Applied Optics
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Lecture 18
Young's Double Slit Experiment 1

Hello everyone, welcome back to my class and today we will talk about the Young's double slit experiment.

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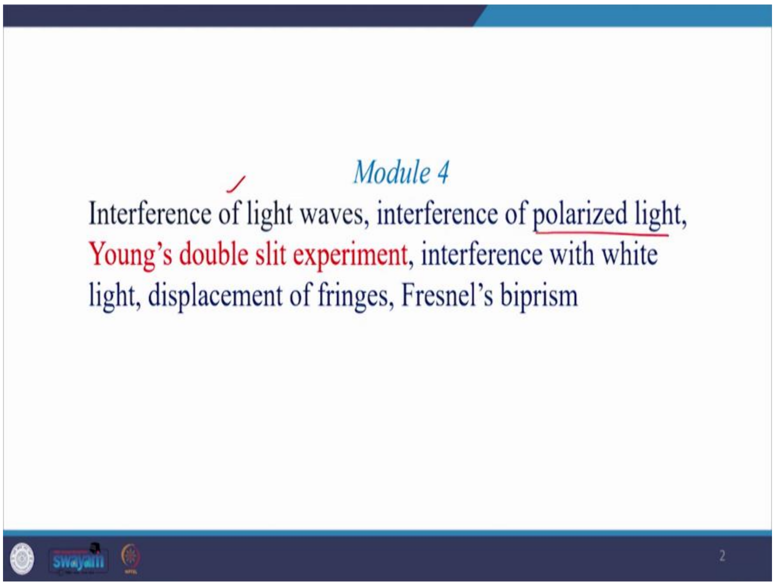

We are in module 4 and we have already discussed interference of light waves and the role of polarization in observing interference fringes is also discussed. And, let us start today the Young's double slit experiment.

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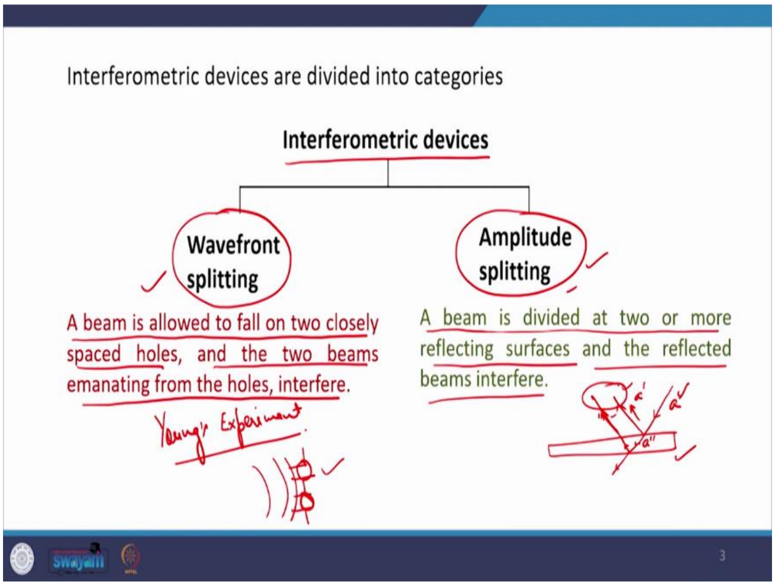

But before moving ahead, let us talk a bit about interferometric devices which people usually use in the labs and to absorb interference we use different devices like Young's used two slits, we in the last two lectures we use two point sources. And they are the part of some experiment and therefore, it is called interferometric devices or device which let us observe a particular type of fringe pattern.

Now, these interferometric devices can be categorized into separate categories. Now, why we are separating them in two categories because there working principle is slightly different. Now, one set of these interferometry devices work on wavefront splitting and the second set work on amplitude splitting. The first set produces interference by splitting the wavefront and then again letting them superimposed. While the second set splits the amplitudes of the incoming wave and these splitted amplitude are then led to overlap, these are later met to overlap in space and thereafter they produce interference fringes.

Now, what are wavefront splitting? A beam is allowed to fall on two closely spaced holes and two beams emanating from the holes interfere, this is how wavefront splitting interferometry devices are defined. Now, the best example of this kind of interferometric devices is our Young's experiment, this is the best example for these type of interferometric devices. Now, in amplitude splitting a beam is divided at two or more reflecting surfaces and the reflected beams interfere.

Now, the best example here is our soap film or thin films. Suppose, we have a thin glass film and then a light falls on this film, a part to get reflected, a part get transmitted, the transmitted part again get reflected back and then it comes out and then it interfere with the initially reflected wave, this is the other way of observing interference and such kind of devices which produces this type of interference is called amplitude splitting device.

Now, here you see amplitude a is falling from top a portion of this amplitude is say a' is getting reflected and the rest a'' is transmitted and this a'' again a part get reflected and again get transmitted and therefore, the interference is happening between two different amplitudes of the same incoming wave.

While in Young's double slit experiment, we launch a wave, and traditionally it is spherical wavefront and then there is openings at these holes, they allow only a part of the wavefront, it means the wavefront is getting splitted here in this device, and therefore, this is called wavefront splitting interferometer.

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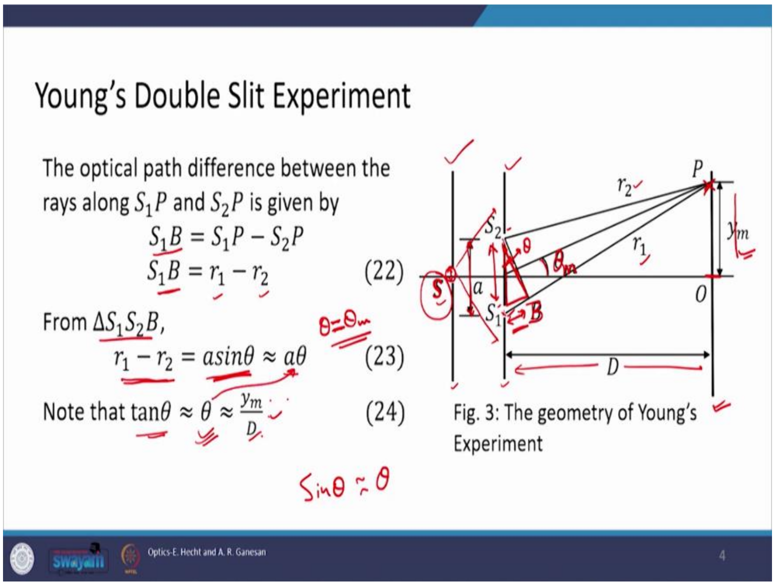

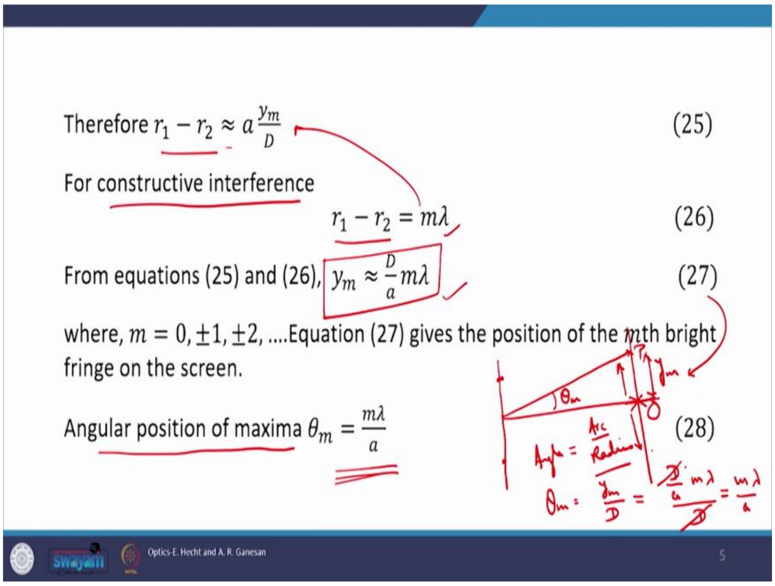

Now, once we have defined these two terminologies let us move ahead to our Young's double slit experiment which is of course a wavefront splitting interferometer. Now, we know that the interference requires coherent wave, once we have a coherent wave then we would be able to produce very nice and beautiful fringes, but, at the time of Young's lasers were not invented.

And since the lasers were not available, then people were not knowing how to perform these experiments with incoherent source. What Young's did is that he took a very small pinhole and put it in front of sun and if the whole dimension is very small, then the light which is coming out of this hole would be spatially coherent.

And, if it spread a lot then it will cover a larger region what Young's did is that he put another screen with two holes S_1 and S_2 again a very small and they spread in the wave which is emanating from source S, or whole S is such that it covers the two slits S_1 and S_2, under this conditions the Young was able to get very nice spatially coherent light. And, when these waves are interfere on the screen, he observed a very nice beautiful fringe pattern. Now, let us do a bit of mathematics and let us try to find out the condition of interference the position of maxima and minima the fringe width.

Now, this was the first plane in which small hole was made and this whole small hole works as a source S. Now, next to this first plane a second screen was kept with the two holes which are named as S_1 and S_2 and these two holes are separated by a distance a, this is a schematic picture of Young's experiment, the light coming from source S falls on slit S_1 and S_2 and from here they propagate in the right direction and then they overlap in the space, the screen is placed at a distance D from S_1S_2 screen.

Now, the point of observation is at observation plane. Now, suppose the distance between source S_1 and P is r_1. While the distance between source S_2, S_2P is r_2. Now, if we want to calculate the path difference of wave which are interfering at point P and starting from source S_1 and S_2 respectively then what we will have to do is that we will have to draw a perpendicular from S_2 and S_1P and this S_2B is the perpendicular.

Now, the path difference would be S_1B. Now, since S_1B is the path difference than this would be equal to $(r_1 - r_2)$, under the approximation that P is very close to O. And S_1S_2 is also very close, the two approximations which we assume here in this experiment is that S_1S_2 is very close that is a is very small and D which is the separation between the source plane and the screen is very large, under these approximation, the angles which are substandard by the S_1P andS_2 P, this can be assumed to be equal and for simplicity we assume it to be equal to θ_m, m stands for m^{th} fringe.

Here we are assuming that m^{th} fringe is being found at the point of observation P which is at a distance y_m from the center O, center of the screen. Now, if we look into the triangle S_1 S_2B, this triangle, then $(r_1 - r_2)$ which is S_1B will be equal to a sinθ, this is your θ, here $\theta = \theta_m$,, m stands for mth fringe. I already told you, m stands for m^{th} fringe and if we do not want to talk about or if we do not want to take into account the fringe number right now, then we can assume that $\theta = \theta_m$ here.

Now, S_1B from this figure, since S_1 S_2 is a and this angle is θ, rθ_m then under the approximation that capital D is very large and small a is very small, $asin\theta \sim a\theta$ because $sin\theta \sim \theta$. Now, let us calculate $tan\theta$, $tan\theta = y_m/D$. Because $tan\theta \sim \theta$ because θ is very small, why θ is very small? because the screen is placed at a very large distance from the source wave.

Now, what does θ represent? θ represents the angular position of a fringe m^{th} fringe which is at a distance y_m from point O and D is the separation between source plane and observation plane.

Now, therefore, once r_1, r_2 is given as $a\theta$ and $\theta = y_m/D$ then we can substitute the expression of θ from here to here then this gives r_1 and, $r_1 - r_2 = ay_m/D$. And, what is $r_1 - r_2$? $r_1 - r_2$ is optical path length difference.

Now, let us implement the conditions of maxima and minima for constructive interference, constructive means for maxima $r_1 - r_2$ the optical path length difference must be integral multiple of λ we have already talked about it. Now, let us substitute $r_1 - r_2 = m\lambda$ and see what happens, here we are substituting it here in equation number 25 and from here we get $y_m \approx Dm\lambda/$a,and what is y_m?, y_m is the position of m^{th} fringe from the center of observation plane.

Once we have this position, then we can calculate the angular position of maxima, we have two sources which are in a plane or along a line and here we have a point of observation P once we know this distance which is y_m which is given by equation number 27. Then we can also calculate the angular position of m^{th} fringe.

How to calculate angular position? Because we know angle is equal to arc by radius or you can also exercise the $tan\theta$ formula and from there this angle $\theta_m = y_m/D$ and what is y_m? y_m from equation number 27. It is $Dm\lambda/aD$ and D and D will go away we are left with $m\lambda/a$ and this is what written here and in equation number 28.

This gives us the angular position of the m^{th} maxima or m^{th} bright fringe, m is integer, which stand for the counting the number of bright fringe. Again the fringes are being counted from the center on the screen here the point O is sitting here, the zero fringe is here. All positive fringes are above O, all negative fringes are below O, we can also treat the fringes below O as positive fringes and fringes above O are negative fringes it is entirely up to us.

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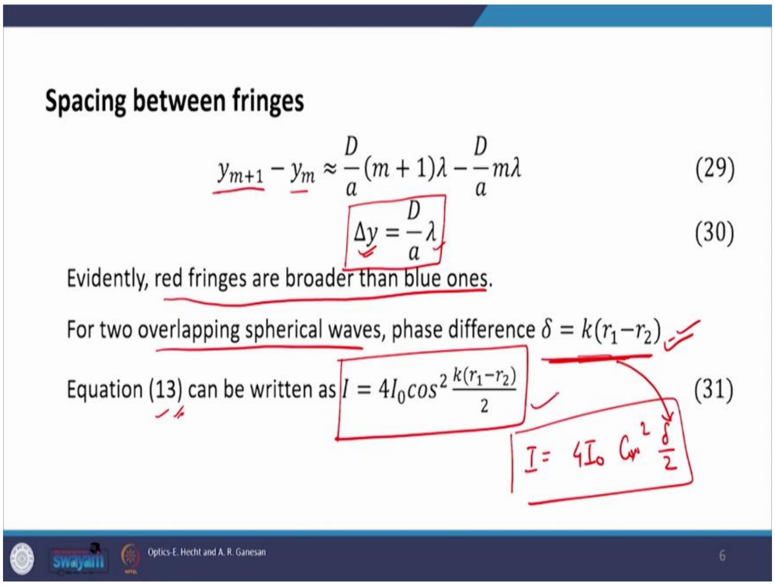

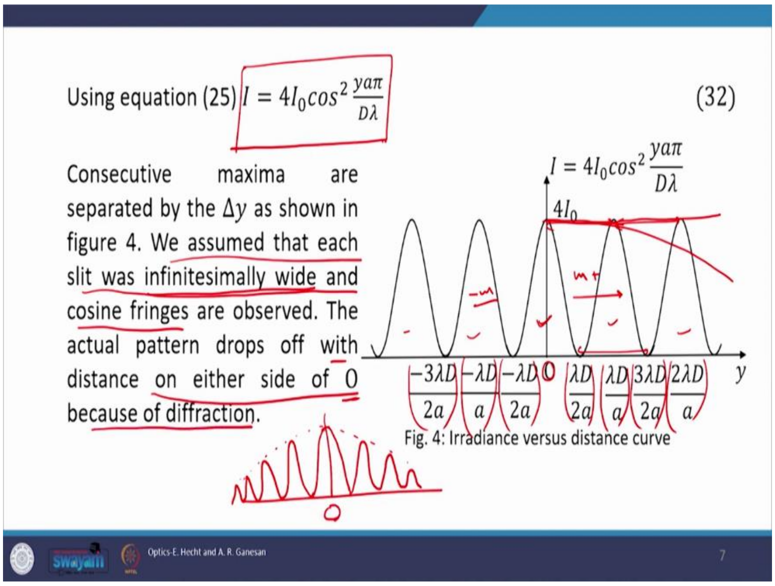

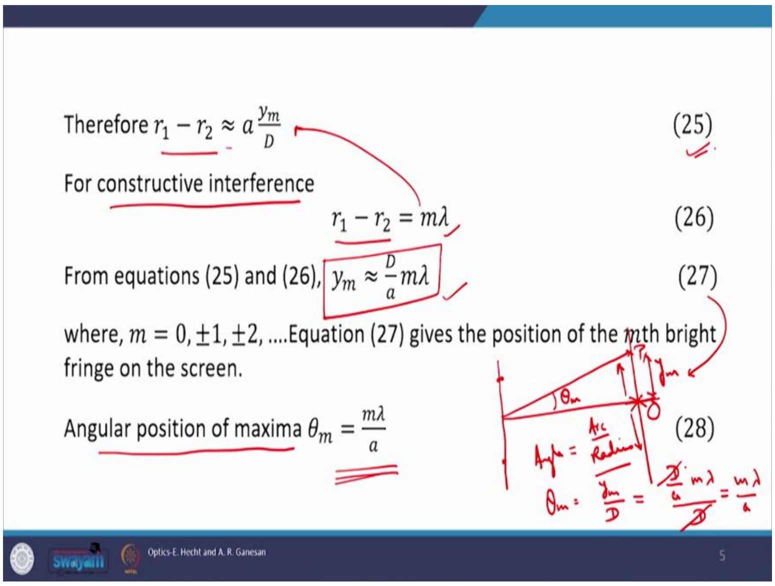

Now, once the fringe positions are known, let us calculate the fringe width or spacing between the fringes, how to calculate this. We know the expression for m^{th} fringe and therefore, we also know the position for $(m+1)^{th}$ fringe. Let us take the difference and this will give us the width of the fringe or fringe spacing, we subtracted $y_m$from y_m +1 and this will give Δy which is fringe width, which is equal to $D\lambda/a$, this fringe width is function of wavelength.

Larger the wavelength larger would be the fringe width and this is why red fringes are broader than the blue ones. Now, we know that for two is overlapping spherical waves the phase difference is given by this relation, this we have already seen in the spherical wavefronts where mid to overlap and then we calculated the phase difference in the case when where the initial phase difference was 0, this is the phase which is equal to $k(r_1 - r_2)$, the same can be exercised here also.

Now, we will substitute this phase expression into equation number 13 which we derived earlier and this will give us the irradiance formula, the expression for irradiance which is $4I_0 cos^2(k(r_1 - r_2)/2)$. If you do not remember what was there in equation number 13. Then for you I can write it here, it was this expression this was the equation number 13. And here we substituted further and this gives equation number 31.

Now in equation 31. We know what is the value of $(r_1 - r_2)$, we calculated it here in equation number 25. We calculated the value of $(r_1 - r_2)$ we will substitute it back and this gives the expression of irradiance which we will observe at the screen in Young's double slit experiment.

Now, we can plot I as a function of y, the position then at the center we get a maxima and then at this value of y we get minimum and then again at this value of y we get next maximum, at this value of y next minimum, next maximum and similarly, for the corresponding negative values we get maxima and minima alternatively on both sides of the central maxima, this is our central maximum and these are the different orders, m equal to plus is on right hand side plus m and minus m are here.

Now, we see that the fringe widths are same here, the peak to peak separation or deep to deep separation is the same and we assume that each slit was infinitesimally wide which means that each slit is considered to be of zero thickness, we have a point source and we kept point source along a line and this point source created a one dimensional object which has a length but no width and therefore it has infinitesimally wide width and this infinitesimal width therefore leads to cosine fringes or periodic variation in the intensity and this, the periodicity, are ideal and there is no change.

But in actual case, we cannot form a slit with zero width. In reality in real life, all the slits will have certain width and if you take the width into account, then the intensity of these peaks which remains constant will drop down.

Once we consider non-zero width of slit in Young's double slit experiment, then intensity will slowly drop down. And the resultant fringe pattern will now look like this. Because the central maxima and for guidance, I am drawing this dotted line, the next maxima would be a bit smaller the next a bit smaller than next more. This is how the intensity will go down here. This is due to diffraction, the actual pattern drops off with a distance on either side of O, this is O. And this is because of diffraction. This is because of the width of the slit. Now, this is all in Young's double slit experiment.

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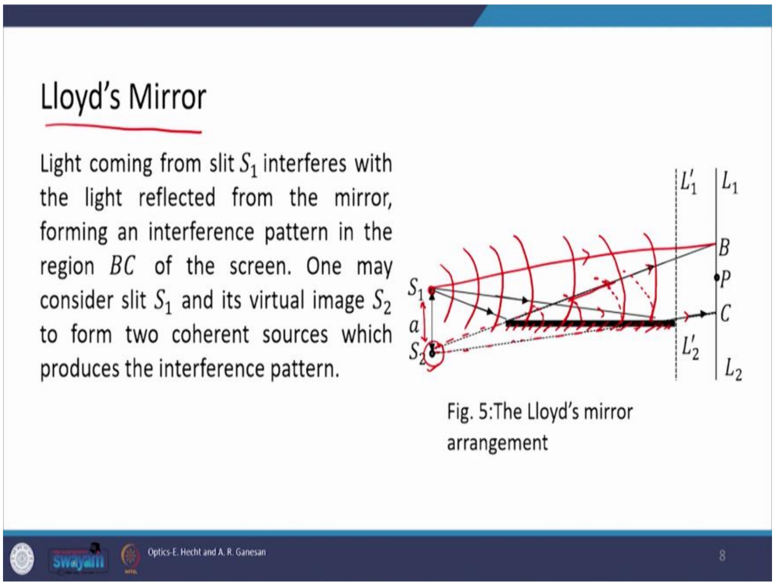

Now, let us again go to next arrangement and this is another type of interferometric setup which is called Lloyd's mirror. In this case, we have a mirror, this dark line represents a mirror and a light from source S_1 falls on this mirror and then it gets reflected here. And this reflected light interfere with the direct light which is coming from the source, we know that the source S_1 is the point source therefore, it would emit spherical wavefront.

Now, let us draw this a spherical wavefront. Now, here at the mirror what will happen this is sphere now collide with the mirror and a part of this sphere will now get reflected and it will generate a reflected spherical wavefront. This reflected spherical wavefront will interfere with the directly incident spherical wavefront, this arrangement can be understood by assuming a virtual image of S_1, as S_2 the light is falling on the mirror and then it is getting reflected. If we trace these lines back then they joins at point S_2 which we assume as virtual image of S_1 or S_2 can be treated as virtual source and the separation between S_1 and S_2 is a.

Since, the interference here is happening between direct light and the reflected light this interference is slightly different from what we observe in Young's double slit experiment, what is the difference here. After reflection there is a phase shift which the reflected light will bear. The light which is falling on the mirror after getting reflected, it will carry an additional phase ofπ , that reflected light will get shifted by a phase of π and due to this additional phase the conditions of maxima and minima will alter, they would actually be exchanged the integral multiple wavelengths will lead to dark fringes.

Therefore, contrary to Young's double slit fringe pattern, the dark here would be treated as bright or at the position of dark fringe we will get bright fringe while at the position of bright fringe we will get dark fringe.

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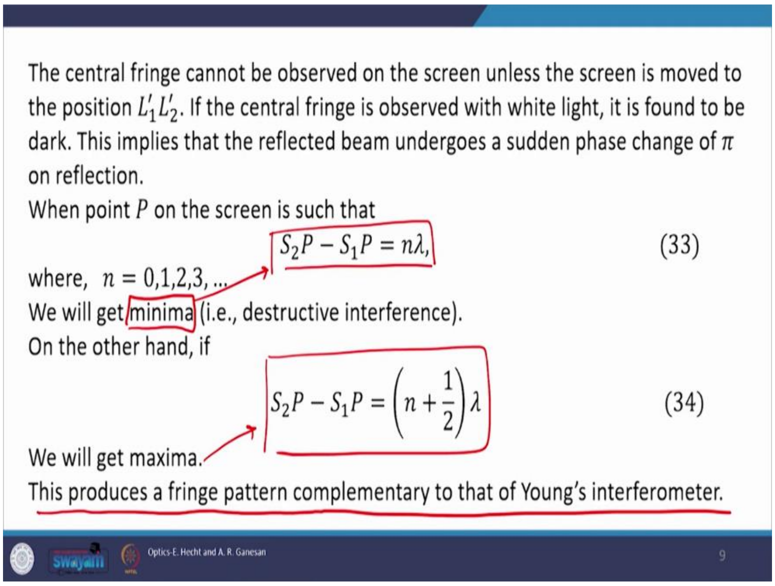

And therefore, the condition $S_2P - S_1P$ which is integral multiple of λ, this is condition of minima. Similarly, this condition is for maxima and therefore, this experimental arrangement produces a fringe pattern which is complementary to that of Young's interferometer. This is all for today and thank you for hearing me.