Applied Optics
Professor Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology, Roorkee
Module: 3
Lecture: 15
Elliptical Polarization

Hello everyone, welcome to my class. Now, we are talking, we are covering topics of Module 3 and till now, we have covered the wavefront, Huygens principle and its applications.

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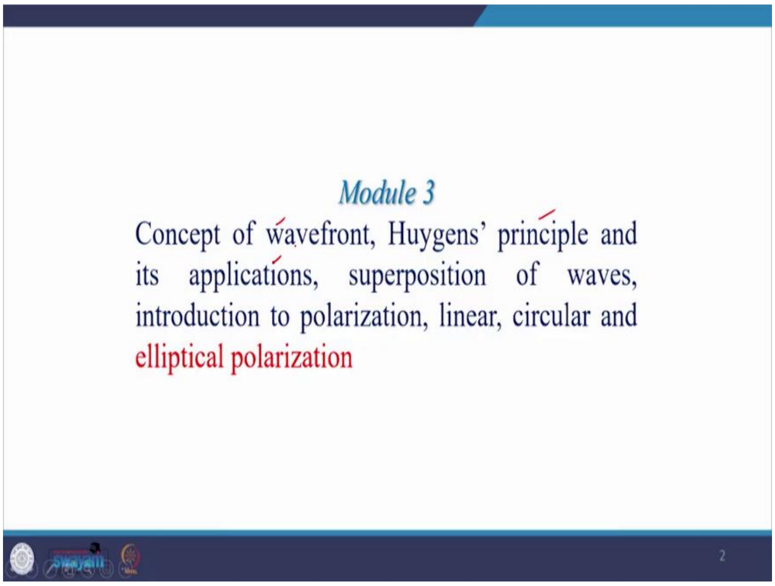

Then we talked about superposition principle and then we learnt about the polarization of light wherein we talked about linear polarization and circular polarization. Today, we will start elliptical polarization.

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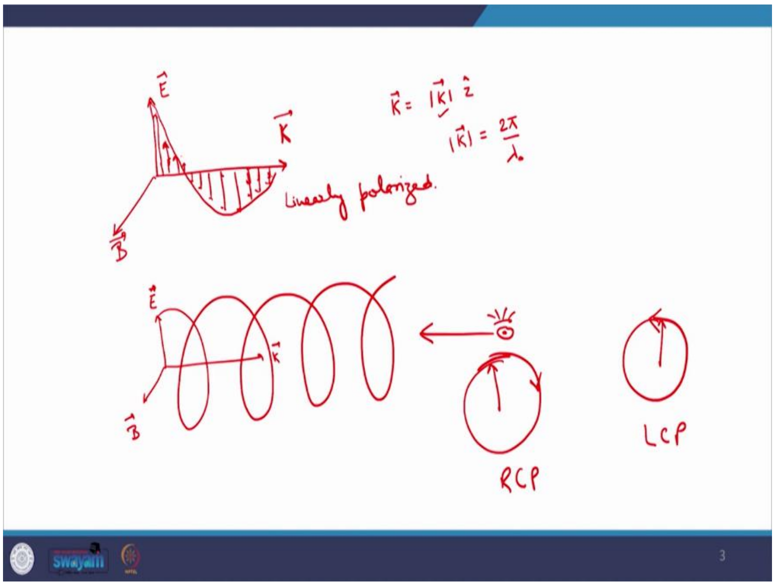

Just to remind you a wave consists of E field and B field. E and B field are perpendicular to each other and these fields are also perpendicular to the direction of the propagation which is usually designated by wave vector K, the K wave vector is a vector quantity and it has amplitude and certain some direction say $\hat{z}$ (z-cap), $\hat{z}$ is unit vector along z-axis.

And we know that this $|K| = 2\pi/\lambda$. If it is in the vacuum then it is designated at $2\pi/\lambda_0$ otherwise it is designated as $2\pi/\lambda$. Now, the tip of electric field associated with electromagnetic wave decides polarization. Now, since E, the magnitude of electric field vector is very large as compared to that of magnetic field therefore, polarization is always associated with the electric field.

Now, if electric field vibrates in a plane defined by E and K vector which is shown here in this figure by these arrows, these are the amplitudes, the time variation of the E vector then this type of electromagnetic wave is called linearly polarized or plane polarized. I repeat if the electric field vector oscillates only in one plane or in other words if the oscillation of the electric field is confined in a single plane then such a wave is called plane polarized wave or linearly polarized wave.

Now, apart from this type of wave there is a second kind of polarization which is called circular polarization and in case of circularly polarized light, the tip of E vector forms a circle, in this case the E vector rotate like this. Now, if you see from this direction if we keep our eyes here and see from here then we will see that there is a circle on the tip of the electric field vector rotates on this circle and in this particular case the rotation is in clockwise direction such a wave is called right circularly polarized wave RCP. If the rotation the rotation of electric field vector is in anti-clockwise direction then such a wave is called left circularly polarized wave like this, these are the two types of polarization we have studied till now.

Now, in case of linearly polarized light, the magnitude of electric field vector is a function of time, it changes with time. While in case of circularly polarized light, the magnitude is constant, it is not varying with time or space because it is the radius of the circle which it draws then what is changing in such kind of polarization, it is the direction of the electric field.

Since the direction of the electric field depends upon the time therefore, whole E field is again a function of time and since E field is a function of time, it will generate B field and satisfies all the criteria's of being of electromagnetic or being an electromagnetic wave. Now, we will move further.

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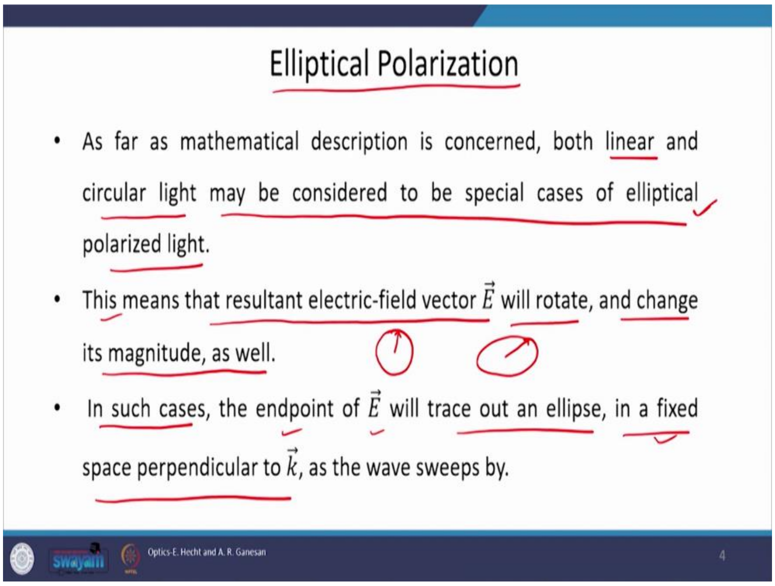

Now, we will talk about an elliptical polarized light or we will talk about Elliptical Polarization. In elliptically polarized light, the electric field tip, it will generate an ellipse and this is already suggested by the name itself and since it is ellipse, both the direction of the E field and the magnitude of the E field change with the propagation or change with time.

Both direction and magnitude will not remain fix and linear and circular light may be considered to be the special cases of elliptical polarized light. We will discuss it this sentence in greater detail in coming slide. Now, this means that the resultant electric field vector E will rotate and change its magnitude as well.

In case of circularly polarized light only the direction is changing, direction of the electric field vector is changing and magnitude remains constant while in an elliptically polarized light both direction and magnitude will change. And in case of elliptically polarized light, the endpoint of E will trace out an ellipse and where would this ellipse be traced, it would be traced in a fixed space perpendicular to wave vector K.

And we have seen that when a circularly polarized light propagates in space, it traces a spiral and when we see it from the front, when we point our eyes towards the source then we see that there is a circle, the locus of the electric field vector is a circle. Now, in the case of elliptically polarized light when we will see towards the source then the locus of the electric field, the tip of electric field vector will trace an ellipse. Now, depending upon some conditions the orientation of the ellipse may change what are those conditions we will analyze right away.

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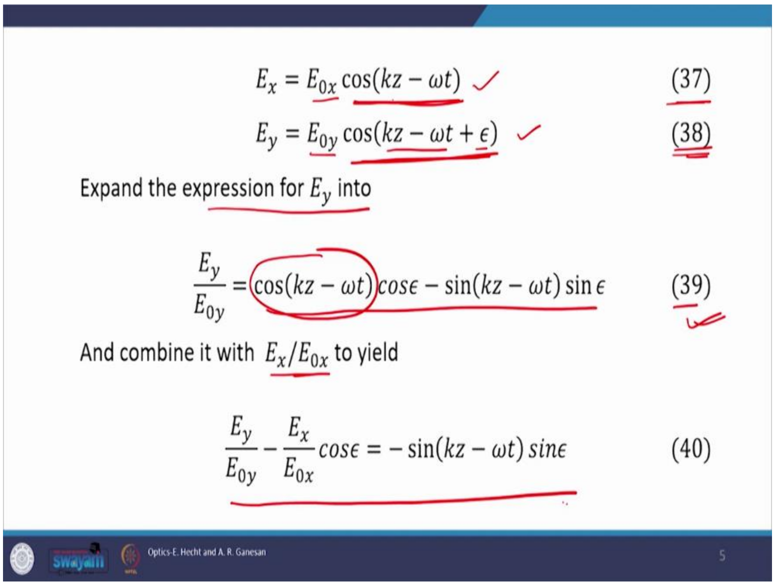

Now, let us again start with the two components of electric field which are E_x and E_y. We also started with these two components when we were analyzing circularly polarized light. The same fields are also taken here. The only difference is that the magnitudes, the amplitudes of E_x and E_y in our case is different now. In case of circularly polarized light $E_{0x} = E_{0y} = E_0$..

But for elliptically polarized light, we will deliberately take these amplitudes different. Now, we will expand E_y, we will just consider equation number 38 and we will expand this cosine function. We see that cosine function is cos (A+B), we will expand this cosine (A+B) and see what we get, after expanding the cosine function we get this expression, a bit bigger, this is given by equation number 39.

Let us also include the expression of E_x how to include we know that from equation number 37, $E_x = E_{0x}cos(kz - \omega t)$. Now $cos(kz - \omega t) = E_x/E_{0x}$. Let us do this let us replace $cos(kz - \omega t) = E_x/E_{0x}$ then this gives us equation number 40. This got a bit simplified.

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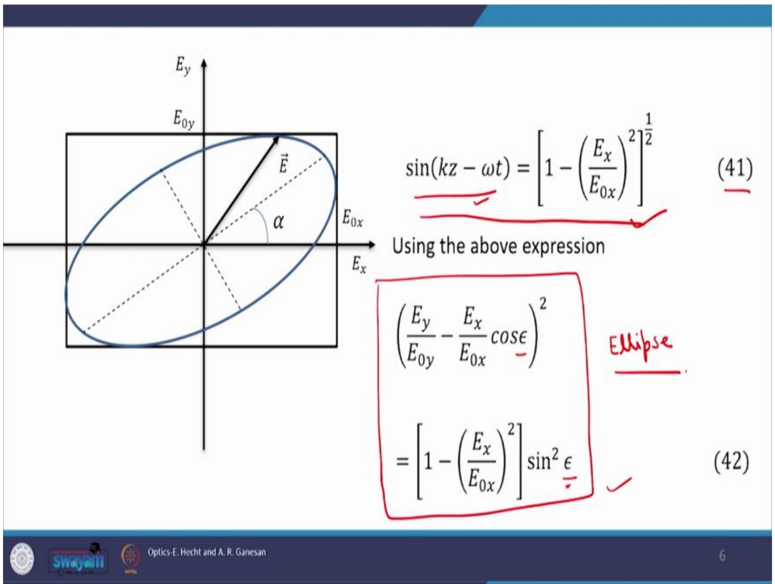

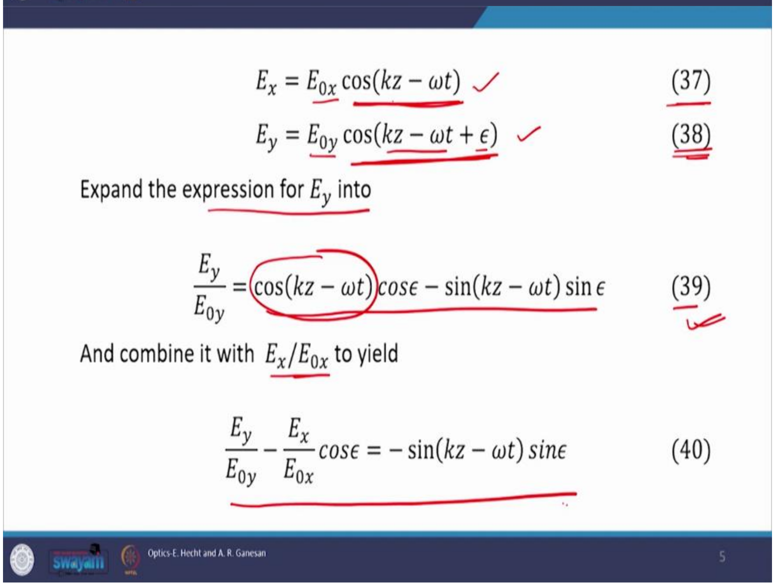

Let us again seek for the expression of $sin(kz-\omega t)$ what would be $sin(kz-\omega t)$? it would be equal to $\sqrt{1-cos^2(kz-\omega t)}$ and we know what is the expression of $cos(kz-\omega t) = E_x/E_{0x}$ then after doing this we get equation number 41.

Therefore, we have now expression for $sin(kz-\omega t)$ also. Once we have this expression too, we will again substitute the expression of $sin(kz-\omega t)$ back into equation number 40 here. Now, this is the final expression, we have substituted for sin function, we have substituted for cosine functions, sin and cosine means $sin(kz-\omega t)$ and $cos(kz-\omega t)$, after these two substitutions, we got rid of $kz-\omega t$ term.

And this is the final expression which we got. Now, this is and if we plot it then we will get an ellipse. This is an expression of ellipse and what is then this ϵ, we know that this ϵ was the initial phase difference between the two electric field components which were E_x and E_y, this ϵ decides the ellipticity or orientation of the ellipse, how does it define?

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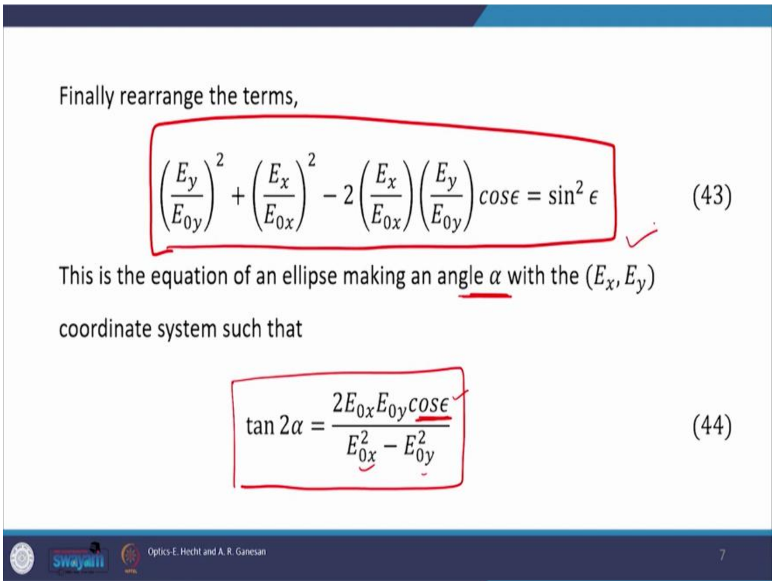

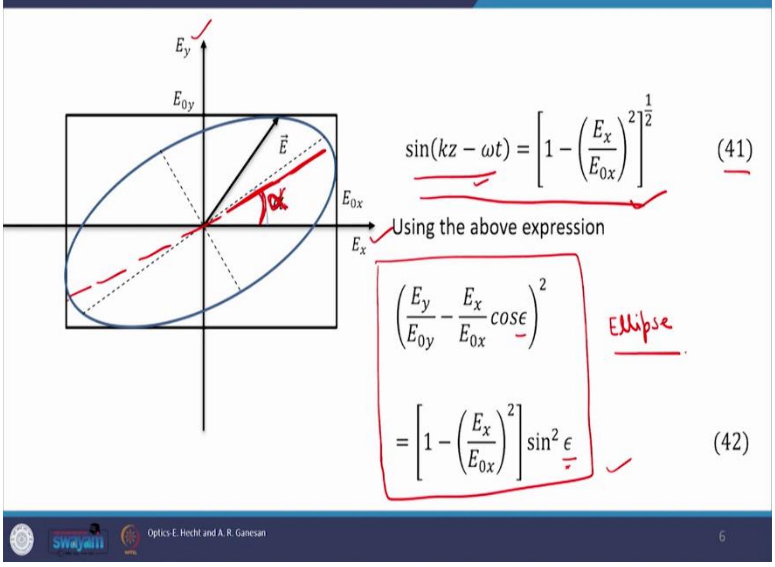

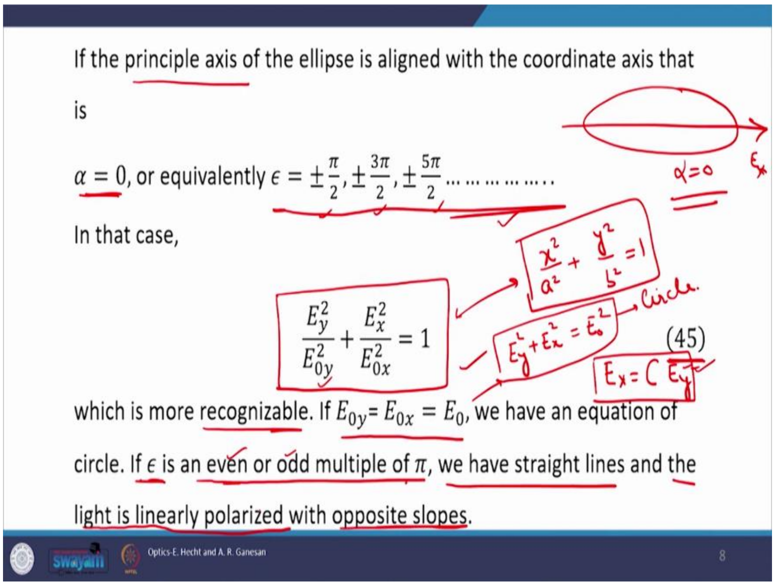

We rearrange the last equation and this is the final equation of the ellipse and this ellipse is making an angle α with E_x and E_y, what is the E_x and E_y? E_x is pointing along x-axis, E_y is pointing along y-axis. Now, the major axis of the ellipse, these drawings should be like this, the major axis of the ellipse is inclined by an angle α with respect to the horizontal with respect to the E_x axis and how this α is given?

α is given by this expression, using plain geometry we can get this expression and $tan2\alpha = 2E_{0x}E_{0y}cos\epsilon/(E_{0x}^2 - E_{0y}^2)$. E_{0x} and E_{0y} are nothing but amplitudes of E_x and E_y fields and cosine of α is here, where α is the initial phase difference between the two waves.

Therefore, using equation 43 we can generate an ellipse whose major axis is inclined at an angle α with the E_x axis, this equation can further be simplified with some assumption. Now let us assume that the principal axis of the ellipse is aligned with the coordinate axis what does it mean is that the major axis of the ellipse.

We assume now that it is a long horizontal direction or it is a long E_x axis or you can say that the ellipse is oriented in such a way that its major axis is along E_x now, that is α is equal to 0. If we consider this, then equivalently we can write that initial phase $\epsilon = \pm\pi, \pm3\pi/2, \pm5\pi/2\ldots$, saying α is equal to 0 is equivalent to saying ϵ has these values.

Once we assume this then this complicated equation, equation number 43, which is an equation of ellipse oriented at certain angle this gets reduced to a very typical equation of an ellipse which we have already studied in our junior classes. And this is now, recognizable x format.

Equation number 45 is the usual expression which is $x^2/a^2 + y^2/b^2 = 1$, the typical expression of an ellipse.

It is now in this format, again we can easily now correlate them, we can easily recognize this expression as an ellipse. Now, let us consider some special cases, what are the special cases? Now, we have given two electric field components, out of these two electric field components we did derived we did some mathematics and then concluded that, this two field components will generate an elliptically polarized light.

Now, this elliptical polarization, it may be a right elliptically polarized light, or it may be a left elliptically polarized light, both the electric field vector may rotate either in clockwise direction or in anti-clockwise direction, we have already devoted the last lecture, what is the clockwise rotation? And what is anti-clockwise rotation?

How to know whether the electric free vector is rotating clockwise or anti clockwise, we already learned this. Therefore, I will not repeat it here. What we know now is that, given two electric field components and through these two components, we derived an equation which says that the tip of the electric field vector will generate an ellipse and therefore, we name this electromagnetic wave as elliptically polarized wave.

Now, let us consider a few special cases, in equation number 45 we see that in the denominator, we have E_{0y} and E_{0x} which are the magnitudes, which are the amplitudes of the field components. Now, let us assume that $E_{0y} = E_{0x} = E_0$.

Then what will equation number 45 represent, in this particular case equation number 45 will take this form $E_y^2 + E_x^2 = E_0^2$ and what is this equation? This equation represents a circle. Therefore, we can safely say that in an elliptically polarized light if the magnitude of the two electric field vectors are the same, then the ellipse will reduce down to a circle. Therefore, we can say that circularly polarized light is a special case of elliptical polarized light and this justifies the claim which we made at the beginning of the lecture.

Now, the second case, if ϵ is an odd or even multiple of π then what will happen? If ϵ is an odd or even multiple of π, where is ϵ? The epsilon is sitting in equation number 43. Now, under this condition what will happen let us see here, now, if ϵ is an even multiple of π, then cosine function, the cos epsilon would be equal to 1, sin function will be equal to 0.

And therefore, we will have $E_y/E_x = E_{0x}/E_{0y}$ which is a constant number therefore, we will have E_x is equal to some constant and multiplied by E_y, we will effectively get E_x is equal to some constant and then E_y which is nothing but an equation of a line. Similarly, if ϵ is odd multiple of π then instead of getting this positive constant we will get a negative constant, the orientation of the line will change.

The line will rotate by some angle decided by the relative amplitudes of E_{0x} and E_{0y}. Therefore, if ϵ is odd or even multiple of π, we have a straight line and therefore the resultant wave is a linearly polarized. With opposite slopes means the odd or even multiple of pi will generate plane polarized light with opposite slopes.

And therefore, it also justifies the claim that the linearly polarized light is a special case of elliptically polarized light, and therefore, we saw that both circularly polarized light and linearly polarized light are special cases of elliptically polarized light. Now, let us keep varying the ϵ value and see what are the shapes we can be generated using a elliptically polarized light.

And the expression of elliptically polarized light which we are using is this equation number 43. In this equation number 43, we will change ϵ and then will trace out the equation and we will see what type of structure what type of curve we get.

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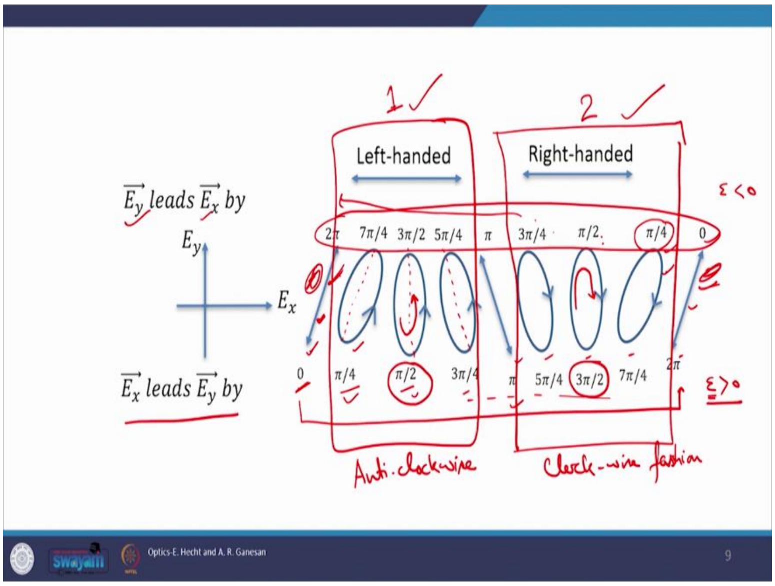

Now, in this figure, first we analyze the case, we are E_x leads E_y by 0 radian, then $\pi/4$ radian, and then $\pi/2$ radian and so on. And what does we see is that when E_x leads by E_y by 0 degree or alternatively, what we can say is that if there is no phase difference between E_x and E_y, then we will get a linearly polarized light which is given by this line, straight line, which is very much obvious we have already talked about it.

Now, if E_x leads E_y by $\pi/4$ then we get elliptically, an elliptically polarized light if it leads by $\pi/2$, then you see that the ellipse got rotated the major axis of the ellipse earlier was in this direction now, it is vertical and now, we keep increasing an angle and you see that the ellipse again got rotated and for π phase difference we again get linearly polarized light and then again elliptically polarized light and again linearly polarized light. Now, we see that this is a full rotation of $2\,\pi$ and after $2\,\pi$ rotation, this linear representation repeated itself. After full circle the polarization repeats itself. Now, I will look here at this $\pi/2$ and $3\,\pi/2$.

They look similar the ellipse which is drawn for phase difference of $\pi/2$ and $3\,\pi/2$, they are vertically oriented ellipse, the major axis of these two ellipses are vertically oriented but the sense of rotation of the electric field tip is opposite, here the rotation is in anti-clockwise direction while it is in clockwise direction for $3\,\pi/2$ case. Similarly, you can see for the other figures also.

Now here the ϵ was positive the same thing will be repeated, if you take epsilon negative, here all the angles are, the top angles are measured in negative, there is a minus sign and in this case E_y leads E_x. Now if the phase difference is 0 then we will get the same type of state of polarization which we got here, this case is similar to this case for $\pi/2$ you get an ellipse, $\pi/4$ we get an ellipse, $\pi/2$ different orientation of ellipse and so on and so forth.

Now, from here till here we see a type of rotation, which is different from these rotations. The 3 ellipse here in the first blog have different sense of rotation, then the 3 ellipses here on the right block. Here the ellipses in block 1 the ellipses are rotating in anti-clockwise direction. Here in the right block, the ellipses are rotating in a clockwise or tip of electric field vector is rotating in a clockwise fashion.

Now, therefore, the second block where the tip of electric field vector is rotating in a clockwise fashion is called a right handed and therefore, this polarization is called right elliptically polarized light while in block one the rotation is in anti-clockwise direction, it is called left elliptically polarized light and this correspond to the definition of left and right circularly polarized light also.

The sense of rotation which we introduced or which we discussed in the last class is the same here, just due to the change in the magnitudes of E_x and E_y components, the circle got converted into an ellipse. And this is all for elliptically polarized light. Thank you for hearing me and we will discuss or we will start interference from the next class. Thank you.