Applied Optics
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Lecture – 16
Interference of Light Waves, Interference of Polarised Light - I

Hello everyone, welcome again in my class. Today, we will start module number 4. In last lecture, we talked about elliptically polarized light, the module 3 covered whole polarization. Now in today's class, we will be formally introduced to interference.

Now in this module 4, interference of light waves will be discussed and then we will talk about interference of polarized light. There is a small application of polarization while discussing interference and therefore we covered polarization before interference.

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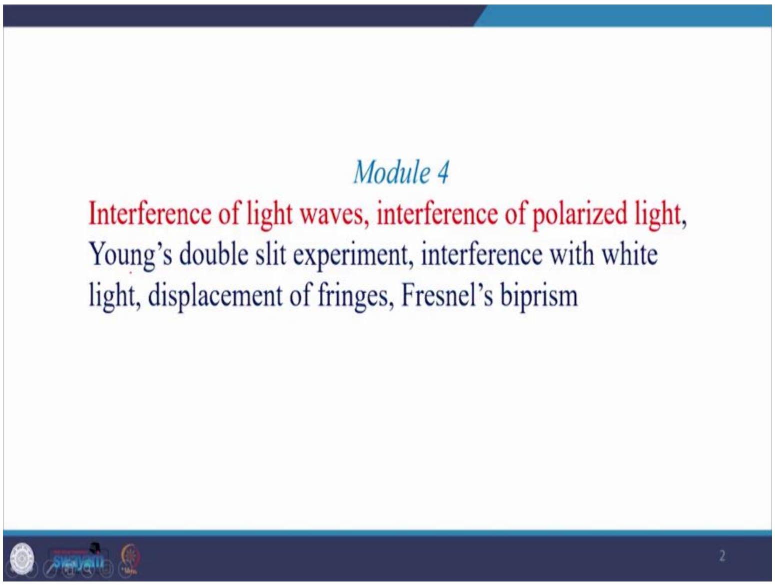

Then we will talk about the famous Young's double slit experiment. Then instead of using the coherent light sources, we will use white light and see how does it affect the fringe pattern of Young's double slit experiment and then we will introduce a thin film or we will cover one slit using a very thin transparent film and then see the changes in the interference fringes. And we will try to calculate either refractive index or thickness of the film using Young's double slit experiment. And in the last we will talk about Fresnel's biprism.

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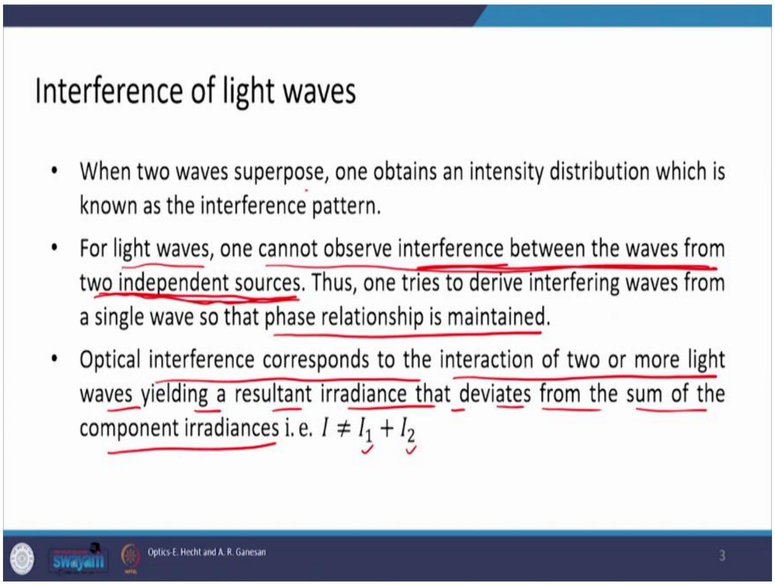

Today we will start formerly with the interference of light waves. We know that when two waves superpose one obtains an intensity distribution which is known as interference fringes. Whenever we talk about fringes we mean that there is a distribution of intensity, we launch to light this superimpose and they automatically create some pattern and this is called fringe pattern.

We see their maxima minima, maxima minima and this particular type of fringe pattern depends upon the wavelength of the light, it depends upon the frequency of the light. It also depends upon the type of the slits which we are using. For light waves one cannot observe interference between the waves from two independent sources. This is the usual sentence which we learn in our textbook.

But this is not the case, usually whenever we say that the two waves are interfering then we expect as a result of interference observation of fringe pattern and whenever we see some kind of fringe pattern, we say that the two waves must have interfered. But this is not the case, the observation of the fringe pattern is for the sake of measurements, irrespective whether we are observing fringe pattern or not, the two waves always interfere.

But usually at junior levels, we say that as long as the two sources are coherent then only we observe interference. But the truth is that or the reality is that irrespective whether the sources are coherent or not, irrespective whether their frequencies are very closely placed or not, the waves will always interfere. But to observe sustained interference fringes, we require waves which are coherent and which have almost same frequencies and the amplitudes are almost equal. These are conditions which are given because we want to observe beautiful fringes because we want to observe sustained interference fringes.

If the interfering waves are not coherent, the interference will happen there too but the generated interference fringes will not be sustained. That is the positions of maxima and minima will keep changing and once the positions of maxima and minima are changing rapidly then we will see a uniform illumination on the screen. Interference fringes are getting formed there but it is in ability of our instrument, in ability of our eyes which are not able to detect them because their positions are rapidly changing. But interference in any case is happening there, the phenomena does exist in all the cases.

Now having said that, to observe beautiful sustained interference fringes we require a phase relationship between the two-interfering wave. And what is this this phase relationship? This phase relationship says that the phase difference between the two interfering wave must be constant and if the phase difference is constant these two waves are called coherent waves and the interference pattern produced by these two interfering waves is very beautiful, you will see very bright maxima, dark minima and their position do not alter with time.

Now optical interference corresponds to the interaction of two or more light waves yielding a resultant irradiance that deviates from the sum of the component irradiances. Suppose, we have two sources which are emanating intensities I_1 and I_2 respectively then after interference the resultant intensity will not be equal to I_1 plus I_2.

We know that interference follows superposition principle and what superposition principle says is that the amplitude of the two waves must be summed vectorially not the intensity and if we sum up the amplitudes then to calculate intensity we will have to square them up and if we square amplitude $A_1 + A_2$, then there would be a cross term also. Therefore, $I \neq I_1 + I_2$.

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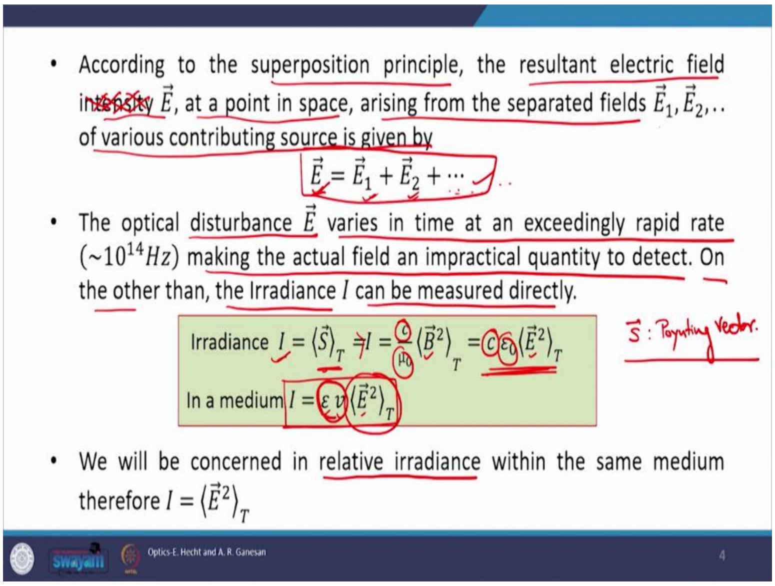

Now, according to super position principle the resultant electric field intensity E, now we are talking about field that is amplitude. The resultant electric field intensity at a point in space arising from the separated field, it should not be intensity here in this area. The resultant electric field E at a point in space arising from the separated fields E_1, E_2 and so on of various contributing source is given by $E = E_1 + E_2 + \ldots..$

This is the statement of superposition principle if we have n number of sources which are eliminating E_1, E_2, E_3 and so on fields then the resultant field would be vector sum of all the contributing fields and this is what exactly being done here. The resultant field E is equal to vector sum of $E_1 + E_2 + E_3$ and so on.

The optical disturbance E varies in time at exceedingly rapid rate. We know that the frequency of optical wave is around 10^{14} hertz which is a very big number. Therefore, E varies very rapidly and it is extremely difficult to detect E and this makes the actual field an impractical quantity to detect. We require very dedicated experimental facilities to detect it.

Then what is easy rather is to detect their intensity, therefore, the irradiance or intensity is measured directly. Then people may ask what is irradiance? It is nothing but it is synonyms of intensity and how it is defined, irradiance is defined as time averaged pointing vector. This S is nothing but pointing vector which people study in electromagnetic theory.

Then, if you take the time average of pointing vector then we get irradiance or intensity which is equivalently, which we can write equivalently as $(c/\mu_0)\langle\vec{B}^2\rangle_T$ time average field square and which is equivalent to $c\epsilon_0\langle\vec{E}^2\rangle_T$. B is your magnetic field and E is your electric field vector. If we take time average of E and B field then from there to we can calculate irradiance.

Now here c is the speed of light in vacuum and ϵ_0 and μ_0, are permittivity's and permeabilities in vacuum respectively, ϵ_0 is permittivity in vacuum and μ_0 is permeability in vacuum.

Now these quantities are defined in vacuum, how would the irradiance be defined in a medium where n is not equal to 1. Now in a medium irradiance is defined by this relation where ϵ is medium permittivity, v is the velocity of light in that particular medium and E is the electric field and we took here the time average of E square. This is how the intensity or irradiance is defined in a medium.

But whenever we calculate irradiance in our derivations or while calculating the maxima and minima, we will always be concerned with the ratio of the intensities. We will always be concerned in relative irradiance and whenever we will take ratio of 2 irradiances, the constant which appears before time average E square which are ϵv, they will go away.

Therefore, from now on whenever we express irradiance, we will neglect ϵv term. These coefficients, these constants would be neglected here on because we would be calculating the relative irradiances in the same medium. Therefore, we will define irradiances I= $\langle\vec{E}^2\rangle_T$, is a vector quantity is our electric field vector.

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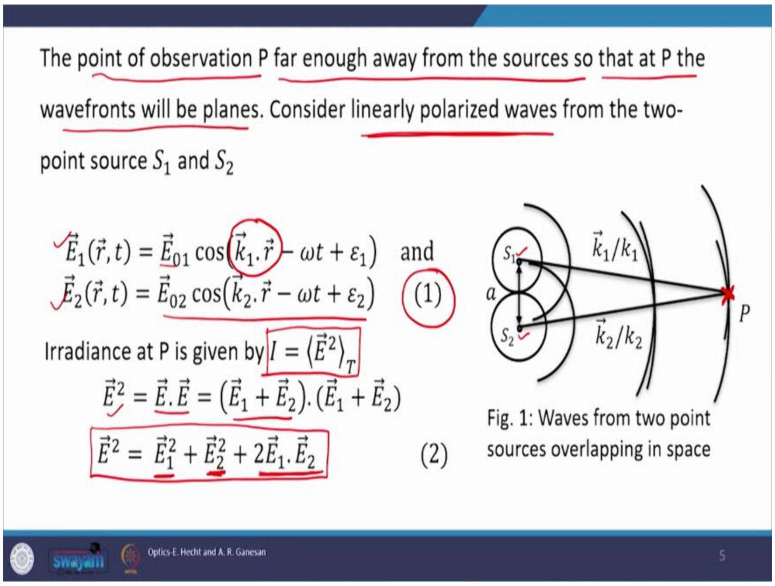

Now having said this, now consider we have two-point sources which are designated by S_1 and S_2. Since the S_1 and S_2 are point sources, they will emanate spherical wave fronts. The shape of the wave front would be spherical. Now suppose we have a point P which is very far from these two sources and distance of P from S_1 and S_2 is such that by the time this is spherical wavefronts reach to point P, they almost become plane.

The P is situated at such a distance from S_1 and S_2 that the spherical wave fronts which get emanated from point sources S_1 and S_2, they can easily be treated as plane wave when they reach at point P, this is what exactly is written here, the point of observation P far enough away from the sources so that at P, the wave fronts will be plane.

Now we also consider that point sources S_1 and S_2, they emit linearly polarized wave. Now we will analyse this situation mathematically. Suppose $\vec{E}_1$ is the field which is emanating from point source S_1 and $\vec{E}_2$ is the field which is emanating from point source S_2 then $\vec{E}_1 = \vec{E}_{01} cos(k_1.r - \omega t + \epsilon_1)$. Where ϵ_1 is the initial phase, $\vec{k}_1$ is the wave vector associated with S_1 and r is the radial vector.

Similarly, for $\vec{E}_2$ we have this expression and they both are given by equation number 1. Now this quantity, you see it is a dot product $\vec{k}_1.\vec{r}$ and therefore, since it is a dot product the resultant would be a number, a constant. $\vec{k}_1$ is the direction of the wave vector and r is the radial vector. Now the resultant would be scalar and therefore we can easily say it is a dimensionless quantity and we can easily take the cosine of this number.

Now if we want to calculate the resultant disturbance at point P, the point of observation P then what we will have to do? We will have to sum $\vec{E}_1$ and $\vec{E}_2$ and then square this expression and the resultant will give the total disturbance or total irradiance at point P and this is what is shown in this expression the resultant irradiance I= $\langle \vec{E}^2 \rangle_T$ where $\vec{E} = \vec{E}_1 + \vec{E}_2$ and how to calculate $\vec{E}^2$? To calculate $\vec{E}^2$ we take dot product between the two vectors $\vec{E}.\vec{E}$ and what is $\vec{E}$? $\vec{E} = \vec{E}_1 + \vec{E}_2$, the resultant disturbance.

After multiplying we get this expression, the resultant field at point of observation P would be equal to $\sqrt{\vec{E}_1^2 + \vec{E}_2^2 + 2\vec{E}_1.\vec{E}_2}$. Now you see here, this is the irradiance due to the first source, this is the second term irradiance due to the second source but apart from these two contributions we have a third term also. An extra contribution which owes its origin in multiplication between $\vec{E}_1$ and $\vec{E}_2$ and therefore this is called interference term.

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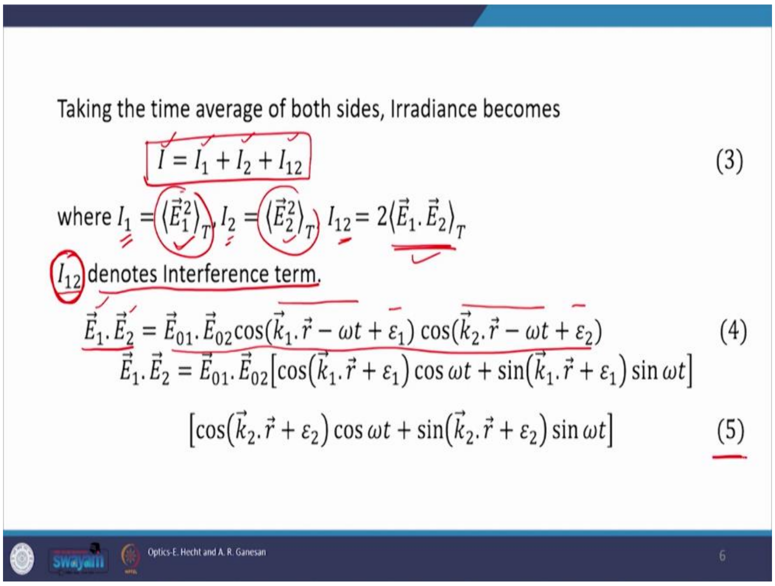

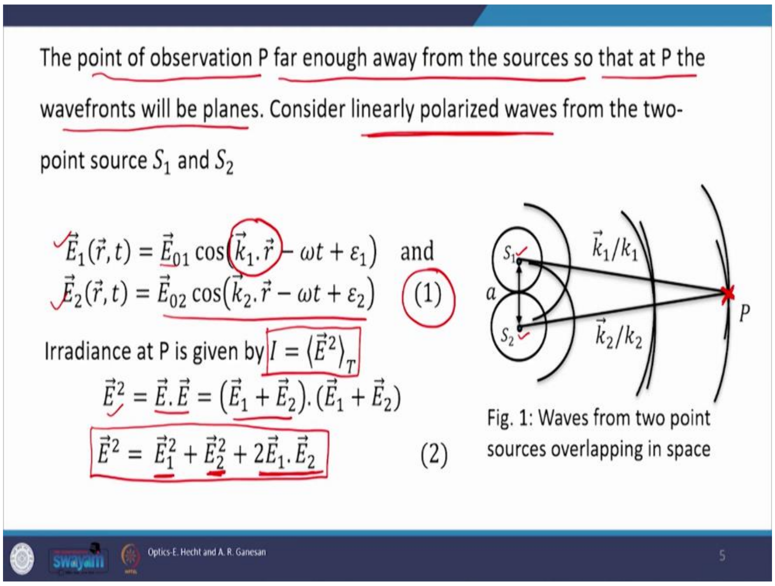

Now let us take time average of equation number 2 to calculate irradiance here. When we take time average then we express the $\langle \vec{E}_1^2 \rangle_T$ by I_1, $\langle \vec{E}_2^2 \rangle_T$ by I_2 and time average of this cross term by I_{12} and therefore the final expression takes this form. The resultant irradiance $I = I_1 + I_2 + I_{12}$.

As said earlier, I_{12} denotes interference term. Therefore, the resultant would not be equal to the sum of magnitudes of the two-interfering wave. There would be a few places the magnitude will go up, at few places the magnitude will go down, at few places it may go to 0. We will analyse these things slowly.

Now, the time average of $\vec{E}^2$ we know, we know the expression of $\vec{E}$, we can calculate the $\langle \vec{E}_1^2 \rangle_T$. Similarly, we can calculate the $\langle \vec{E}_2^2 \rangle_T$ and $\langle \vec{E}_1 . \vec{E}_2 \rangle_T$. But this is an interesting part, I_{12} is interference term, let us see how what are the quantities on which I_{12} depends on. To calculate the expression for I_{12} let us first calculate the dot product between vector $\vec{E}_1$ and vector $\vec{E}_2$. We just substituted the expression for $\vec{E}_1, \vec{E}_2$ here and then we got this big expression. And here again, the cosine function is expanded in using the formula cos (A+B) and after this expansion we get expression 5.

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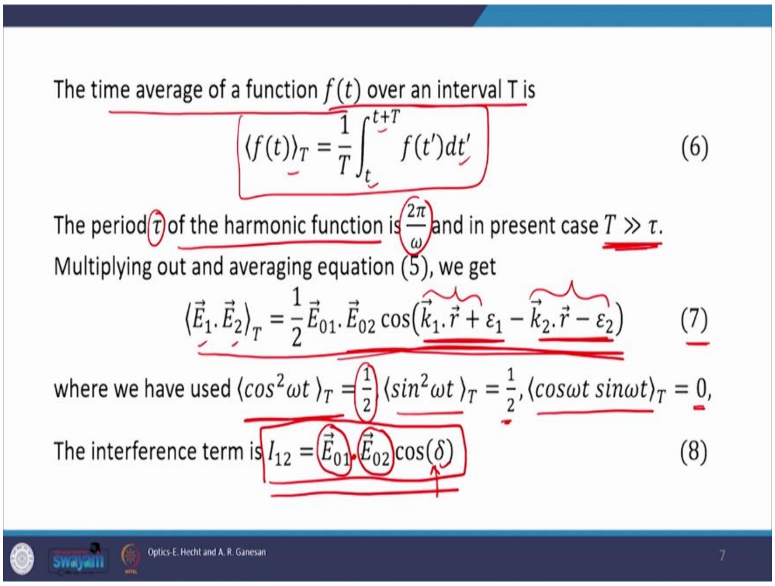

Now, once these things are done. Let us take time average, so then the question arises what is time average? The time average of function f(t) over an interval T is defined through this relation, $\langle f(t)\rangle_T = \frac{1}{T}\int_t^{t+T} f(t')dt'$, where period τ of the harmonic function is given by $2\pi/\omega$ where it is assumed that τ is the period of f which is a harmonic function and this period must be much much smaller than capital T.

The interval over which the function is being integrated, this interval must be much larger than the period of the harmonic function. Once these criteria are met, we can easily take the time average. So, after taking the time average, we get this expression. The time average of $\vec{E}_1.\vec{E}_2$ is expressed here by equation number 7. And how did we calculate it? While taking $\langle cos^2(\omega t)\rangle_T$, we substituted it with half.

Similarly, the $\langle sin^2(\omega t)\rangle_T = 1/2$, $\langle cos(\omega t)sin(\omega t)\rangle_T = 0$. This we can easily do, it is a simple integration.

Therefore, the resultant interference term which is half of $\langle \vec{E}_1.\vec{E}_2\rangle_T = \vec{E}_{01}.\vec{E}_{02}cos(\delta)$, where δ is the phase difference. This is from first wave and the second term this $\vec{k}_2.\vec{r} + \epsilon_2$, this is from the second wave. We started with two waves $\vec{E}_1$ and $\vec{E}_2$ and this belong to first wave, while this belong to second wave and we are taking difference here between the two. This means this δ which is appearing in equation number 8, it is a phase difference.

And this is what is written here, $\delta = \vec{k}_1.\vec{r} + \epsilon_1 - \vec{k}_2.\vec{r} - \epsilon_2$ and this is defined as phase difference arising from the combined path length and initial phase angle differences. If you rewrite this expression of δ then you get this, $\delta = (\vec{k}_1.\vec{r} - \vec{k}_2.\vec{r}) + (\epsilon_1 - \epsilon_2)$. This term, this is the phase difference arising out of initial phases, ϵ_1 is the initial phase of wave one, ϵ_2 is the initial phase of wave two and therefore $\epsilon_1 - \epsilon_2$ is phase difference arising out of initial phases.

Similarly, $\vec{k}_1.\vec{r} - \vec{k}_2.\vec{r}$ stands for phase difference arising out of path length difference. We have already defined what path length is, $\vec{k}_1$ is the wave vector, r is the radial coordinate of the point to which the wave is travelling then $\vec{k}_1.\vec{r}$ is the path trivial by path length trivial by first wave, similarly $\vec{k}_2.\vec{r}$ is the path length trivial by the second wave. The difference is path length difference between the two waves and this also contribute to a phase difference.

Therefore, we can say that the resultant expression of phase difference has two components, one corresponds to path length difference and the second correspond to initial phase angle difference or initial phase difference. Now again come back to the expression, final expression of interference term. Now here you see the first term is vector $\vec{E}_{01}$ and the second term is vector $\vec{E}_{02}$ and between the two, we have kept a dot.

We are taking dot product between two vectors $\vec{E}_{01}$ and in $\vec{E}_{02}$ are the amplitudes of the two waves and the information of direction is also embedded in it and what is the information of direction? The information of direction means the polarization information. The polarization information is embedded in these amplitudes.

Now we are taking the dot products between the two vectors. Therefore, polarization also comes into the picture, polarization is associated with $\vec{E}_{01}$ in form of vector. Similarly, the polarization of wave two is associated in $\vec{E}_{02}$ in form of a vector. Now we are taking the dot product between these two vectors. Therefore, the relative orientation of these two polarizations which are associated to the two waves will affects the output, will decide the value of I_{12}, the interference term.

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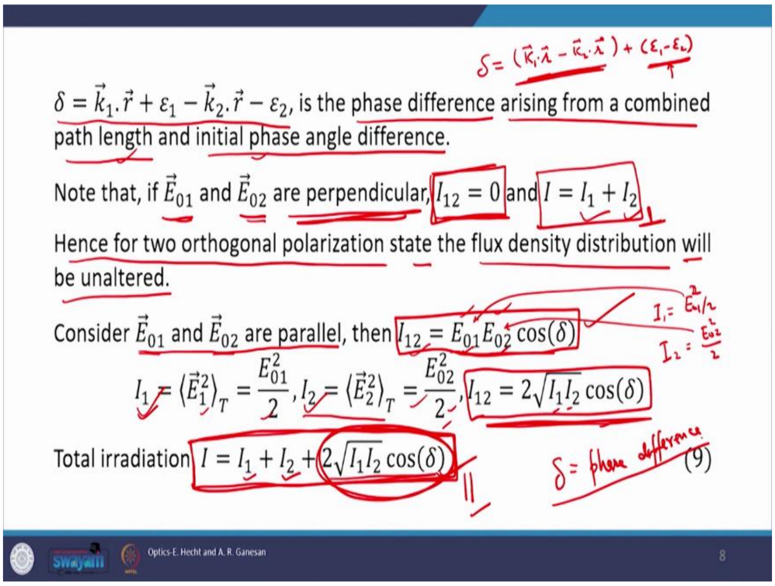

Now, let us consider a special case, what is this special case? This is special case is, if $\vec{E}_{01}$, vector $\vec{E}_{01}$ and vector $\vec{E}_{02}$ are oriented at 90 degree, then what would be the interference term? If vector $\vec{E}_{01}$ and vector $\vec{E}_{02}$ were perpendicular, then what will be the dot product? The dot product would be 0, $\vec{E}_{01}.\vec{E}_{02}$ if they are oriented at 90 degrees, they will give 0 value and therefore, cross term or the interference term I_{12} would be 0.

In this particular case, the resultant irradiance would be sum of irradiances of wave 1 and wave 2. But this is not the general expression, this expression is valid if the interfering waves are orthogonal to each other, the polarization is oriented at 90 degrees. Hence for two orthogonal polarization state, the flux density distribution will be unaltered. There would not be any alteration at the observation plane, the intensity would be uniformly distributed on the screen. We will not be able to see any kind of fringe pattern because the intensity which we are observing at the screen is sum of the intensities of the two waves.

Therefore, the resultant intensity would be higher than the individual intensities of the two sources or two waves which are falling on the screen. But there would not be any kind of fringe pattern, we cannot observe any fringe pattern there. It would be a uniform distribution of intensity.

Now, let us consider a different case. Now let us consider that the vector $\vec{E}_{01}$ and vector $\vec{E}_{02}$ are parallel or they have the same linear polarization. To reminds you, we started with the two waves which are linearly polarized. The first case which we discussed here is the cross-polarization state, where the polarizations of the two waves are perpendicular.

Now in the second case, we are considering that the polarizations are parallel. The two vectors are oriented at 0 degree now, then what will happen? We will have maximum value out of the dot product now. The interference term would be equal to now $E_{01}E_{02}cos\delta$. Now in this particular case $I_1 = \langle \vec{E}_1^2 \rangle_T = \langle E_{01}^2 \rangle_T/2$.

Similarly, we can define I_2 which is $I_2 = \langle \vec{E}_2^2 \rangle_T = \langle E_{02}^2 \rangle_T/2$ and similarly we can define the interference term. Once these are known then the total irradiance will have this expression. Once the cross term is known we can easily calculate what is I_1, what is I_2.

Once these are known, then we can express E_{01} and E_{02} in terms of I_1, I_2, what I mean is that I_1 you can always express as ${E_{01}}^2$ in case of parallel polarization case and similarly I_2, you can express as ${E_{02}}^2/2$ as is given here and once you know this expression, you can substitute for E_{01} and E_{02} here.

Then you can express I_{12} in form of I_1 and I_2 which is given here, once you know the expression of I_1, I_2, the resultant irradiance, now is given by equation number 9. In this resultant expression where the polarization are parallel, you see that this is the first term which is I_1, the second term is I_2 and we have added term which is $2\sqrt{I_1 I_2}cos\delta$.

Cosine function means there would be some variation, sinusoidal variation. Sinusoidal variation means there would be a background intensity and this intensity would be modulated by a cos function. It means at some places on the screen we will have maxima, at some places on the screen, we will have minima. And this would be periodic, there would be periodic variation in the intensity and this is ensured by this interference term, the third term which is appearing here in equation number 9.

While when the polarization was perpendicular, we saw that there were no fluctuation in the intensity at the screen. The intensity or irradiance at the screen where uniform earlier when two cross polarized light were interfering but when the polarization is parallel, then we see a case where intensity is periodically varying and therefore it is forming of particular type of fringe pattern.

Now what would be the position of maxima, what are the conditions under which we will get maxima? These all depends upon the value of δ, the phase difference. Now what are the values which will give maxima, what are the values of δ which will give minima, these are things, we will analyse in our next class. This is the final expression which we obtained today and I stop my lecture here.

In next class, we will move further from equation number 9, analyse it for different values of δ and we will see what type of fringe pattern we get. Thank you all for listening me.