Applied Optics
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Lecture 02
Introduction to Geometrical Optics

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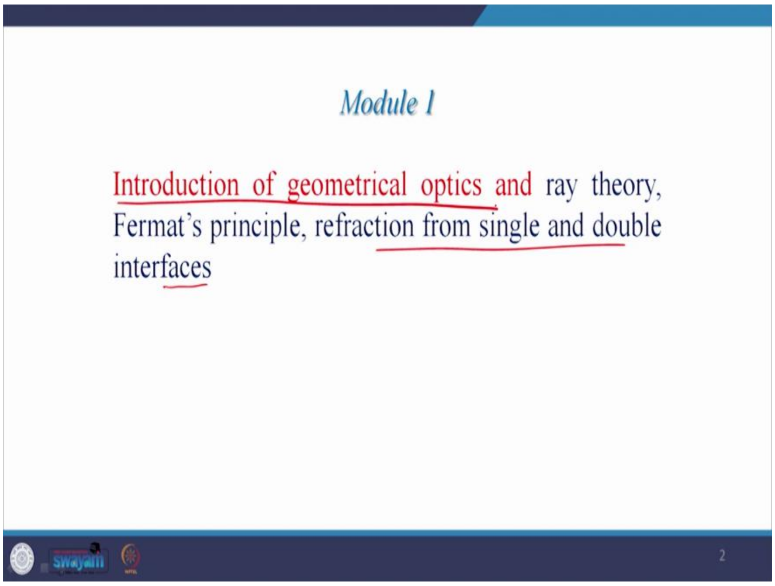

Hello everyone, today, we will formally start the course and I will start with module one. The module one is about introduction of geometrical optics and ray Theory, Fermat's principle, refraction from single and double interfaces. Today we will talk about introduction of geometrical optics and a part of ray theory.

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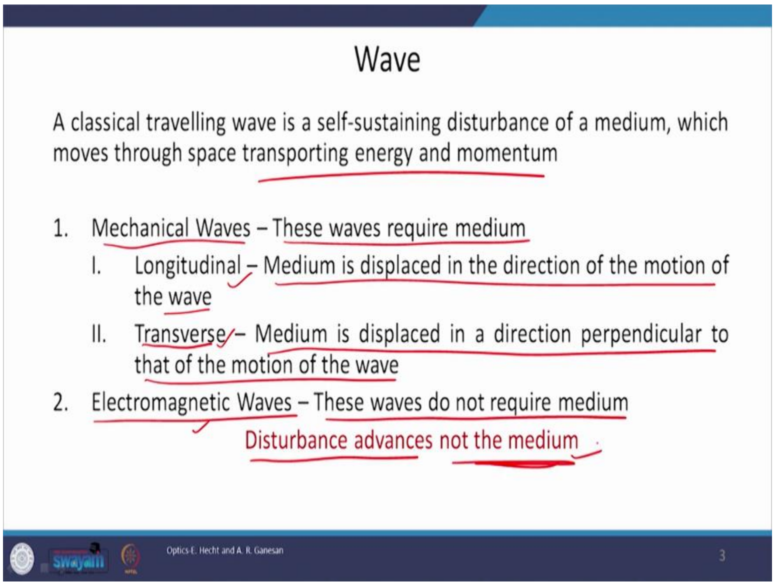

Now, what is a wave? this question brings some oscillations in our mind. Whenever we say wave then the waves in a sea appears to mind, the way we wave our hand holding a rope, this appears to our mind. Than the most basic definition would be wave is the disturbance. This is the shortest and best definition, okay, but firmly if you want to bind these words in the best possible way then it would be like this: Classical traveling wave is a self-sustaining disturbance of a medium, which moves through space transporting energy and momentum okay. Now, these waves are of two types: the first one is mechanical wave, sound is the best example of the mechanical wave and second is electromagnetic waves where our visible light falls. Now mechanical waves, these waves require medium to propagate. And these waves are subdivided further as longitudinal wave and transverse wave.

In longitudinal wave, the medium is displaced in the direction of the motion of the wave. Sound waves propagating in air comes in this category okay. When we speak then contraction and rarification of air happens and through this contraction and rarification, this sound energy propagates and then it goes to our ears, then it resonates with our eardrum and then we hear the sound.

Now, the second kind is this transverse wave, in transverse wave. Medium is displaced in a direction perpendicular to that of the motion of the wave, if you drop a cork ball in water, the dropping of pebble will generate water waves and this surface wave will make the water move up and down and this wave propagate radially outward and if you drop a water cork ball, you will see that due to this movement of the wave the cork moves up and down slowly and this represents that the medium is displaced in a direction which is perpendicular to the motion of the wave, the wave is moving radially outward in the water at the water surface and the cork ball is moving up and down. This is an example of a transverse mechanical wave.

Now coming to the electromagnetic waves, which is what we will be studying about in this course, these waves do not require any medium since there is no association of the medium the wave will propagate with a very high speed and this is why the speed of electromagnetic wave is very huge and which is generated by, which is described or which is represented by c and its value is value in vacuum is 3×10^8 meter per second okay.

And as I said disturbance advances not the medium and since this medium does not necessarily require a medium it propagates very fast.

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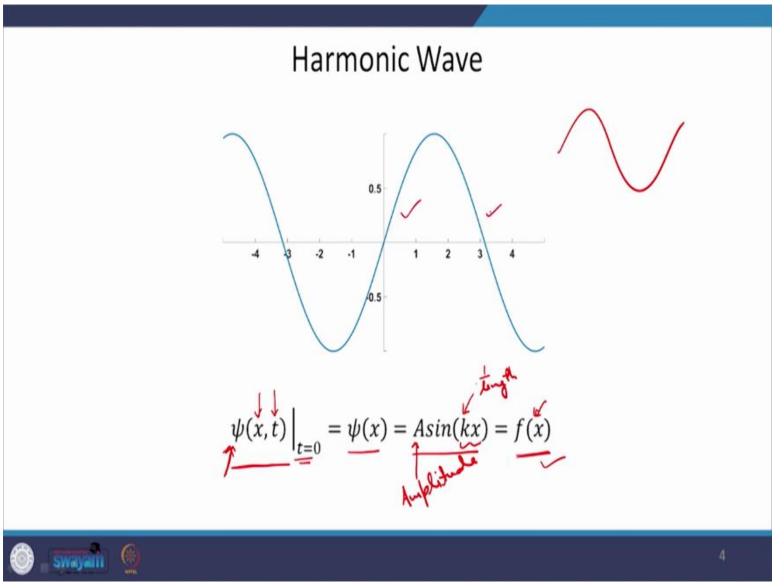

Now, whenever we say wave, this harmonic wave comes into our mind the sinusoidal wave comes into our mind and all types of waves are represented by some wave function. And this wave function is given by this expression,

$$\psi(x,t)|_{t=0} = \psi(x) = Asin(kx) = f(x)$$

ψ is the wave function and this wave function ψ is function of position variable x and time variable t. Now, this wave function at t is equal to zero would be represented by ψ as a function of x.

Since, it is a harmonic wave or since it is a sinusoidal wave, generally we can write it as A sin(kx), okay. Now you may ask that initially there was no k, how come k is appearing, how come A is appearing, A represents amplitude of the wave, okay. Now, the insertion of k is necessary. Why? then if you take sin(x) then that this would be not allowed. Why? because x is dimensional quantity, x is distance, x is length and you cannot take sine of some dimensional variable you will have to make a dimensionless.

And how to make this x dimensionless you will have to multiply x with one by length unit therefore, k will have unit of 1 by length and this makes kx a dimensionless quantity and once it is dimensionless we can easily take sine of this quantity and we will have something, so, now allowed.

And therefore, we can see that A sin(kx) is a function which is x dependent, x is a variable. Here now, if you vary x and see how does ψ look then you will find this type of curve which is given in blue here this is a sinusoidal curve.

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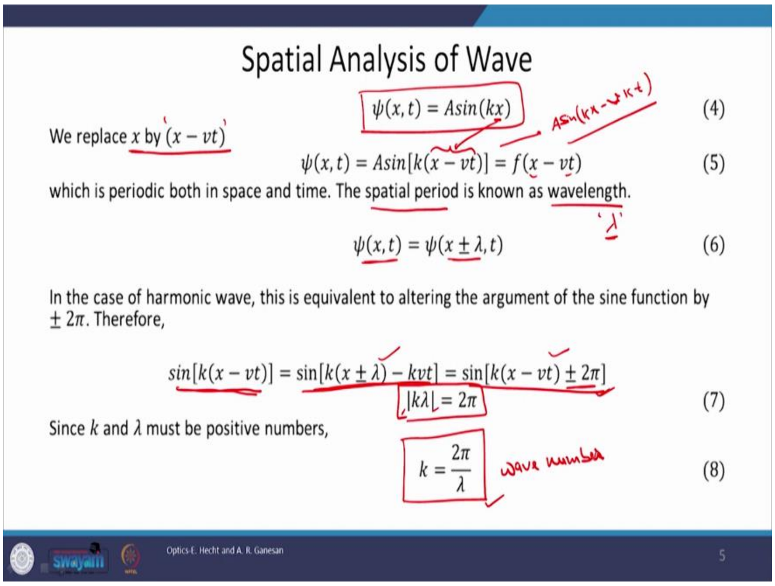

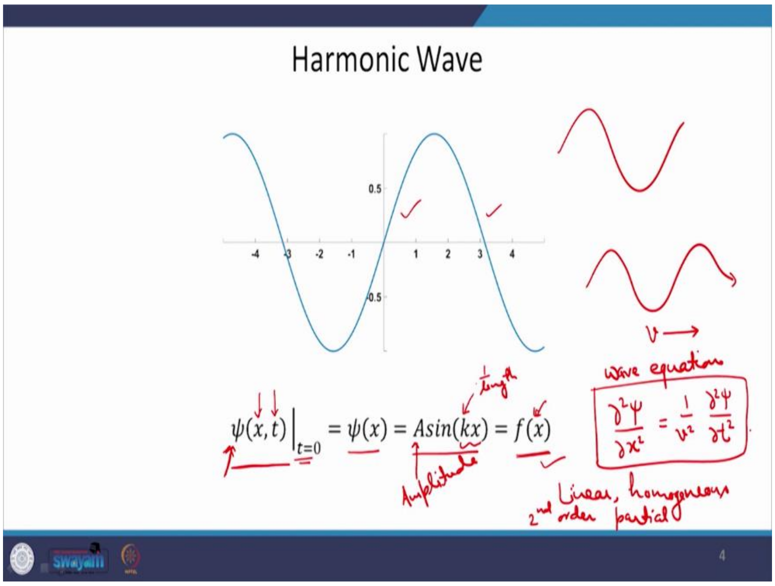

Now, the general expression for a wave is A sin(kx) okay, But, we know that a wave moves in space okay and since a wave is moving it must be having some velocity okay. All sort of waves are represented by some wave function and these wave functions, this evolution of the waves is governed by a certain type of mathematical expression which we call wave equation and what is the form of wave equation?

Wave equation is represented like this. This is the wave equation this is our differential equation, which is linear, homogeneous, second order partial differential equation okay, linear, homogeneous second order partial differential equation and all kinds of wave function obey this second order partial differential equation, they constitute the solution of this equation okay.

Now, as we discussed before, since a wave is moving therefore, velocity v must appear in the expression of wave function, how to insert the expression of velocity in the expression, just by replacing x by x-vt. v is the velocity and this x-vt will now introduce the expression of velocity into the wave expression. Now, in this expression if you replace x by x-vt then you will get this okay which is a function of x and t both now okay.

Now, the resultant wave function will look like A sin (kx-vkt) and this is sinusoidal both in space and time, this expression is periodic both in space and time okay. Since there is a periodicity in space, we can define or we can name this period and in a spatial domain this spatial period is known as wavelength and this wavelength is expressed by a Greek letter λ (lambda) okay. As I said the wave repeat itself after λ therefore, if you add or subtract λ in x, the resultant expression would be unaltered okay. If you add or subtract λ in x, it would not cause any variation into the resultant expression okay.

Now, we know that ψ is a harmonic function the wave function is harmonic and by adding or subtracting λ there is no variation in the expression then what we can do is that we will write this wave function and then we will add and subtract lambda in this okay and we also know that in the expression of sin in the argument, if we add or subtract 2π, this will also not cause any variation. This will also not alter the argument of the sine function therefore, this would be equivalent to $sin(kx - vt \pm 2\pi)$ because sine is a cyclic function. If you add 2π in its phase, it would come again to its original value therefore, this will not make any variation in the resulting function.

Now, if you compare the argument of the two, then we will find that $|k\lambda| = 2\pi$, what is λ (lambda)? lambda is a spatial period which is having the dimension of length and which will of course be a positive quantity. what is k? k is a parameter which is having the dimension of 1 by length (inverse of length) therefore, it would also be a positive quantity therefore, we can safely remove this mod and therefore, we can write k as 2π by lambda, lambda is having the dimension of length therefore, k would be having the dimension of inverse of a length okay which is aligned with the initial definition of the k okay and this k is called wave number.

Now, since the wave function is periodic both in space and time and here we have studied the spatial periodicity let us now analyze the temporal periodicity.

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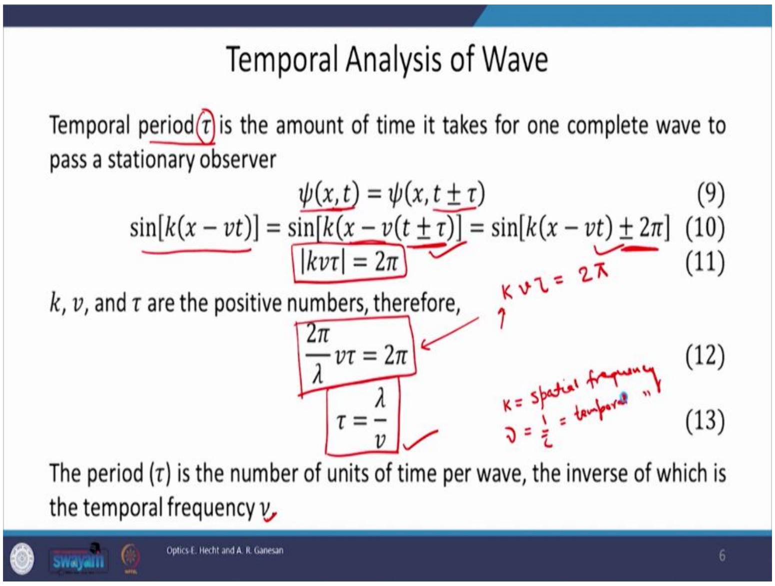

Now, let us assume that temporal period is τ (Tau), τ is the temporal period. It means, the wave takes τ time for completing one cycle okay. Now, if you add or subtract τ in time then you should also end up to the same expression because τ is the time period in time or periodicity in time. Therefore, we will just applying the same analogy what we have used in case of spatial analysis, $sin[k(x - vt)\}{\sim}sin[k(x - v(t \pm \tau)]$ and since it is a sinusoidal function, this would be equivalent to adding plus minus 2π.

Upon comparing these two arguments, we will get $|kv\tau| = 2\pi$, again k is a positive number as we have already discussed τ is time positive quantity v is positive number therefore, we will get $kv\tau = 2\pi$. Now, in the last slide, we saw $k = {}^{2\pi}/_{\lambda}$, substituting for k leads to this expression which is ($2\pi/\lambda$) (v× τ)= 2π, and from here we can get the expression for τ which is time period.

Therefore, in equation 13 we see that $\tau = {}^{\lambda}/_{v}$, ratio of lambda to v gives the expression for τ and the inverse of τ defines temporal frequency υ (mu) okay I would also like to mention here is that k is also called spatial frequency okay and inverse of τ is our temporal frequency.

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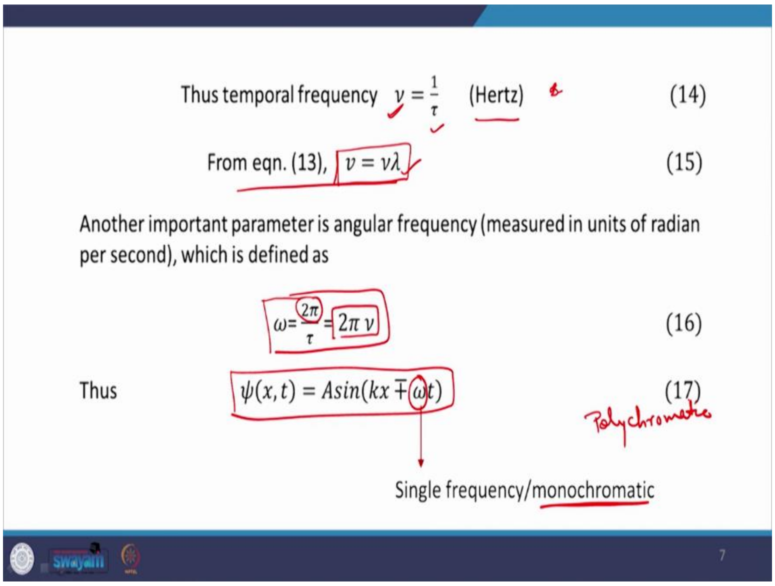

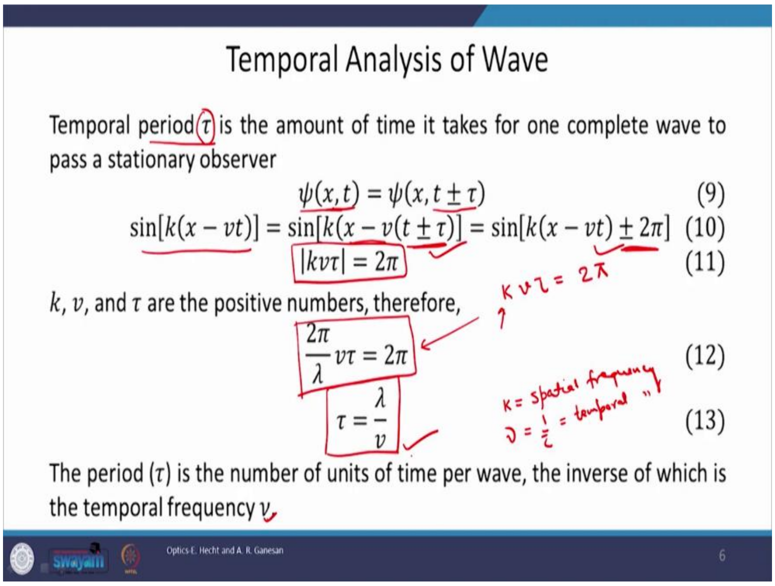

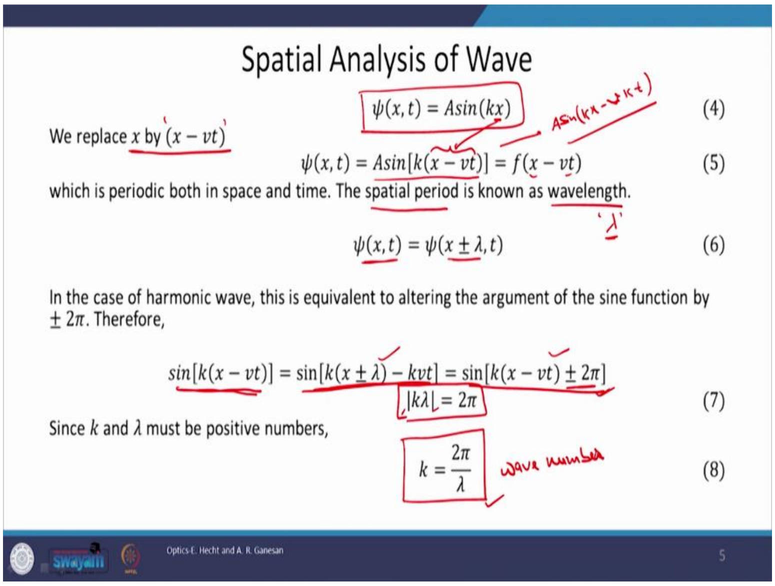

Once temporal frequency is defined. Now, τ is time, it is measured in seconds, hours and so on. Now, if τ is measured in second then the unit of υ (nu) is Hertz which is inverse of second. Now, from equation 13 which is in our last slide, where $\tau = \lambda/v$, we can write velocity $v = \upsilon\lambda$, υ (nu) is our temporal frequency and lambda is the wavelength okay this is a very important relation.

Now, apart from these terms another very important term is angular frequency which is often used in optics and how angular frequency is defined it is analogous to k, angular frequency is defined by inverse of τ but it has 2π term in the numerator okay, $\omega = 2\pi/\tau$ and we know that 1/ τ is equal to υ, therefore, we can write $\omega = 2\pi\upsilon$ again.

Now, we have so many definitions. We defined k, which is wave vector, we define ω (omega) which is angular frequency therefore, we can write the initial expression this $\psi = Asin(k(x - vt))$ in terms of k and omega.

Now, if you write it in terms of k and omega then the final expression of the wave function would be like this $\psi\ (x, t) = Asin(kx \pm \omega t)$ okay. what does plus and minus represent? plus and minus represents the direction of the wave. If it is propagating, whether it is propagating in positive x direction or it is propagating in minus x direction it is defined or taken care of this minus and plus symbol.

Now, here we see that this wave function has only one frequency omega and such a wave is called monochromatic wave okay a wave which has only one frequency, single frequency is called monochromatic wave, but, in practice it is impossible to have a monochromatic wave. All the waves have certain bandwidth and these waves are called polychromatic wave okay. But if this bandwidth is very narrow then these waves is very close to be a monochromatic one and these are called pseudo monochromatic.

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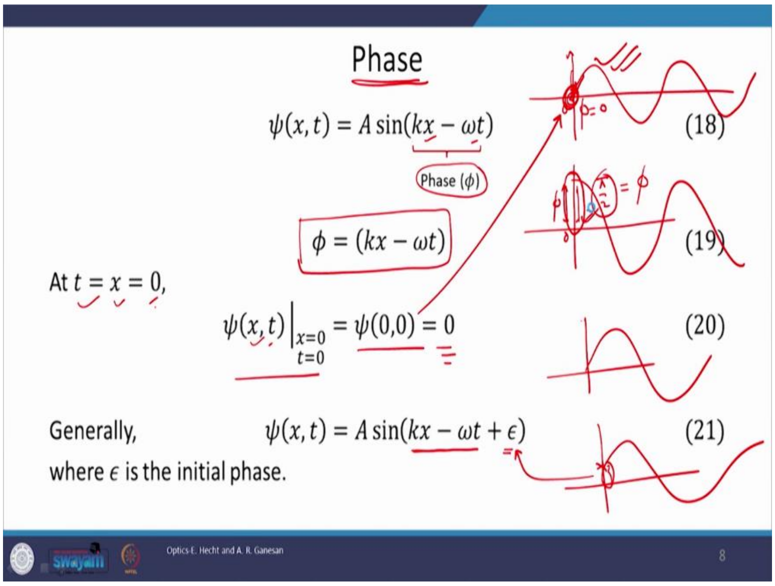

Now, once we have defined all these, we can move on defining phase, what is phase? now, the argument associated with the sine function is called phase okay and if we want to understand it figuratively then, if you plot a sine function and this is the type of the plot which we get okay. We see that the sine function is start with the zero, it is a zero here on the horizontal axis then we see that the sine function is start from exactly from zero and therefore, we can say that phase here is zero.

But sometimes we may start not from zero but from certain value which is not equal to zero and this decides the phase what is the value at the beginning here but more accurately phase is a relative concept. It is always measured with respect to some reference value like here in this first case the reference value is zero, here the reference value is again zero but there is a deviation here.

And then you can see that this is deviated by $\pi/2$ and therefore, the phase here would be $\pi/2$ okay anyway, let us go with the definition of phase here. The phase $\varphi = kx - \omega t$. Now, at t and x is equal to zero. We can define the wave function which would be zero, because when both x and t are zero the sinusoidal function would be zero and therefore, the wave function would be zero.

Therefore, we can see that phase plays a very important role here. Now, when both x and t is equal to zero, then this correspond to this point the wave function itself is zero which is here, but here when phase is equal to $\pi/2$ the wave function has certain non-zero value okay. But generally in addition to $kx - \omega t$, there is some extra term ϵ (Epsilon) in right along with this

$kx - \omega t$, which is called initial phase because, when both x and t is equal to zero, the wave should look like this, but sometimes what happens is that even at x and t is equal to zero, wave start from this point and here it means that some nonzero phase is there at the beginning this non-zero phase is represented by this epsilon nonzero phases also here.

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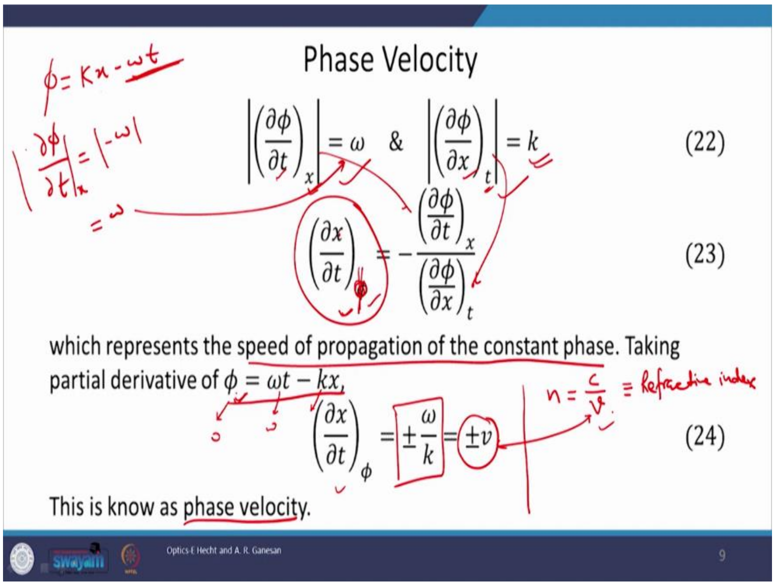

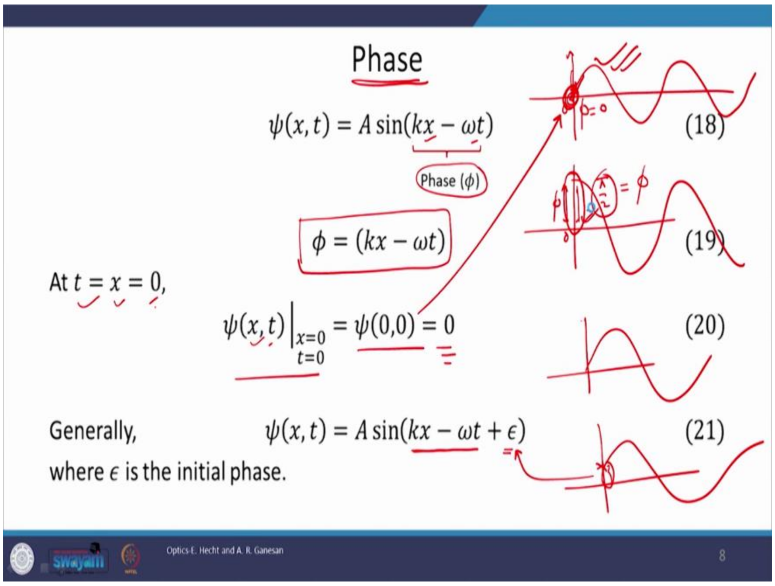

Okay, once we know what a phase is, what a wave, what is ω is, what a k is? Now, let us define phase velocity now, we have this expression of phase $\varphi = kx - \omega t$. Now, let us take time derivative of this phase keeping x constant if we take time derivative of phase keeping x constant then since $\varphi = kx - \omega t$, then $\partial\varphi/\partial t = -\omega$. because we are keeping k, sorry x constant and if you take the mod of this then you will get omega this is what is being represented here.

Similarly, if you take the space derivative of a φ keeping time constant now, then you will get k from the same expression this expression okay. Now, if you divide this and this, this is coming here in the numerator this is in the denominator then we will get del x by del t at constant phi. This quantity is called phase velocity and this represents speed of propagation of the constant phase. Now, if we know that $\varphi = \omega t - kx = kx - \omega t$ and if you take the time derivative of this quantity then keeping φ constant here.

Here it says that we are taking time derivative of x and φ is kept constant okay and if you treat φ as a constant and take the derivative of this φ with respect to time this then this term would be zero, here you will get omega and here you will get minus k and from here you can calculate $\frac{\partial x}{\partial t}$ at constant φ which would be equal to $\pm\frac{\omega}{k}$ and this is defined as velocity or phase velocity.

Now, here we can also talk about the refractive index, what is index of refraction? usually people define refractive index n as c the speed of light in vacuum by v, speed of light in a certain medium and this ratio gives index of refraction or refractive index. The v which is used here is this v, the phase velocity, okay and this definition is valid as long as our wave is monochromatic okay.

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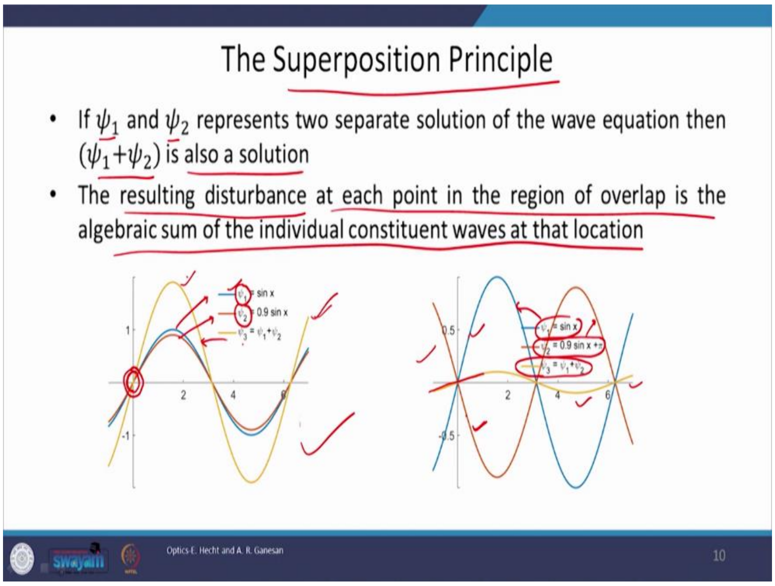

Now, suppose we have more than one waves or we have more than one wave functions which overlap in space, then what will happen? What will be the resultant of this? this would be decided by superposition principle. And this principle says that if ψ_1 (Psi1) and ψ_2 (Psi2) represents two separate solutions of the wave equation, or if ψ_1 and ψ_2 are two wave functions then $\psi_1 + \psi_2$ is also a solution of wave equation or we can say that if ψ_1 and ψ_2 are two disturbances which are overlapping in space, then the resultant disturbances would be the addition of the two and this is what is being written here the resultant disturbance. Disturbance at each point in the region of overlap is the algebraic sum of the individual constituent waves at that location.

Now, here this is what is written here is shown schematically with a blue color we show wave function ψ_1 and with the orange color we show wave function ψ_2, the wave function ψ_1 is here, wave function ψ_2 is here and with this yellow color the resultant is being expressed. Now, we see that resultant is sum of ψ_1 and ψ_2.

And we see that all these waves start, they are in phase. What I mean by saying that the waves are in phase, in phase means, the crest and trough of these wave come at the same time all these waves start from zero the initial phase is zero here and they passes through their maxima and minima simultaneously. But, if we consider the opposite case, then what will happen is shown here in this diagram again the blue curve represents the wave function ψ_1, the wave function ψ_2 is given by this color and the yellow curve color represents the resultant.

Now, here you see what you see is that the blue color and this red color is opposite in phase and therefore they cancel out and therefore, the resultant which is given by this yellow color is a very low amplitude. While here the resultant is of very high in amplitude, it means the resultant will always depend upon the relative phase of the waves which are overlapping in time or which are overlapping in space okay, this is all superposition principle tells.

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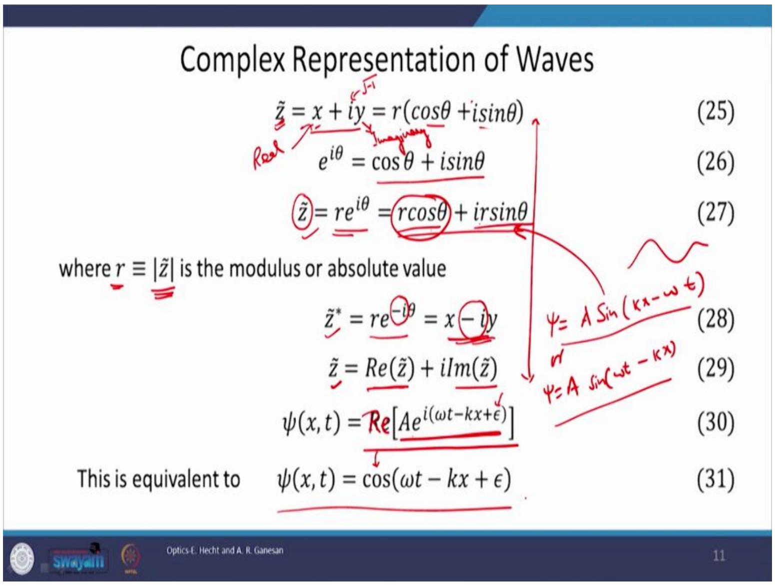

Now, it is while doing mathematics it is always very easy to move to complex domain. Therefore, in this slide we will learn how to represent a wave function in complex representation. Now, we know that any complex number can be expressed as x + iy. where x is a real part, y is imaginary part okay, and i is iota which is equal to a $\sqrt{-1}$. In polar coordinate, this can equivalently be written as $rcos(\theta) + irsin(\theta)$ okay.

And this can equivalently be written this $cos(\theta) + isin(\theta) = e^{i\theta}$, this all we know. Therefore, complex number which is expressed by

$$\tilde{z} = re^{i\theta} = rcos\theta + irsin\theta.$$

If you want to calculate the absolute value or the modulation of this complex number, then you have to take the mod and this mod will be equal to r, r is modulus of z tilde ($\tilde{z}$).

Now, how to calculate complex conjugate. Now, to calculate complex conjugate you will have to just replace iota by minus iota. Therefore, here it the in the exponent i is removed, replaced by minus i and here in this form i is again replaced by minus sign this represents the complex conjugate of $\tilde{z}$ (z tilde).

Now z tilde which is a complex number as I said before it consists of a real part and the imaginary part, the real of the z tilde is expressed as x and imaginary of z tilde is y, now, with these information, we can write our wave function in complex note form, how to write this? We know that our $\psi = Asin(kx - \omega t) = Asin(\omega t - kx)$. The minus sign will just alter the phase okay, this is a real part, if you plot it you will see some variation here, but how to go to the complex domain we will follow this, that a complex number can be represent as $rcos\theta + irsin\theta$

Then once $cos\theta$, $sin\theta$, and r is known then everything can go into a complex notation which is $re^{i\theta}$. And the real part of this complex number is r which will represent our wave function. Knowing this we can represent our wave function here.

We have $Ae^{i(\omega t-kx+\epsilon)}$which is our wave function which is written in exponent complex form. But the imaginary part of a wave function does not hold any significance. It is the real part which is of our interest. Therefore, we put Re here, the real part of $Ae^{i(\omega t-kx\pm\epsilon)}$ will represent the wave function, ϵ is some arbitrary phase. If you take the real part of $Ae^{i(\omega t-kx\pm\epsilon)}$you will get cosine function. And this is which you will get the $cos(\omega t - kx \pm \epsilon)$.

This is all for today. Now in the next class we will move further in the module one and we will learn about the ray theory and then Fermat's principle. Thank You.