Applied Optics
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Lecture 19
Young's Double Slit Experiment - II

Hello everyone, welcome to my class. Today we will move further in investigating interference.

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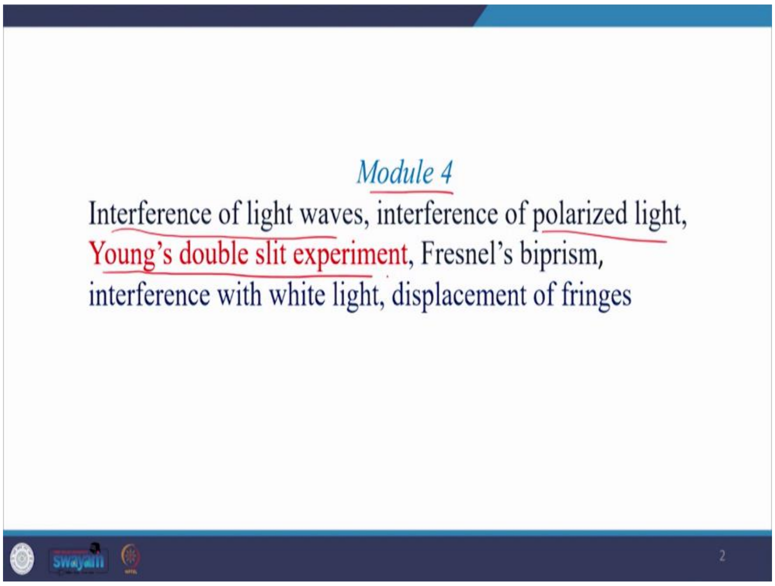

Now, we are in module 4, and we have already studied interference of light waves. We also talked about the role of polarization in the formation of interference fringes. And we also touched upon Young's double slit experiment in our last class. We studied the experimental arrangement, experimental setup and we talked about the fringe positions and then we talked about fringe width. Now, once these things are known, in today's class, we will solve a few examples.

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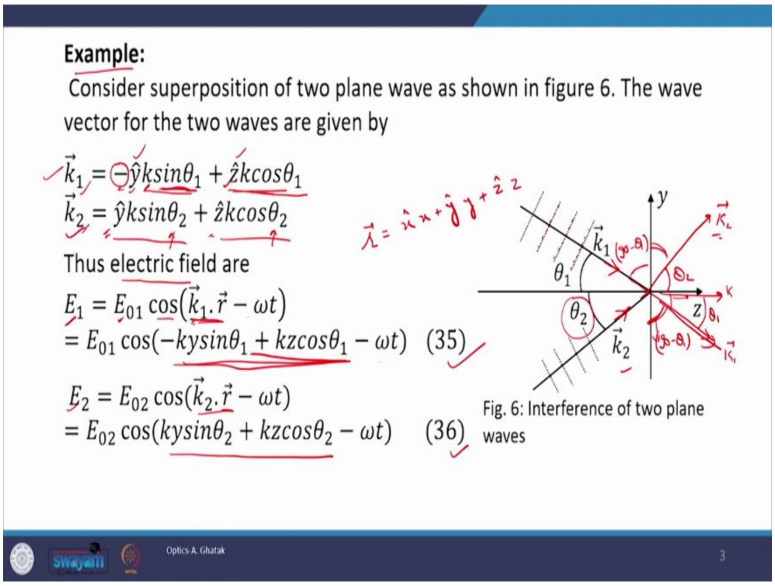

The first example is given here in this problem, we are given 2 plane waves, which are super imposing in a space and this plane waves are inclined with some angle and this angle is shown here in this figure, the $\vec{k}_1$ represents the wave vector of first wave and $\vec{k}_2$ represents the wave vector of the second wave and $\vec{k}_1$ is inclined at angle θ_1 with respect to the horizontal axis, which is z axis in our case and $\vec{k}_2$ is inclined at angle θ_2 with respect to the horizontal axis.

Now, let us figure out the expression for wave vector $\vec{k}_1$ and wave vector $\vec{k}_2$. Now, as you see in the figure, the $\vec{k}_1$ is making an angle θ_1 with z axis and therefore, $90 - \theta_1$ with y axis here this angle is $90 - \theta_1$ and this dash line they represent the wave front or phase front which is plane in our case. Therefore, they are represented by dashed straight line. Now, let us go into the expression of $\vec{k}_1$ now, $\vec{k}_1$ is expressed as $ksin\theta_1(-\hat{y})$, $\hat{y}$ is unit vector along y axis.

Now, you see that $\vec{k}$ is directing in this direction. Since $\vec{k}$ is pointing in this direction and in this direction z axis is positive while y axis is negative it is going down. Therefore, there is a negative sign here for $\hat{y}$ and θ_1 is the angle which the ray makes with the horizontal z axis as stated before and since, we are now finding the component of the wave vector along y axis therefore, this angle would be of importance and this is our $90 - \theta_1$.

Therefore, $cos(90 - \theta_1) = sin\theta_1$. Therefore, we have $sin\theta_1$ here, k is the magnitude of the wave vector and ŷ is the unit vector and minus sign represents that y is in opposite direction.

Similarly, in the z component of the vector $\vec{k}_1$ is expressed by the second term where $\hat{z}$ is the unit vector along the x axis and $k_1cos\theta_1$ is the component of this vector along z axis. It is k_1

here and this angle is θ_1 therefore, the component would be $k_1 cos\theta_1$, and here we can take mod of k_1 that is equal to k and $\hat{z}$ represents the unit vector along z.

Similarly, we can express the wave vector $\vec{k}_2$ in its component form which is shown here. Now, instead of θ_1, it is θ_2 here because the second wave is inclined at angle θ_2 with respect to the horizontal and now you can see that $\vec{k}_2$ is going in this direction. Since $\vec{k}_2$ is in this direction, this is your θ_2, this is your $90 - \theta_2$ and for $\vec{k}_2$ in its component form, both the components are aligned along x sorry, along y and along z which are in positive y and z direction and therefore, we do not have minus sign before $\hat{y}$ or before $\hat{z}$.

Now, once we have the expression for wave vector $\vec{k}_1$ and wave vector $\vec{k}_2$, we can write the corresponding expression for electric fields. Now, for the first wave electric field can be written as $\vec{E}_1 = E_{01}$, E_{01} is the amplitude of the field and then the phase part $cos(\vec{k}_1.\vec{r} - \omega t)$.

Let us, write $\vec{k}_1$ in its component form and then this is our $\vec{k}_1$, this is the first expression. Now, similarly, we can write the expression for $\vec{E}_2$ and $\vec{E}_2 = E_{02} cos(\vec{k}_2.\vec{r} - \omega t)$ here. And now you see that once you take the dot product of $\vec{k}_1$ with vector $\vec{r}$ then the unit vectors are gone. Here which is obvious.

Because r is a vector which also have its component here it has x component, y component, z component and if you write this vector in its component form then you can write $r = x\hat{x} + y\hat{y} + z\hat{z}$ here, if you write it in this form and then take the dot product $\vec{k}_1.\vec{r}$ then you will get this expression.

Similarly, for $\vec{k}_2.\vec{r}$ you get this expression here. Therefore, we have the expression for electric field of both the waves as $\vec{E}_1$ and $\vec{E}_2$ which are given by equation 35 and 36 respectively. Once the expressions for electric fields are known then to have the resultant expression but we will have to do is that we will just sum them up.

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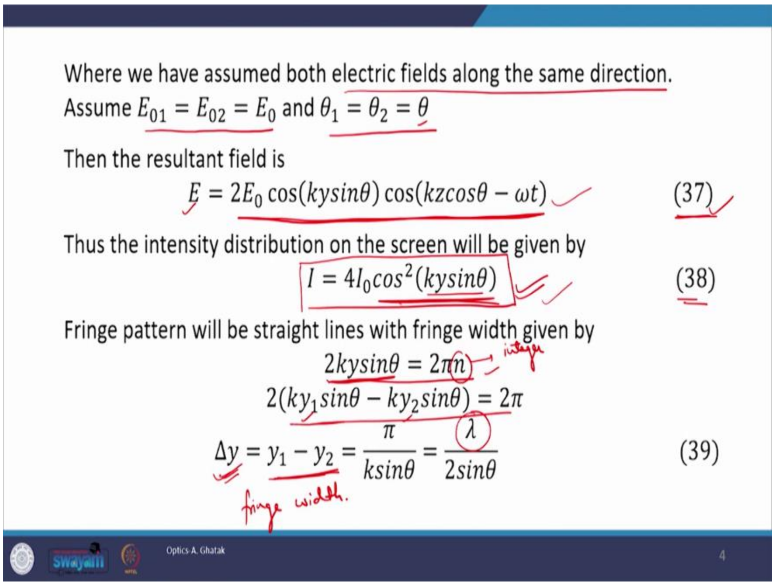

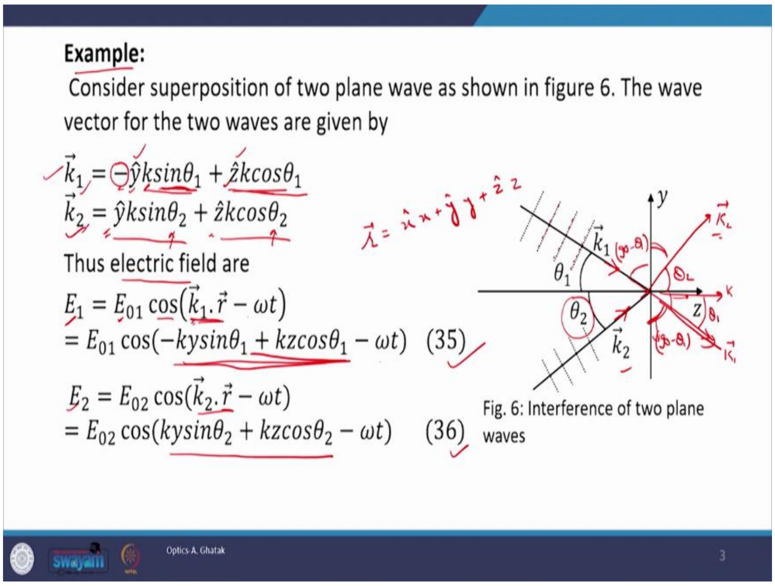

But before that let us assume our let us simplify our problem and for this simplification, we assume that the field amplitude the field magnitudes of the two waves are the same and which is equal to E_0 and the angle of inclination θ_1 and θ_2 is also same and we assumed the two to be equal toθ and we are also assumed that the electric field are along the same direction, they are the state of polarization is same in both the waves.

Under these assumptions, our problem become very simple and we can easily calculate the resultant field, how to calculate the resultant field? Add the 2 vectors or add the 2 field, and what are the 2 fields? $\vec{E}_1$ and $\vec{E}_2$. Therefore, the resultant $\vec{E} = \vec{E}_1 + \vec{E}_2$ and if we perform some simple mathematics then this is the final expression which we would be left with and the final the resultant disturbance, or a resultant field would be given by equation number 37 here.

Once the resultant disturbance is known, we can quickly calculate irradiance, how to calculate the irradiance? Take the time average of E^2 and this is what is done here and the irradiance are the intensity on the screen is given by equation number 38, this is the final intensity. Once the intensity is known, then we can calculate. Once the intensity distribution is given, then we know the fluctuations in intensity. We know the places where intensity is maximum and we know the places where intensity is minimum and thus forming the fringe pattern. Once fringe pattern is known, we can calculate the fringe width here.

Now, how to calculate the fringe width? We know that if you plot it you will get straight line fringes on the screen, for maxima this condition holds good. Because you know that this expression is nothing but $I = 4I_0 cos^2\delta/2$. Where δ is your phase difference. Now, $kysin\theta = \delta/2$ therefore, $\delta = 2kysin\theta$ and for maxima this phase must be equal to integral multiple (n) of 2π. Here n is nothing but an integer.

Now, from here since we know the condition of maxima. Now, we will pick 2 values of y on the screen y_1 and y_2 which represents the position of 2 consecutive maxima. Once the position of 2 consecutive maxima are known, we can subtract that and find out the fringe width here, this is what is done here. This we know from here we can calculate $y_1 - y_2$, which is nothing but your fringe width here, this is your fringe width. This we have already done in Young's double slit experiment therefore, I am not going too much detail.

Now, once $y_1 - y_2$ is known, fringe width is known, and therefore, what we saw is that, given the direction of wave propagation, we can calculate the expression for field by summing them up, we can calculate the total field distribution, from there we can calculate irradiance distribution and from there we can calculate fringe width and fringe width is proportional to λ. Larger λ is larger would be the width of the fringe. This is all about problem number 1.

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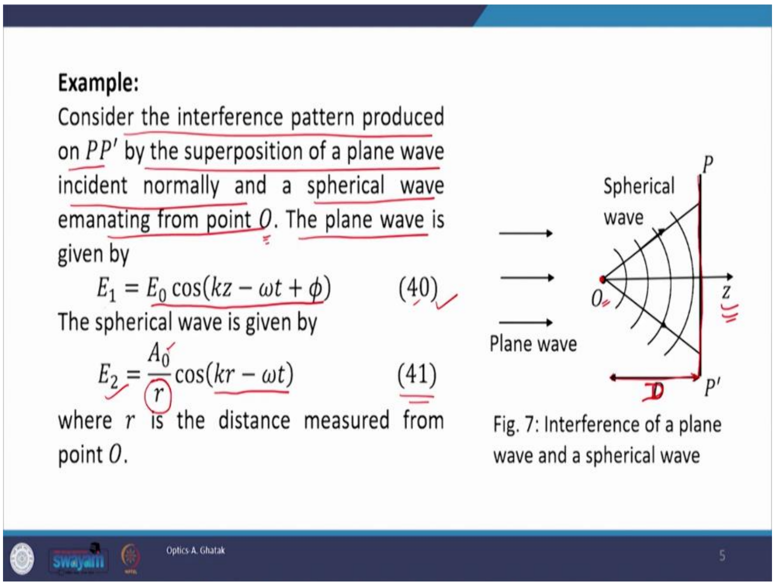

Now, we will move to second problem. The second problem is that we have a screen PP' which is represented here by this vertical line. Now, a plane wave is propagating normally towards the screen PP' and there is a point source O which is kept here and from this point source spherical waves are getting generated and they are also propagating towards the screen. The question is when the plane wave interfere with this spherical wave then what type of fringe pattern will be generated on the screen.

Now, as a solution now, consider the interference pattern produced on PP' by the superposition of plane wave incident normally and a spherical wave emanating from point source O, this is what is given. Now, to find out the expression for the resultant distribution or to find out the intensity distribution of the resultant disturbance, what we will have to do? We will first have to write the expression of electric field for plane wave and then write the expression for electric field for a spherical wave and then sum them up and from this resultant field we will have to calculate the irradiance.

And then we will plot the irradiance to see the fringe pattern or the type of the fringes which the observable will observe on the screen PP'. We all know that the plane wave is given by this expression here $E_0 cos(kz - \omega t + \varphi)$, φ is initial phase and k is the wave vector or wave number and z is the direction of the propagation, this is given here, z is the axis which is pointing towards the screen PP' and is normal to the plane of the screen.

Therefore, equation number 40 represents an expression for electric field of a plane wave. Once the expression of plane wave is known, now, our next target is to find out the expression for a spherical wave. We have already studied about this spherical wave and the expression for electric field for a spherical wave is given by equation number 41.

And here you see that in the denominator we have r, A_0 is the magnitude or amplitude of the wave, r is sitting in the denominator and in the phase part we have $(kr - \omega t)$. It is not kz but it is kr now, here it is because spherical wave propagate in all possible directions. Now, r is measured from origin or source O.

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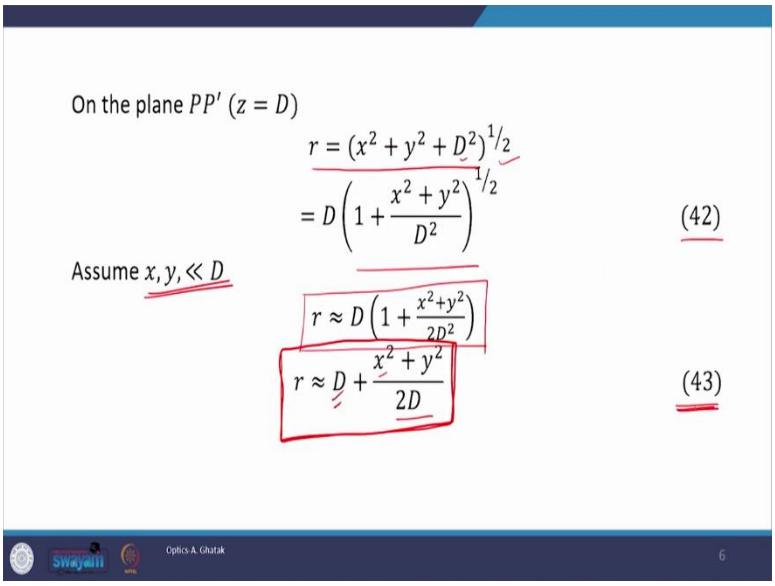

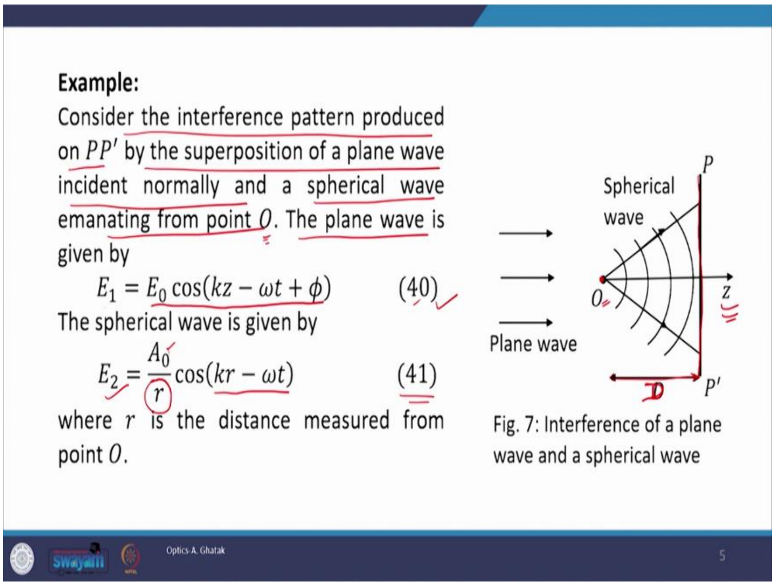

Now, once we have the expression for field, we will do some mathematics to simplify things to write to express r in terms of x and y. Because the plane wave is written in terms of z, we will also try to reduce the expression for the electric field of a spherical wave in Cartesian coordinate system.

Now, r can be expressed by this relation, $r = \sqrt{x^2 + y^2 + D^2}$. What is D then? D is the distance between point source O and the screen, as shown here in this figure number 7. Once D is known, it means the value of z where our screen is kept is D. And we know that $r = \sqrt{x^2 + y^2 + z^2}$ and at place of z, we will write D here.

And this D is very used as compared to x and y. If you remember, then we always try to observe interference fringes close to the centre of the fringe sorry centre of the observation plane. Now, in this expression of r, we will take D out and this is the final expression of r which we are left with.

Now, applying this condition which says that D is much much larger than x and y. In this particular case, we can do this binomial expression here and neglect higher order term because D is a very large quantity and therefore, the next order additive term would be very small and we can safely neglect them and the final expression for r in this particular case is given by equation number 43. Now, let us substitute the expression of r into the expression of field of spherical wave.

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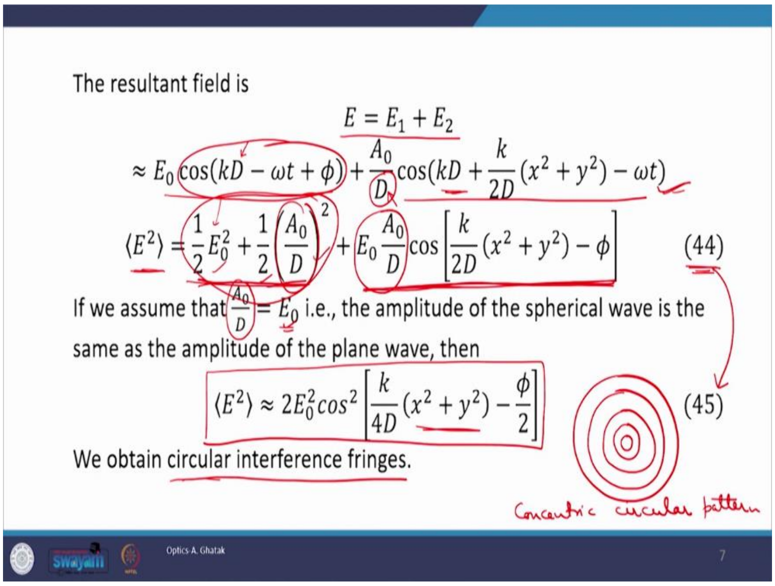

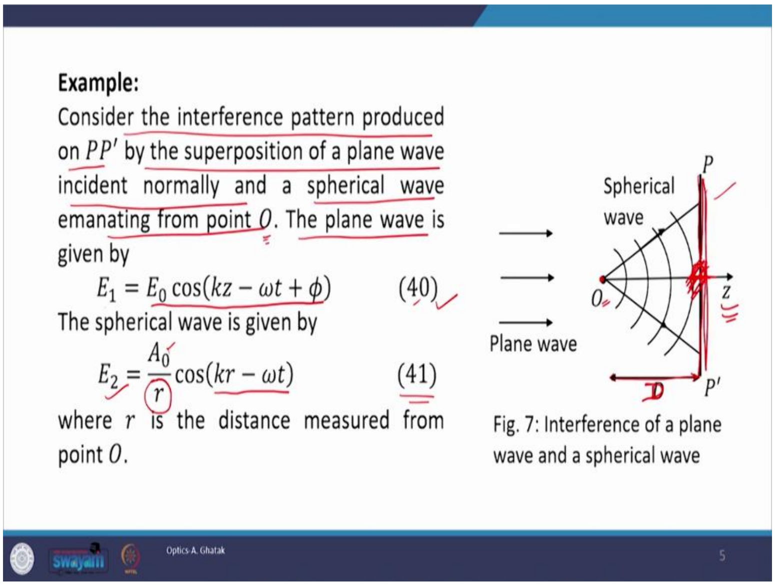

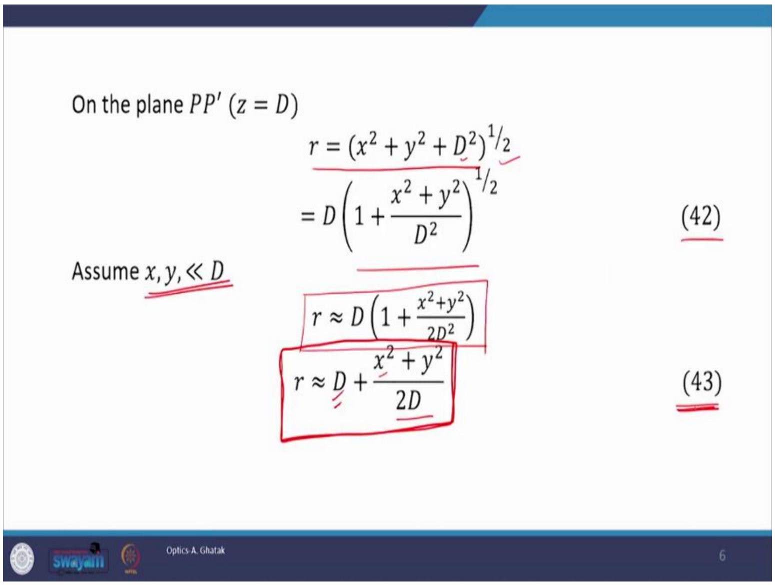

Here it is the resultant field, the expression of E_1 is given by this relation where we have replaced z by D, again here r is replaced by D here because we are interested in screen and particularly in the region of the screen which is close to the centre point of the screen means this region we are interested in.

Therefore, r here is replaced by D in this expression. In the phase part r is replaced by this equation number 43 and therefore, if you square this equation and take the time average then we will get equation number 44. Because the time average of square of $cos\theta$ is 1/2, therefore, here we are getting $E_0^2/2$.

Similarly, this term time average of $cos^2\theta$ is 1/2. Therefore, we get $E_0^2/D^2 2,$ this is correct. Now, we will have a cross term also here because it is (a+b) any if you are squaring it then you will get a^2+b^2+2ab, 2ab information is here this is the cross term.

And then if you take the time average of this term, then the resultant expression looks like this. This is the final irradiance at the screen PP'. Now, let us further simplify this equation number 44 by assuming that A_0/D which is nothing but magnitude of a spherical wave at the screen is equal to E_0.

We are assuming that the magnitude of electric field and magnitude of electric field of spherical wave and the magnitude of electric field of the plane wave are the same at the observation screen which is PP' in our case here, which are shown here in this figure here, PP' is this screen. Under this assumption, what we can do is that we can replace A_0/D with E_0, with this equation number 44 reduces to equation number 45. Because this is again equal to E_0^2 and therefore, the first 2 terms would be equal to E_0^2. Because the first one is $E_0^2/2,$ second is also $E_0^2/2$.

Therefore, these 2 term collectively would be equal to E_0^2, here too these two terms would be equal to E_0^2 and what is left is $cos[k/2D\ \ (x^2+y^2)-\varphi]$. Just by looking at this expression, you can see that this is symmetric in x^2+y^2, it is the forming a circle here in the phase part.

Therefore, we will obtain a circular interference fringes, just by looking at equation number 45 we can guess this. If you vary an x and y then you will rotated on a circle. Therefore, if you interfere plane wave front with a spherical wave front then on the screen we will have concentric circular fringe pattern, which would look like this, this is concentric circular fringe pattern. Now, this is all about problem number 2.

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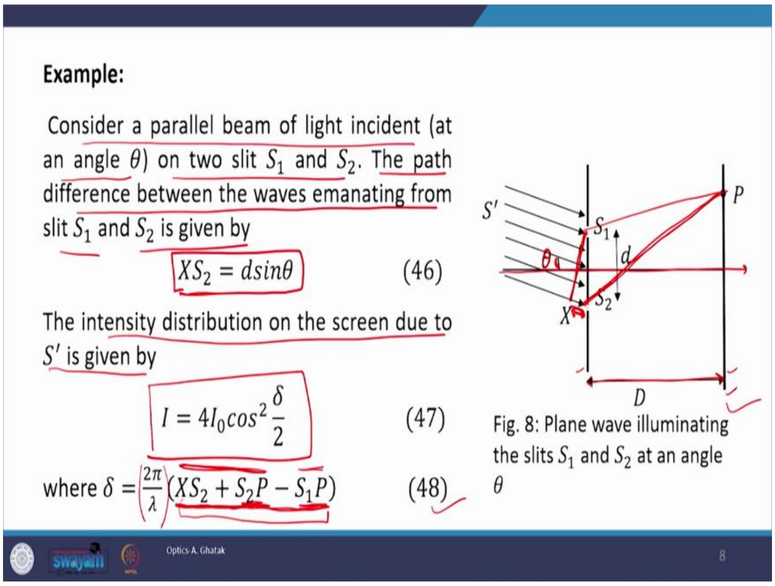

Now, we will move to our next problem and the description of next problem is as follows. We have 2 slits, which are S_1 and S_2 and a plane wave is make to incident on this slit at an angle θ with respect to the horizontal line and this is the horizontal axis and with this axis the plane waves make an angle θ.

And now, we are observing the fringe pattern at the screen which is at a distance capital D from the source plane. The plane in which sources S_1 and S_2 are called source plane and the plane in which we are observing the fringes is called observation plane. The source plane is at a distance D from the observation plane or plane of observation.

Now, consider a parallel beam of light incident at an angle θ on 2 slit S_1 and S_2. Now, since the light is not being launched perpendicular to the source plane therefore, there would be a path difference between the lights which are falling at point source S_1 and the light which is falling at point source S_2.

How to calculate this path difference? We will draw a perpendicular from S_1 to the ray which is falling to S_2. And the length XS_2 is the extra path which the light ray travels when it falls on sources S_1 and S_2. The light which is falling at source S_2 is traveling an extra path of XS_2 as compared to the light which is falling at source S_1.

Therefore, the path difference between the waves emanating from slit S_1 and S_2 is given by XS_2 and which is equal to $dsin\theta$, what is d? d is the separation between the two sources S_1 and S_1 and θ is the angle which the incident light makes with the horizontal. Now, how to calculate the intensity distribution?

The intensity distribution on the screen due to S', S' is some source which is very far from S_1, S_2 and when the light travels this distance it becomes almost plane wave and this plane wave is falling at some angular with respect to the horizontal. Now, we know that the intensity distribution is given by equation number 47, $I = 4I_0 cos^2 \delta/2$. Our task is to calculate or to find the expression of phase δ.

Now, how to calculate δ? To calculate δ or to calculate the phase difference we have to find the path difference through this schematic diagram, once the path difference is known just multiplied it with k, what is k? k is nothing but $2\pi/\lambda$. It means that this is our path difference. In traditional Young's double slit experiment path difference is equal to S_2P $-S_1$P. We draw a line from S_2 to P and from S_1 to P and we take the difference between these 2 paths and this difference gives us the path difference between the 2 ray paths.

But in the present case when the ray emanates from the 2 sources it is having a priori path difference which is XS_2, which is d$sin\theta$. Therefore, the S_2P path will have an additional path difference which is X_2S, therefore, to calculate the final path difference we have to sum these two quantities. What are these two quantities? The first is XS_2 which is the extra path which light ray trivial when it reaches to S_2 and then S_2P and therefore, equation number 48 gives us the phase difference.

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$$\delta = \frac{2\pi}{\lambda}[(S_2P - S_1P) + dsin\theta] \qquad (49)$$

$$= \frac{2\pi}{\lambda}\left[\frac{yd}{D} + dsin\theta\right] \qquad (50)$$

Thus the intensity distribution is given by

$$I' = 4I_0 cos^2\left[\frac{\pi}{\lambda}\left(\frac{yd}{D} + dsin\theta\right)\right] \qquad (51)$$

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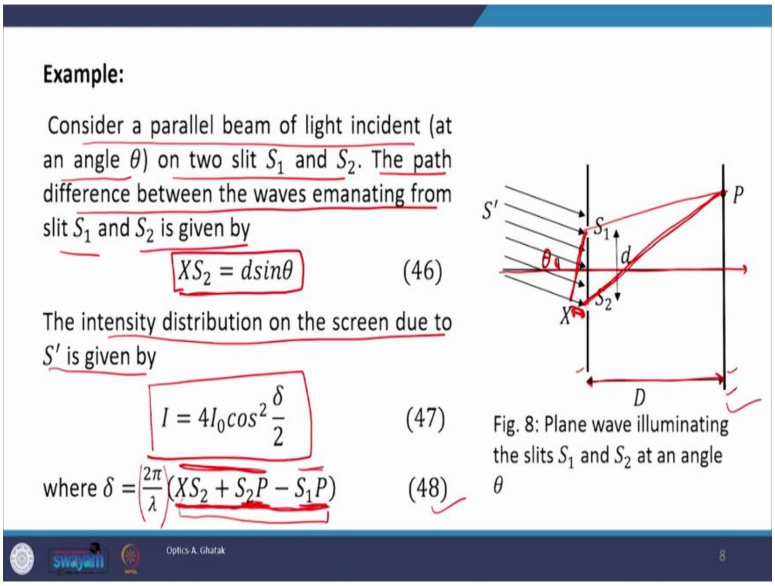

Now, let us substitute this the phase difference δ is equal to $2\pi/\lambda$ S_2P $-S_1$P and d is your XS_2, this is the expression for XS_2 which is d $sin\theta$. But we know from Young's double slit experiment S_2P $-S_1$P is equal to wide small d/D, d is the separation between the 2 sources and D is the separation between the source plane and observation plane, this we have already derived in Young's double slit experiment.

Once these phase difference is known, you can easily calculate the intensity distribution. Then we will substitute equation number 50 back to equation number 47 and this expression gives us the irradians or intensity distribution at the screen and this term you see there is this additional phase term due to the inclination of the phase front, due to the inclination of the plane wave which falls on the sources screen, this is all for today. And thank you. See you all in next class.