to calculate the path difference we will have to subtract these two optical path lengths and this is what is being done here.

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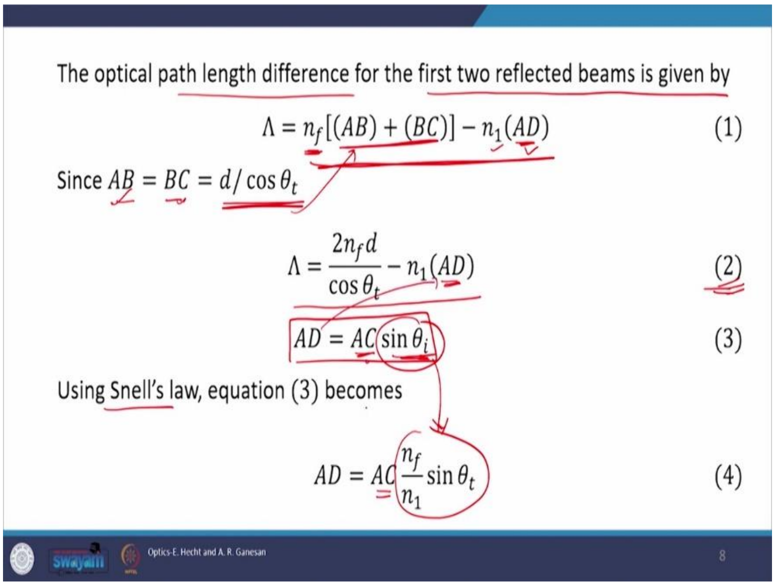


The optical path length difference for the first two reflected beams is given by

$$\Lambda = n_f[(AB) + (BC)] - n_1(AD) \qquad (1)$$

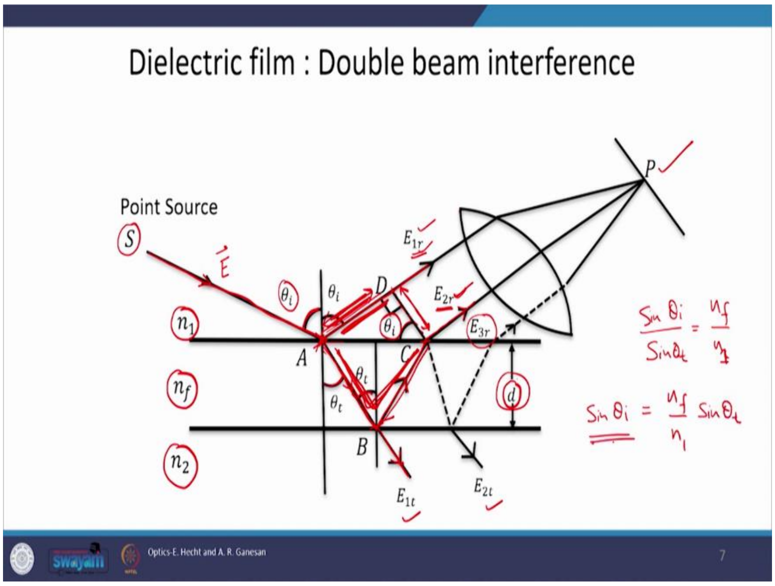
Since $AB = BC = d/\cos\theta_t$

$$\Lambda = \frac{2n_f d}{\cos\theta_t} - n_1(AD) \qquad (2)$$

$$\boxed{AD = AC\sin\theta_i} \qquad (3)$$

Using Snell's law, equation (3) becomes

$$AD = AC\frac{n_f}{n_1}\sin\theta_t \qquad (4)$$



Dielectric film : Double beam interference

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_f}{n_1}$$

$$\sin\theta_i = \frac{n_f}{n_1}\sin\theta_t$$

The optical path length difference for the first two reflected beams is given by AB+BC which is nothing but this path AB+BC, this is the extra part which second rays travelling within the film, and AD is the path with the first rays travelling outside the film, this is the AD. But to calculate the correct optical path we will have to multiply these lengths with the corresponding refractive indices and what are the corresponding refractive indices?

In film the refractive index is n_f therefore, the extra path, which is inside the film that has to be multiplied by the refractive index of the film, while the extra path for ray one which is outside the film that has to be multiplied by the refractive index of the first medium which is n_1 here. Therefore, this difference will give us the optical path length difference between the two rays E_{1r} and E_{2r}.

Now, from geometry, the AB is equal to BC, since θ_t is known and D is known the thickness of the film is known therefore, AB which is equal to BC can be calculated using this trigonometrical relation which is equal to $d/cos\theta_t$. Once expressions for AB and BC is known, we can substitute them back into the equation number 1 and from there we get equation number 2. Now, unknown quantity here in equation number 2 is AD, how to calculate AD?

Let us go back into the diagram, AD is this length. Now, θ_i is known, of course, AC is unknown, but we can calculate AC, once AB is known and thickness d is known. Therefore, let us express AD in terms of AC and this is the relation between AD and AC, AD= AC sinθ_i and θ_i is here this angle. Now, AC is unknown, how to calculate AC and θ_i is the angle of incidence.

Suppose we want to represent everything in terms of θ_t, then we can use Snell's law and what is Snell's law? Now, you see in this figure $sin\theta_i/sin\theta_t = n_f/n_1$, if this is known, then from here we can get the expression, you see here we want expression of θ_i, and $sin\theta_i = sin\theta_t n_f/n_1$ therefore, we can replace $sin\theta_i$ by $sin\theta_t n_f/n_1$ and this is what is being done here. $sin\theta_i$ in equation number 3 is replaced with $sin\theta_t n_f/n_1$. Once it is done the next unknown is AC.

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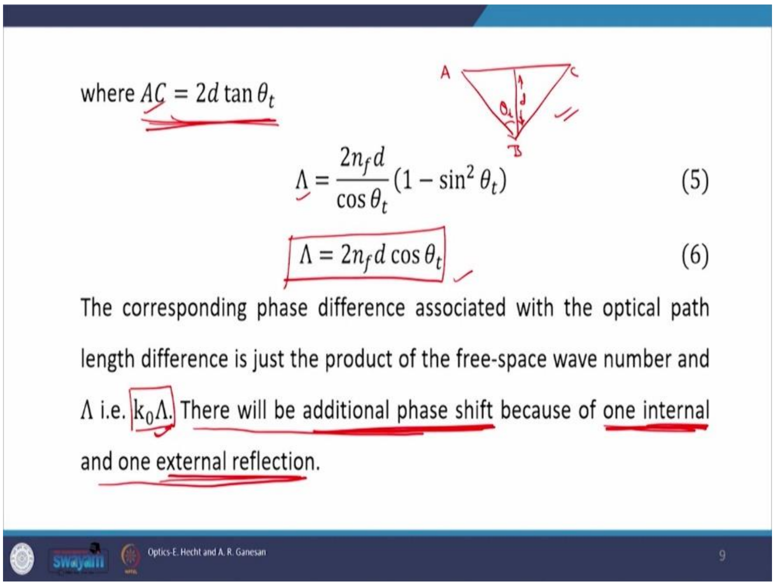
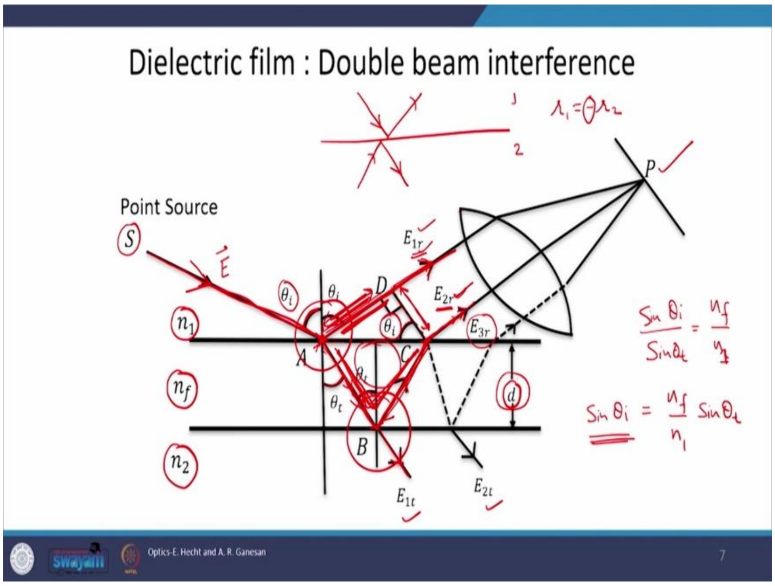


where $AC = 2d\tan\theta_t$

$$\Lambda = \frac{2n_f d}{\cos\theta_t}(1-\sin^2\theta_t) \tag{5}$$

$$\boxed{\Lambda = 2n_f d\cos\theta_t} \tag{6}$$

The corresponding phase difference associated with the optical path length difference is just the product of the free-space wave number and Λ i.e. $\boxed{k_0\Lambda}$. There will be additional phase shift because of one internal and one external reflection.



Dielectric film : Double beam interference

Now, from the figure itself AC= $2dtan\theta_t$, AC is this distance and B is situated here and this is your d and this is θ_t, using very simple trigonometry we can calculate that AC=$2dtan\theta_t$, this is nothing but this part of the diagram. Now, there is no unknown quantity now, we can substitute the expression of AC into equation number 2, both AD is replaced with this expression and AC with this expression $2dtan\theta_t$, this ultimately give us the expression for optical path length difference which is represented here by Λ .

Now, a bit of simple mathematics will give this very simplified expression for this optical path length difference and which is equal to $2n_f dtan\theta_t$, n_f is the refractive index of film, d is its thickness, and θ_t is angle of transmission, the first transmission. Now, once the optical path length difference is calculated, we can easily calculate the corresponding phase difference, how to calculate the phase difference, just multiply it with k_0, what is k_0? k_0 is free space wave number and this is what is being done here this is the corresponding phase difference $k_0\Lambda$.

But apart from these phase difference there will be some additional contribution, what is that contribution? The additional contribution would be because of the different nature of reflection involved here, what are different nature of reflection? Here one reflection is internal while the other is external, what is the difference?

Now, you see here in this figure, the ray is falling here at the first interface and then it is getting reflected. this is a happening at the first interface while the transmitted rays, they gets in and then they get reflected back and then transmitted back into the first medium. Now, if the film is kept in air then the first reflection is happening here and the second reflection is happening here, it is similar to the case where we are having one interface here it is first medium it is here it is second medium and first reflection is happening like this while the second reflection is happening like this.

This is similar to the concept which we discussed while studying optical reversibility and optical reversibility says that $r_1 = -r_2$, it says that between external and internal reflection there will be a phase difference of π and this is why, in this case also, there would be an additional phase shift of π that has to be taken into account while calculating the conditions for maxima and minima.

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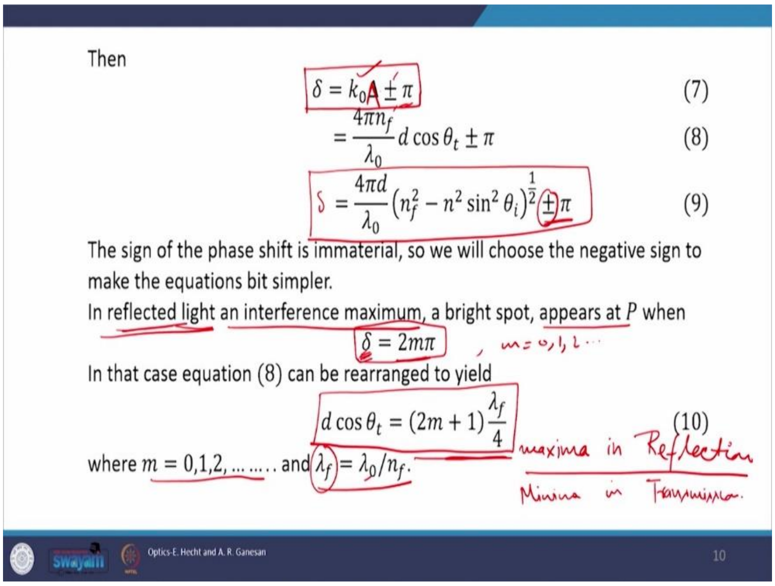


Then

$$\boxed{\delta = k_0\Lambda \pm \pi} \quad (7)$$

$$= \frac{4\pi n_f}{\lambda_0} d\cos\theta_t \pm \pi \quad (8)$$

$$\boxed{\delta = \frac{4\pi d}{\lambda_0}\left(n_f^2 - n^2\sin^2\theta_i\right)^{\frac{1}{2}} \pm \pi} \quad (9)$$

The sign of the phase shift is immaterial, so we will choose the negative sign to make the equations bit simpler.

In reflected light an interference maximum, a bright spot, appears at P when

$$\boxed{\delta = 2m\pi}\ , \quad m = 0, 1, 2 \cdots$$

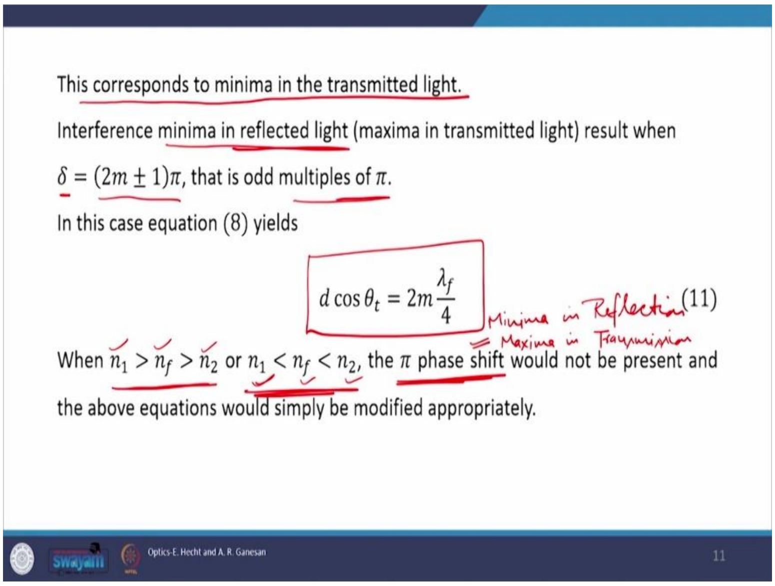
In that case equation (8) can be rearranged to yield

$$\boxed{d\cos\theta_t = (2m+1)\frac{\lambda_f}{4}} \quad (10)$$

where $m = 0,1,2, \dots\dots$ and $\lambda_f = \lambda_0/n_f$.

maxima in Reflection

Minima in Transmission.



This corresponds to minima in the transmitted light.

Interference minima in reflected light (maxima in transmitted light) result when $\delta = (2m \pm 1)\pi$, that is odd multiples of π.

In this case equation (8) yields

$$\boxed{d\cos\theta_t = 2m\frac{\lambda_f}{4}} \qquad (11)$$

Minima in Reflection

Maxima in Transmission

When $n_1 > n_f > n_2$ or $n_1 < n_f < n_2$, the π phase shift would not be present and the above equations would simply be modified appropriately.

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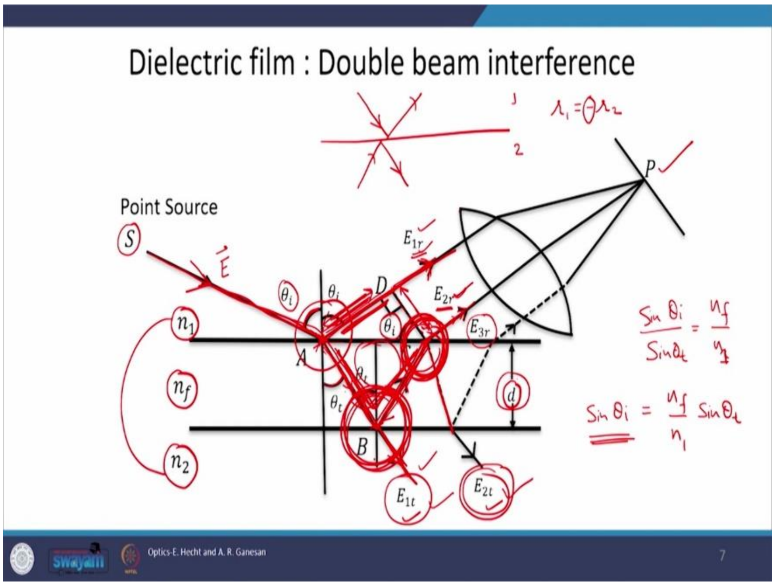

Dielectric film : Double beam interference

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Therefore, the total phase difference would be equal to $k_0\Lambda$ and $+\pi$ or $-\pi$, we can choose any, either $+\pi$ or $-\pi$. Now, let us substitute for the expression of Λ then we get $k_0 = 2\pi/\lambda_0$ and after a bit of simplification this is the expression for phase difference. The sign is immaterial and therefore, let us pick one particular sign and here we pick minus because this gives a bit simpler expression.

Now, we are talking the reflected light here, in reflected light and interference maximum will occur when the phase difference is equal to $2m\pi$, here m is equal to 0, 1, 2, it is an integer therefore, maxima occur at P when the phase is equal to integral multiple of 2π.

Now, let us substitute the expression of this phase difference from equation 9 in here with a minus sign before π, if you do this after a bit of rearrangement, we get this condition, this is the condition of maxima, this is the condition of maxima in reflection, where m is an integer and $\lambda_f = \lambda_0/n_f$, λ_f is the wavelength of light in film which differs from the wavelength of light in vacuum. Now, once the maximum in reflection is known, we can calculate the condition of maxima and minima in transmission too, what would be the difference?

Let us go back into the first figure, in this figure, you see the first reflection is external one now, this ray is getting transmitted and this is giving a field of magnitude E_{1t}, this is the first transmission and second transmission is E_{2t}, in transmission the first ray or first beam is not suffering any reflection while in second transmission the first reflection is here and the second reflection is here, these reflections are internal, these are happening within the same film, the conditions are the same, it is happening from film to the external medium and this second, this one, is also happening from film to the second medium, the interfaces is of same nature provided these two mediums are the same n_1 and n_2 are equal.

If this is the condition then the extra π shift in phase which we consider further reflected right that can be neglected. It means the same condition, this condition of maximum would be condition of make minima in transmitted arm, why? Because we will have to take away π, the extra phase shift, because now the reflection which is involved here are both internal, they are occurring on same type of interface.

Therefore, this previous relation also corresponds to minima in the transmitted light. Now, once the maxima is known, let us calculate the minima in the reflected light, how to calculate the minima? The phase should be $(2m + 1)\pi$, that is odd multiple of π and if you simplify equation 8, then you get this condition and this is minima condition in reflection which also correspond to maxima condition in transmission here, and this means maxima in transmission. And similarly, here this correspond to minima in transmission.

Now, there is two special cases, what is this special case? If the refractive index of the first medium is larger than that of the film and that is again larger than the refractive index of the third medium or if the refractive index of the first medium is smaller than the refractive index of the film and which is again smaller than the refractive index of the third medium. In both these cases, this π phase shift will not appear, it would not be present and therefore, the relations the condition of maxima and minima in transmission and reflection would be modified, this equation would be different because now we will have to take away the π phase, the extra π phase shift from equation number 10 and 11.

Now, this is the basic concept like and this is the very simple example of an interferometer which work on division of amplitude. And this is all for today. See you all in my next class. Thank you.