Applied Optics
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Module 3
Lecture 12
Concept of Wavefront, Huygens' Principle – II

Hello everyone, welcome back to the class, and today we will learn about the applications of Huygens's principle, we will see what are the applications of Huygens's principle, how successful it was, and we will see the phenomena, how it explained the phenomena of reflection, sorry refraction and total internal reflection.

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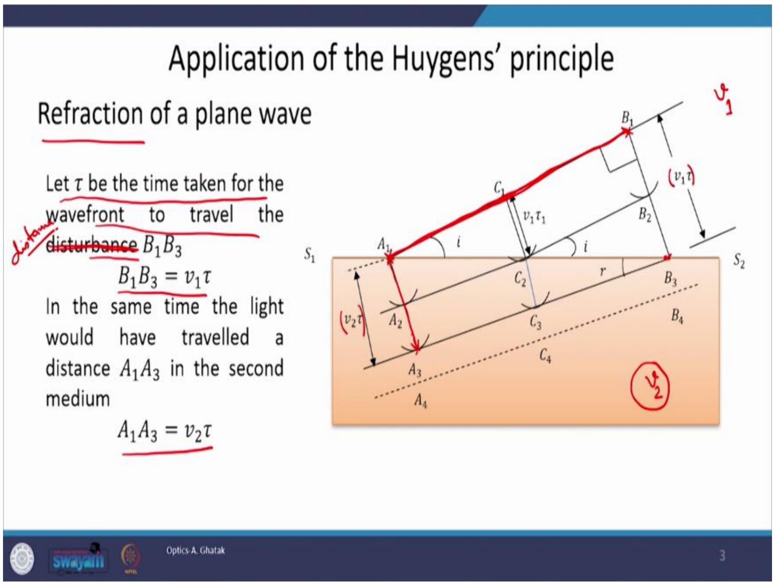

Now, the first application which we will learn today of Huygens's principle is in refraction, we will see using Huygens's principle how to verify the law of refraction. Now, we will start here with a plane wave, and the wavefront of this plane wave is represented by this AB line here.

Suppose this is the, this is a cross-section of a plane which is looking which is in the form of a line, and this A_1B_1 wavefront falls at an angle, at an interface S_1S_2, S_1S_2 divides the two media. Above S_1S_2, there is a medium of different refractive index and below S_1S_2, there is a medium of different refractive index. And the light or the wavefront falls from the top medium at this interface, and then it gets refracted into the second medium, the second medium is drawn here with a shaded colour, the brownish colour.

Now, let τ be the time taken for the wavefront to travel the distance B_1B_3. Sorry, it is not disturbance, it is distance. Now, this is the $A_1C_1B_1$ is the wavefront which is falling out the interface, and since it is this wavefront falling at some angle, the part A_1 of the wavefront will fall earlier as compared to the point B_1. I repeat, the point A_1 of the wavefront will fall at the interface earlier as compared to point B_1. Now, it will the point B_1 will take some time to reach at the interface, and this point B_1 will fall at point B_3 at the interface.

Now, suppose point B_1 takes time τ to reach at B_3, and suppose the speed of the wave in first medium is v_1. While, that in second medium is v_2. Since point B_1 is taking τ (tau) time in reaching to point B_3, the total distance travel by point B_1 will be equal to $v_1 \times \tau$, τ is the time and v_1 is the speed in the upper medium.

Therefore, the total distance would be $v_1 \times \tau$. During the time period τ, the A_1 would have gone deeper into the second medium, it would travel, the A_1 will reach to point A_3 within the medium, and this distance, this much distance would be travelled by point A_1 in medium 2, in the second medium.

And since medium 2 is of different refractive index, the wave will propagate with different speed in this medium. Suppose the wave speed is v_2 here. Therefore, in time τ it will travel a distance $v_2\tau$. Therefore, A_1A_3 would be equal to $v_2\tau$, and this is what is written here $B_1B_3 = v_1\tau$, while $A_1A_3 = v_2\tau$. And A_1A_3 would be different from B_1B_3, because of different velocities.

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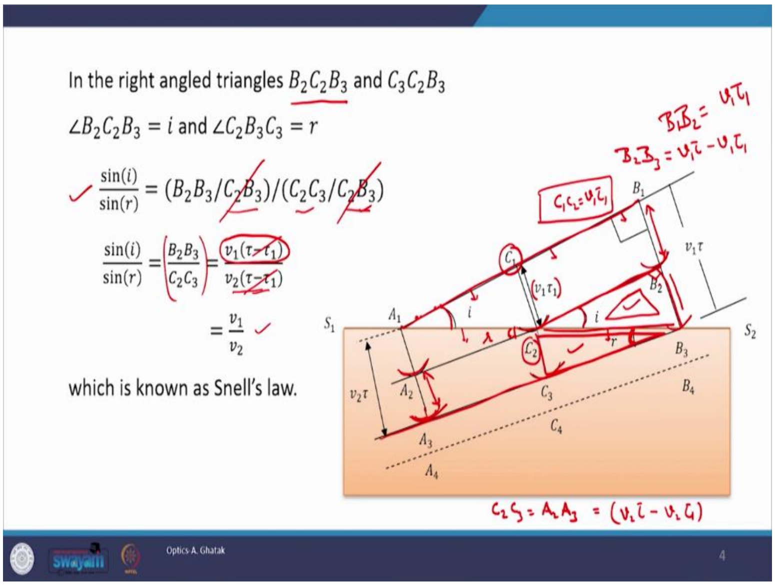

Now, let us consider 2 triangles. The first one is $B_2C_2B_3$. This is the first triangle, and this is our second triangle , $C_3C_2B_3$ is the second triangle. But before moving ahead, let me point out a few things. Now, this is the wavefront which is moving in this direction, and then after refraction it got a bit tilted and then it is traveling in this particular direction, we picked some random point C_1 on the incoming wavefront, and we assume that it took τ_1 time to reach at point C_2, and therefore $C_1C_2 = v_1\tau_1$. We also assume that angle of incidence of this wavefront at S_1S_2 interface is i. Therefore, this angle would be i, which is our angle of incidence, and suppose the angle of refraction is r. Therefore, this angle would be r, or this angle would be r.

Having known this, let us write the expression for sin i/sin r using these two triangles, what would be sin i in this upper triangle, this is our upper triangle, this is 90 degree angle, and how did we form this triangle? We just drew a perpendicular from here to here. Now, what we know is that sin i would be equal to B_2B_3 by the hypotenuse which is C_2B_3. Similarly, sin r would be C_1C_3/C_2B_3. And since this is common, C_2B_3 is common in both the denominators, it will go up and we will be left with B_2B_3/C_2C_3.

Now, as we know $C_2C_3 = v_1\tau$, and this is equal to C_2C_3. Therefore, in $B_1B_2 = v_1\tau$. And therefore, $B_2B_3 = v_1\tau - v_1\tau_1 = v_1(\tau - \tau_1)$..

Now, similarly for C_2C_3, $C_2C_3 = A_2A_3$. Here $C_2C_3 = A_2A_3$, and from here we can get that $C_2C_3 = v_2(\tau - \tau_2)$., this is what is written here. Now, from here the $\tau - \tau_1$ will go away, and we will have v_1/v_2 and which is our Snell's law.

Now, you may see that in the figure we have this type of structures, and which shows the wavefront, the secondary wavelets emit from the wavefront. This is the primary wavefront, A_1B_1 is the primary wavefront, and from each point secondary wavefront emits and these are the traces of the secondary wavefront.

And then we draw an envelope on the secondary wavefront and this envelope is $A_2C_2B_2$, from $A_2C_2B_2$ again new wavefront, new secondary wavefront emanates and these are the part of the new secondary wavefront, and then we draw this envelope on top of it. Similarly, this dashed line also $A_4C_4B_4$ again represents the newer position of the wavefront. And this is how the wavefront propagates, and this is falling from the principle Huygens's principle, this is how the wavefront propagates. The wavefront started from $A_1C_1B_1$ then it leads to $A_2C_2B_2$, then $A_3C_3B_3$ and then $A_4C_4B_4$ and so on. Now, having known the expression of sin i/sin r=v_1/v_2, we know that it verifies the Snell's law.

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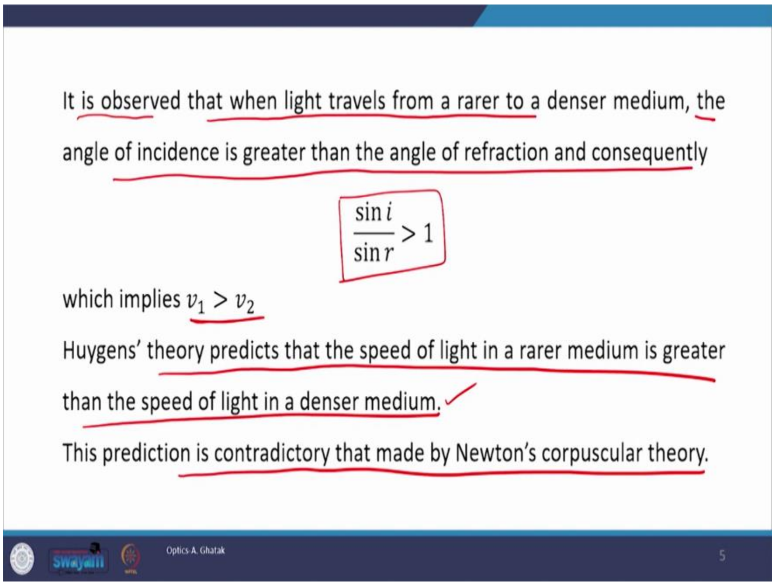

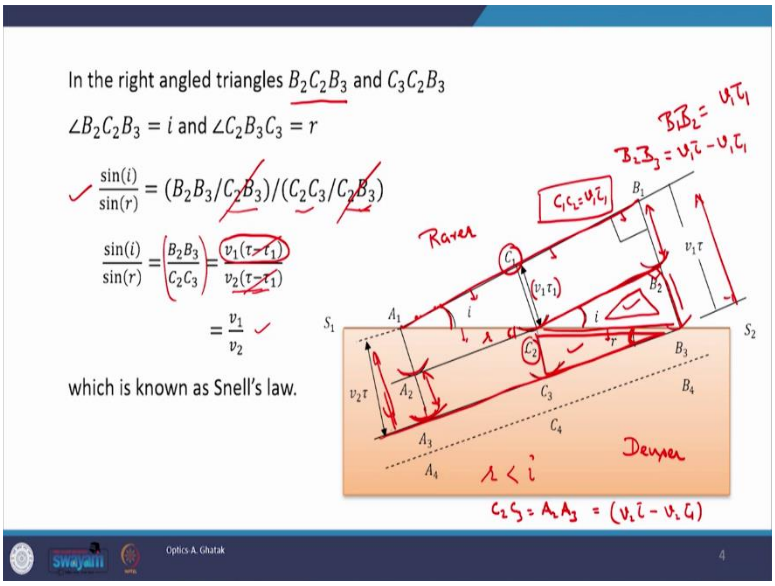

And now, it is absorbed now in the figure itself you can see that here since this is a denser medium. We assume that it is denser, and this is a rarer medium then what will happen? This distance $v_2\tau$, this distance would be smaller than this distance $v_1\tau$. And therefore, r would be smaller than i, angle of incidence would be larger than angle of refraction.

This we can, just from the geometry we can get this information. And this is true for case when light is traveling from rarer medium to the denser medium, opposite would be true for the case when light travels from denser to rare medium. And therefore, the angle of incident would be larger than the angle of refraction, and consequently sin i would be larger than sin r. Which implies that v_1 is larger than v_2, which says that if angle of incidence is larger than angle of refraction, then v_1 is larger than v_2, it means the light will travel faster in the rarer medium as compared to the denser medium.

And therefore, Huygens's principle predicts that speed of light in a rarer medium is greater than the speed of light in the denser medium, and this prediction is contradictory to that made by corpuscular theory. And the prediction made by Huygens theory is correct. And it turned down the Newton's corpuscular theory. Newton's corpuscular theory failed here. And Huygens's principle correctly predicted the relative speed of the light, or the relative speed of the wave in different kinds of medium, if a medium is denser, the light will propagate slower there.

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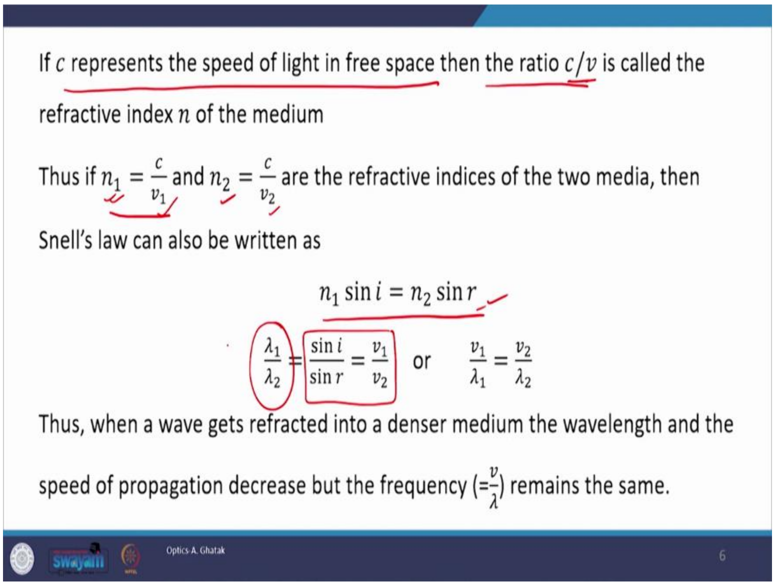

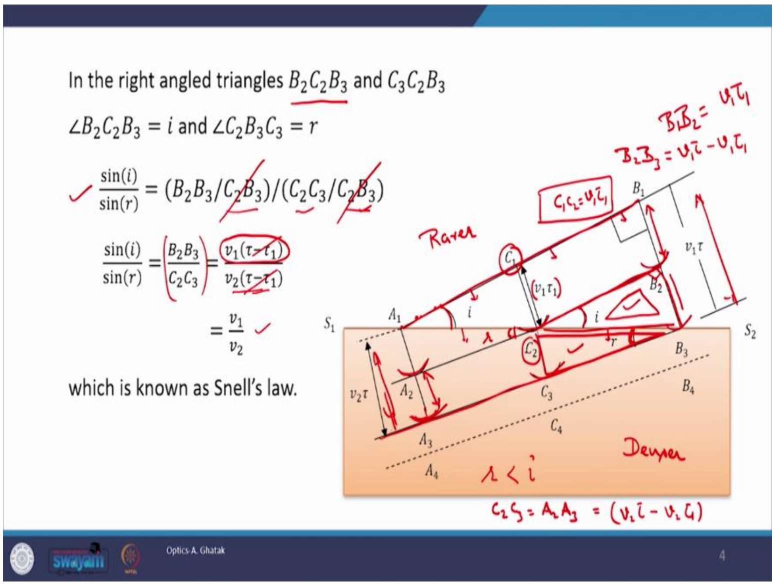

Now, if c represents the speed of light in free space, then there is c/ v we know it is called a refractive index of the medium, n is equal to c/v. We have already discussed it. Now, since the phenomena of refraction, they are involved two medium. Therefore, we will have to define 2 index of refraction. The n_1 and n_2 are the refractive indices of the two medium, n_1 is for the first medium, n_2 is for the second medium, and v_1 and v_2 are the corresponding velocities. Then, we know that Snell's law is sin i by sin r is equal to n_2/n_1 which is written here, but same thing can be written in terms of v_1, v_2 as we derived earlier, and we can also involve wavelength here too, how to involve wavelength?

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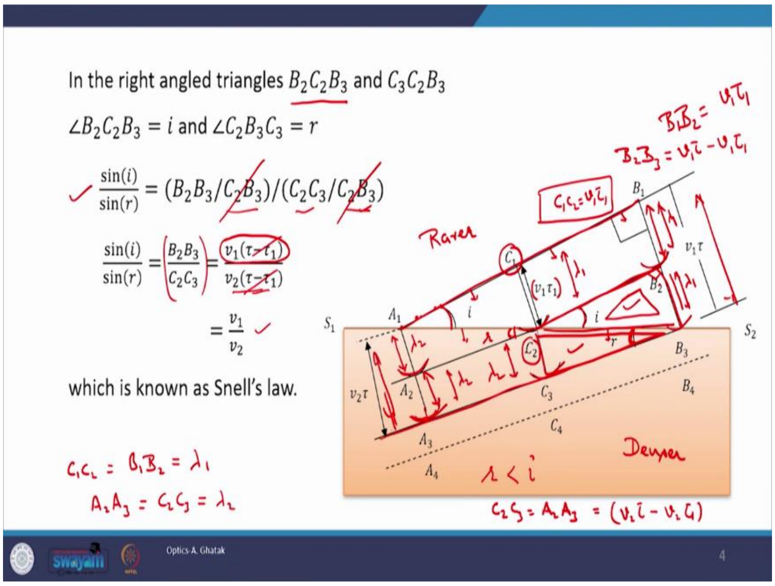

Now, suppose in this figure. Now, you see you will see that in this figure 4 plane waves are drawn, and suppose all these waves, wavefronts are separated by some distance, and suppose this are λ_2, this is again λ_2, this is again λ_2 and here it is λ_1, this will also be equal to λ_1. Because this is in the first medium, λ_1 what we can say is that $C_1C_2 = B_1B_2$, which would be equal to λ_1. And $A_2A_3 = C_2C_3 = \lambda_2$, λ_1 is the wavelength of the wave in upper medium, and λ_2 is the wavelength of the wave in the lower medium, and so on.

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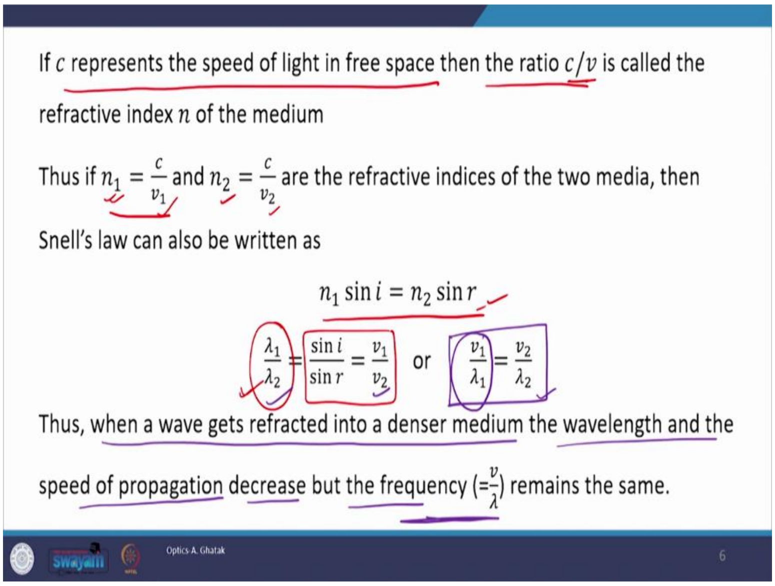

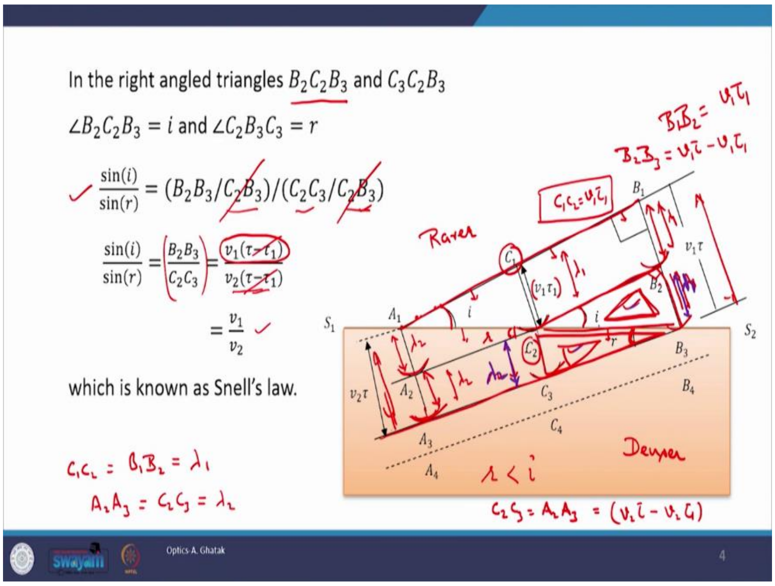

If this is the case, then what we can do is that, we can write this from the geometry in these triangle. In these 2 triangle ,using the geometry, if this distance is λ_2, and this distance is λ_1. And with the assumption that all these wavefronts are separated by their respective wavelength, with these assumptions, or we can say that all these wavefronts represent the constitutive crest with this assumption $C_2C_3 = \lambda_2$, while $B_2B_3 = \lambda_1$. And using these 2 triangles, we can say that sin i/sin r=λ_1/λ_2, and which is again equal to v_1/v_2 and from here we get $v_1/\lambda_1 = v_2/\lambda_2$.

And if they are several layered medium then we can equivalently write that $v_1/\lambda_1 = v_2/\lambda_2$, which is again equal to v_3/λ_3, and so on and so forth. And we can write v_i/λ_i for any medium. It means that the ratio v/λ is constant in this particular case. But λ changes when you go from one medium to the another medium, λ changes as well as v also changes. When we go from 1 medium to another medium, λ changes and the velocity also changes, but the ratio of the two does not change. Therefore, this ratio must be representing some important parameter, what is that parameter let us see.

When a wave gets refracted into a denser medium, the wavelength and the speed of propagation decreases, but this ratio v/λ it remains the same and we call this frequency, frequency of the source, it is a property of the source. v/λ represents a quantity which we name as frequency, and this is the property of the source, it does not have to do anything with the medium. It is not the property of the medium. This is all about refraction, and we successfully explained the phenomena of refraction using Huygens's principle.

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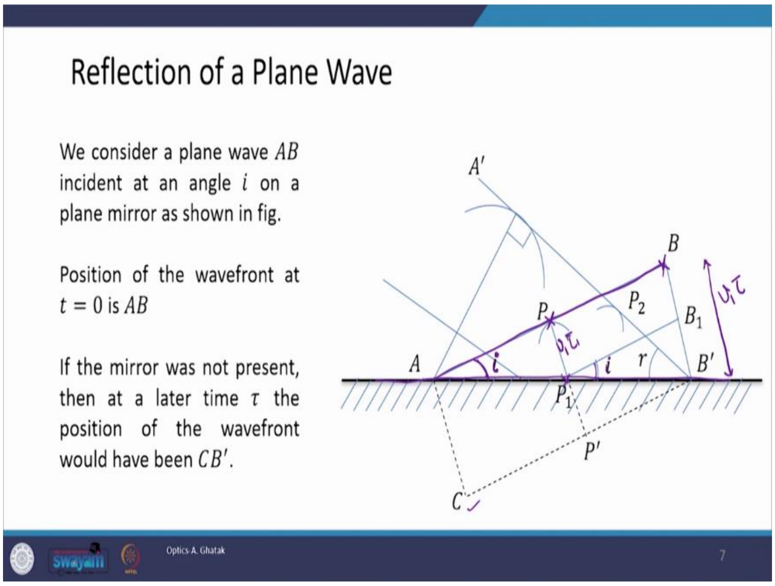

Now, we will try to understand reflection using Huygens's principle. We will start with the same concept, suppose this APB represents a planewave which is falling obliquely, which is falling at an angle i, here i is the angle of incidence on a mirror AB', this horizontal dark horizontal line is a mirror, had there been no mirror this wavefront AB would have propagated further and would have reached to position CB', CB' would have been the new position of the wavefront after certain interval, but due to the presence of this mirror, this incident wavefront could not reach to CB'.

Now, in this figure, let us again do what we did in refraction, let us assume a random point P on the wavefront, and then this point P propagate to point P_1 here on the mirror, and this angle i would be the angle of incidence, and suppose the time the point B takes to reach B' is τ. Therefore, this distance we call it as $v_1\tau$, and suppose here the time is τ_1. Therefore, this distance would be $v_1\tau_1$.

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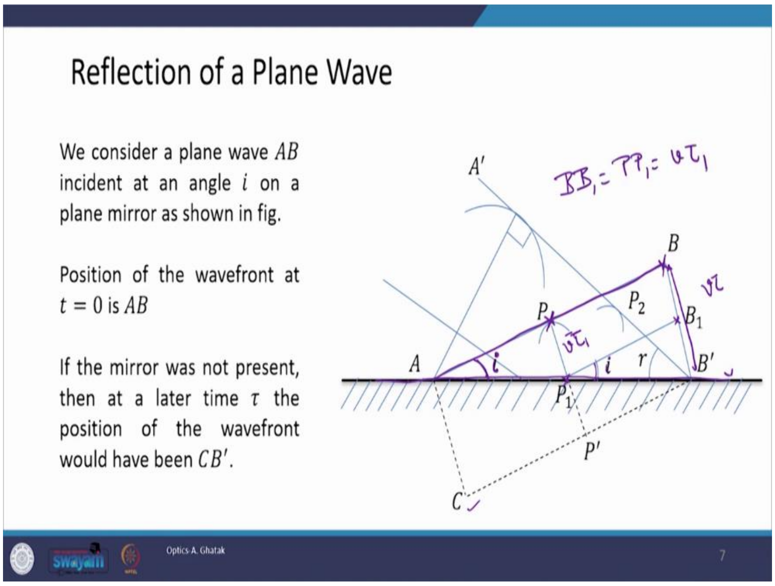

Now, there is one correction, instead of writing v here, let us write let us instead of writing v_1 here let us write v only, let us repeat it, similar to the previous discussion, we assume that the point B takes τ time to reach at point B', and the velocity in this medium of the wave is v.

Therefore, this distance would be $v\tau$, and if we pick a random point P on the wavefront AB then point P will reach to point P_1 in time $v\ \tau_1$. τ_1 is another time, the τ_1 would be shorter than τ. And then we draw a perpendicular from P_1 to BB' line, and this will meet here, and this distance BB_1 would be equal to PP_1 and which is equal to $v\tau_1$.

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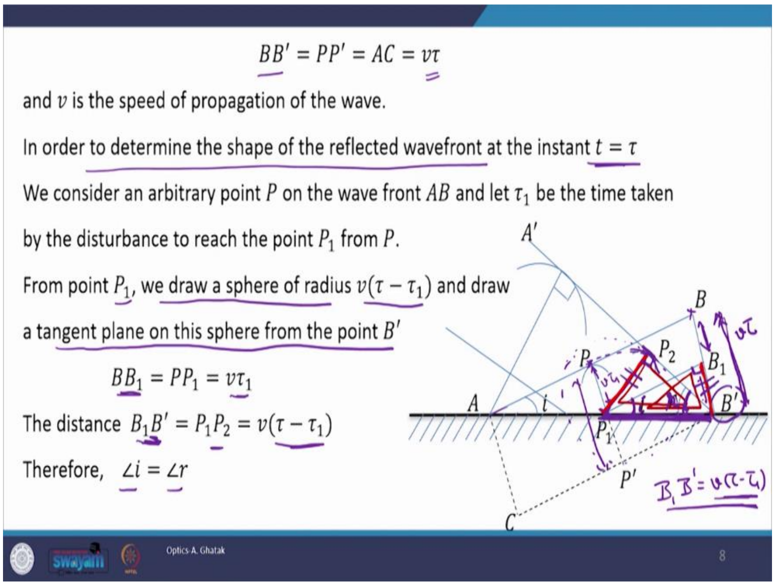

We will see this is what is exactly written here BB. Now, BB' is this distance here, BB' distance is equal to PP' distance and PP' is shown here. This is PP', and this is your BB'. Had there been no mirror BB' would have been equal to PP', and which is equal to AC which is $v\tau$, v is the velocity of the light in that medium and τ is the time it takes. Therefore, $v\tau$ represents the distance. Now, in order to determine the shape of the reflected wave at some time t is equal to τ, we consider this arbitrary point as we stated before, and we assume that this distance is $v_1\tau$, τ_1 is the time which P takes to reach P_1.

Now, from point P_1, we draw a sphere of radius $v(\tau - \tau_1)$ this whole distance $BB' = v\tau$, and the shorter distances $v\tau_1$. Now, we just take the difference between these two distances which is nothing but B_1B'. $B_1B' = v(\tau - \tau_1)$, and we will draw a sphere of radius $v(\tau - \tau_1)$ considering P_1 as the centre of the sphere.

And once the sphere is drawn, we will draw a tangent plane on this sphere from point B'. Now, from P_1 we drew a sphere and with this sphere, we drew a tangent and this tangent is fall passing or touching this sphere at point P_2. We know that BB_1 which is this distance, this is equal to $PP_1 = v\tau_1$, and the distance $B_1B' = P_1P_2 = v(\tau - \tau_1)$.

P_2 is this distance, this is P_1P_2 distance, this distance would be equal to B_1B'. Now, let us consider two triangles here to, what would be those triangles? This is the first triangle, and this is our second triangle, we will consider these two triangles. In these two triangles, this is 90 degree angle, these are right angled triangle, these are the 90 degree angles. P_1B', this line, this the base P_1B' line is common to both of these triangles, P_1B' is common and this side is equal to this side, $P_1P_2 = B_1B'$. The base is common, one side of these two triangle angles is again equal.

Therefore, 2 sides are equal and since they both are right angle triangle, one angle is also equal, and therefore, the angle of incidence would be equal to the angle of refraction, i would be equal to r therefore, and which is nothing but the law of reflection. Therefore, using Huygens's principle, we again proved that the angle of incidence is equal to angle of reflection, and which is the law of reflection.

And what did we use? We use only the concept of wavefront, we used only the Huygens's principle. We started with a wavefront AB and then we assume that the one point at wavefront is taking certain time in reaching to the mirror, and a random point again taking certain other time in reaching to some other point at the mirror, and then we drew some geometry and we found that angle of incidence is equal to angle of refraction. And this is how Huygen principle explained the reflection.

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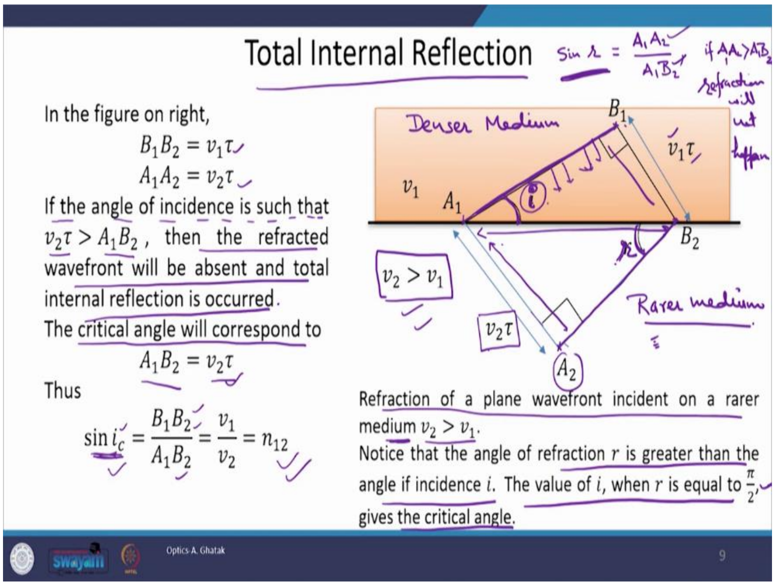

What is else which is left? Total internal reflection. Now, let us try to understand total internal reflection using Huygens's principle, using Huygens theory. In this case, we know that total internal reflection happens, when light travels from a denser medium to a rarer medium. Therefore, this shaded region, we call it as a denser medium. And this we call a rarer medium, and within the denser medium wavefront A_1B_1 is traveling, and it is met to incident on the interface between the two media, at an angle i, which is our angle of incidence. Had there been no TIR, had there been usable refraction, this wavefront A_1B_1 would have travel to the rarer medium, and in the second medium it would have been at position A_2B_2.

The new wavefront position in the rarer medium would have been A_2B_2. And following from our previous discussion, we here again assume that point B_1 sitting at the input wavefront takes τ amount of time in reaching to point B_2 at the interface, and we also assume that v_1 is the velocity or speed of the wave in the first medium, in the denser medium then $B_1B_2 = v_1\tau$. And similarly, in the second medium, $A_1A_2 = v_2\tau$.

And we also know that velocity in the rarer medium would be larger than that into the denser medium, why? Because, Huygens prove it, in our first example where we were studying refraction, we saw that the velocity of light in denser medium is smaller therefore, v_1 would be smaller than v_2, or v_2 would be larger than v_1.

Now, in time τ, B reaches to B_1, reaches to B_2, and during this time only A_1 will reach to A_2, the new position of A_1 would be A_2, and what will be the distance A_1A_2? This distance will be equal to $v_2\tau$, and since v_2 is larger than v_1, therefore, $v_2\tau$ would be larger than $v_1\tau$, or A_1A_2 would be larger than B_1B_2. Now, angle of incidence is given here, and this would be the angle of refraction, assuming that it is refraction happening here. Now, let us go to the text, if the angle of incidence is such that $v_2\tau$ is larger than A_1B_2, A_1B_2 is this distance. Then the refracted wavefront will be absent, and total internal reflection will occur.

Let us repeat, if we want TIR to happen here, then this A_1B_2, this must be smaller than $v_2\tau$, let us take sin of r, r is the angle of refraction, then sin of r would be equal to A_1A_2/A_1B_2, this is from the figure. Now, if we want $v_2\tau$ or A_1A_2 to be larger than A_1B_2, then what will happen?

Now, if A_1A_2 is larger than A_1B_2, then what will happen is that, r the refraction will never happen. The refraction will not happen if A_1A_2 is larger than A_1B_2, why? Because the maximum value of sin r is equal to 1, and if A_1A_2 is larger than A_1B_2, then the value of sin r would be larger than 1, which is not allowed. Therefore, refraction will not happen, and the wave will go into the first medium itself which is TIR, total internal reflection.

Now, the critical angle will occur when $A_1A_2 = A_1B_2$, that is the maximum allowed value of A_1A_2. The maximum allowed value of A_1A_2 would be that it become equal to A_1B_2. And in this case, $A_1B_2 = v_2\tau$. In this particular case, we can define the critical angle, and which is represented by sin i_c. Now, let us calculate sin i_c, sin i_c would be equal to $B_1B_2/\ A_1B_2$, what is B_1B_2? It is $v_1\tau$, and what is A_1B_2? It is equal to $v_2\tau$. And therefore, sin i_c would be v_1/v_2 which is nothing but n_2/n_1. Which is the n_1, n_2 the refractive index.

Now, refraction of a plane wavefront incident on a rarer medium is being considered here. Therefore, v_2 is larger than v_1. The angle of refraction r is greater than the angle of incidence. Now, in this case you can see here, because this is the rarer medium, and in rarer medium, this v_2 is larger. Therefore, r would be larger than i, the value of i when r is equal to $\pi/2$, we get critical angle. And this is all for total internal reflection, and we saw now here that we cannot get refraction if i is larger than a certain value, and that value is i_c here. We keep increasing i, let me reframe this.

Wavefront is made to incident on some non-zero angle, and it falls from a denser medium to the interface, to an interface and this interface separates a denser medium from the rarer medium. Now, this wavefront falls in an incident angle i, as long as this i is small, that angle of refraction is such that A_1A_2 is smaller than A_1B_2, and we have refraction.

Now, situation comes when $A_1A_2 = A_1B_2$, and this is called critical angle. And in this case, we get the TIR, and if we increase i, such that A_1A_2 becomes larger than A_1B_2, then refraction is not allowed. Because the angle of refraction is such that sin of r is now larger than 1. If A_1A_2 is larger than A_1B_2, then sin r would be larger than 1, which is not allowed. Therefore, refraction will not happen. And the wave will go into the first medium itself which is your TIR, which is defined as total internal reflection. This is all for today, I end my lecture here, and see you in the next class. Thank you for being with me.