Nuclear Astrophysics

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Module # 02

Lecture # 09

Thermonuclear Reactions and Reaction Cross-Section

Welcome back, students, to today's lecture, the ninth lecture in this course. I have been talking about thermonuclear reactions. So, before discussing more about thermonuclear reactions, which are the processes responsible for energy production from the stars and the synthesis of elements in the whole universe. Before discussing more properties, let me quickly summarise the previous lecture.

So, in the recap, I have discussed the mass and luminosity relationship of a star. It is linear, and M is proportional to $L^{3.5}$. Then I have quickly discussed the salient features of Hubble's law, its isotropic nature, expansion of the universe and then remnants of 2.7 Kelvin, which is the echo of the big bang 14 billion years ago. Whatever radiation was created after 14 billion years now when we can detect this 2.7 Kelvin from the distant object that has confirmed that the big bang happened around 14.3 billion years ago.

These are some of the salient features of selected general properties of the universe. Then I started discussing the thermonuclear reactions and how many burning stages in stars, hydrogen burning, helium-burning, carbon, oxygen, nitrogen burning, silicon burning, S process, r-process and p-process and then l-process. So, these are the 6 burning stages that constitute the synthesis of the majority of the elements in the universe.

The thermonuclear reactions can be divided into these 6 types. Each type will be discussed in detail in due course. So, let us start today's lecture.

Let me start from the basics with which you are very much familiar. To maintain the flow of the course, let me quickly refresh some concepts. What is the source of nuclear energy? The mass defect, ΔM_n, n corresponds to the nucleus, N corresponds to neutrons.

$$\Delta M_n = M_n - ZM_p - NM_N$$

Multiplying the mass defect with c^2 gives us the binding energy. A nuclear reaction which usually is a fusion of two nuclei releases energy. In the fission process, a heavy nucleus splits into two medium nuclei. It emits some more particles like neutrons and gamma. It also releases energy that is the basic thing for the construction of any nuclear reactor or the production of nuclear power. You are all familiar with the nuclear reactors based on nuclear fission. Where is the reactor based on nuclear fusion? Commercially no fusion-based reactor is available at present. Very interesting research is going on in the project ITER, which is the acronym for International Thermonuclear Experimental Reactor located in France.

And if you are interested more in computational engineering or science-oriented research internship, this is the ITER website(https://www.iter.org/). Please go through it. You will get many opportunities to do project works, doctoral studies and post-doctoral studies. So, this is one of the important facilities for the fusion-based nuclear reactor to produce the power out of nuclear fusion, not from nuclear fission.

And this is the world's largest magnetic confinement plasma physics experiment. So, basically, in ITER a sun is being created. We are trying to create a sun inside the laboratory by using the magnetic confinement of the plasma.

So, the basics of nuclear reaction start like this. The reaction between 1 and 2 leads to the production of 3 and 4.

$$1 + 2 \rightarrow 3 + 4$$

This is a general representation of a nuclear reaction where all these numbers denote some nuclides. The Q-value of a nuclear reaction n stands for nuclear.

$$Q_n = M_{n,1} + M_{n,2} - M_{n,3} - M_{n,4}$$

Now, it is not possible to measure the mass of the bare nucleus. Q-value can be calculated by taking the nuclear masses because it is a reaction between the nuclei. It is not a reaction between the atoms. So, ideally, we should take the difference in the sum of masses of reaction products and nuclides which are initiating the reaction, but it is not possible to measure the nuclear mass directly. That is the reason when you try to calculate the Q-value of a nuclear reaction, you go for the atomic masses.

So, Q_n stands for nuclear Q-value Q_a stands for atomic Q-value.

$$Q_a = M_{a,1} + M_{a,2} - M_{a,3} - M_{a,4}$$

The mass of an atom is the mass of the nucleus plus the number of protons multiplied by the mass of one electron because the number of electrons and number of protons is the same in a stable atom. Binding energy has to be subtracted from the sum of the mass of the nucleus and Zm_e.

$$M_a = M_n + Zm_e - B_e(Z)/c^2$$

Now we can relate the nuclear Q-value and atomic Q-value.

$$Q_a = Q_n + m_e c^2(Z_1 + Z_2 - Z_3 - Z_4) + B_e(Z_1) + B_e(Z_2) - B_e(Z_3) - B_e(Z_4)$$

$$Q_a = Q_n + \Delta B_e$$

Atomic Q-value is the sum of two terms, nuclear Q-value and ΔB_e. In general, one can neglect the ΔB_e. As it is very, very small. However, let me present before you one exciting calculation.

For example, a proton interacts with the ^{17}O emitting α and ^{14}N.

$$^{1}_{1}p + ^{17}_{8}O \rightarrow ^{14}_{7}N + ^{4}_{2}\alpha$$

So, how do you find out the Q-value of a nuclear reaction using mass excess? What is mass excess, ΔM ?

$$\Delta M = (M - Am_u)c^2$$

I suggest you download an app named IAEA, International Atomic Energy Agency. Mass excess and many more properties of each nucleus are provided in the app.

$$Q = ME_p + ME_{^{17}O} - ME_{^{14}N} - ME_\alpha$$

$$= 7288.97 + (-808.81) - 2683.42 - 2424.92 = 1191.83 \text{ keV}$$

Another method is to find out the Q-value, using only atomic masses.

$$Q = M_{a,p} + M_{a,^{17}O} - M_{a,^{14}N} - M_{a,\alpha}$$

$$= [1.007825 + 16.99913170 - 14.0030740 - 4.0026032]931.5 \text{ MeV} = 1.19185 \text{ MeV}$$

No need to remember all these values. The values will be provided in the exam.

Another point which I would like to highlight regarding the importance of binding energy correction. This correction cannot be neglected when you come across positrons.

Proton plus proton gives rise to deuterium (hydrogen with mass number 2) and electron with positive charge and neutrino.

$$p + p \rightarrow d + e^+ + \nu_e$$

For Q-value calculation, using atomic masses

$$Q_a = (M_H + M_H - M_D)c^2 = 1.44\ MeV$$

However, if you go for the nuclear masses.

$$Q_n = (M_p + M_p - M_d)c^2 = 0.93\ MeV$$

So, this highlights the importance of taking atomic masses and nuclear masses while dealing with the binding energy correction. So, this is all about some basics of Q-value of the nuclear reaction and the calculations. So, I have discussed two different methods to calculate the Q-value of nuclear reactions. Both give us the same value.

When nuclear masses and atomic masses are important in calculating the Q-value, that is the involvement of positrons. There you cannot neglect the binding energy correction. So, I have given you some hints. Please try to do it on your own to get a better idea, but remember this particular reaction proton + proton giving rise to deuterium, positron and neutrino.

This is the fundamental nuclear reaction which has not been measured till now because of various challenges, which I will discuss several times when the situation demands in the course.

So, let us go forward. The cross-section is the most crucial property or quantity related to the nuclear reaction. The whole Nuclear Astrophysics course lies in understanding this concept of the cross-section of nuclear reaction. What is the cross-section of nuclear reaction? I hope you are aware of the word cross-section. Even then, to maintain the flow of the course, let me give you a quick idea about the cross-section of a nuclear reaction because it is essential to find out how many reactions occur per unit volume.

When any nuclear reaction happens, you want to find out per unit volume, per unit time. It may be within the star or when you carry out any nuclear reaction in the laboratory on the earth. It is important to see how many nuclear reactions are taking place per unit time per unit volume. It is important to measure the cross-section of a nuclear reaction to find out.

How to find out. What is the basis of the nuclear reaction. In cross-section calculation, is it okay if you go for classical criteria, or shall we go to quantum mechanical case. Let us see what a cross-section of nuclear reaction looks like. Classically,

$$\sigma = \pi(R_p + R_t)^2$$

If you take the sum of radii of the projectile and the target and take its square and multiply pi into it. This value is nothing but the area seen by the reaction. Bigger the area, more the probability for the reaction to happen. If it is smaller, then the probability will be very less. Of course, some more parameter is also involved. I am trying to give you some general case of the classical treatment of the nuclear reaction cross-section value. So, the simplest definition is you go for sum of the radii of target and projectile. In stars, when nuclear reactions happen, there is no meaning to say this is a target, this is the projectile because both are moving within the star.

So, there is nothing like a target and projectile inside stars. On earth in the laboratory when you perform the same nuclear reaction, we have to accelerate one entity and the other is at rest. So, that kind of situation within the star it is not possible in earth's laboratory. So, the nuclear reaction has to be carried out by placing a target at rest mostly.

All those things regarding the measurement techniques and how many types of accelerators can be available which are relevant for the Nuclear Astrophysics I will discuss in due course. For the time being, let us try to understand the concept of nuclear reaction cross-section. So, this is a classical treatment. For example, if I go for proton plus proton classically, it is 0.2 barn.

$$^{1}H + {}^{1}H \rightarrow \sigma = 0.2 \times 10^{-24}\ cm^2$$

In terms of m^2, 1 barn is 10^{-28} m^2. So, 0.2 barn is the cross-section for proton plus proton. If I go for proton plus uranium 238.

$$^{1}H + {}^{238}U \rightarrow \sigma = 2.8 \times 10^{-24}\ cm^2$$

Projectile is proton only then you see the change in the cross-section from 0.2 to 2.8 barns order of magnitude is the same. It is barn only.

Now, let me go for another combination of target and projectile. Uranium 238 plus uranium 238

$$^{238}U + {}^{238}U \rightarrow \sigma = 4.8 \times 10^{-24}\ cm^2$$

Then I am getting the same order of magnitude, same barn, but instead of 2.8, it is 4.8.

However, because it is a quantum mechanical process, the nuclear reaction cannot be treated by classical physics, which is well known to all of you. So, one has to go for quantum mechanical treatment.

And quantum mechanically, how to calculate the cross-section? What are the parameters that come into the picture?

$$\sigma = \pi \lambda\!\!\!^{-2}, \qquad \lambda\!\!\!^{-} = \frac{m_p + m_t}{m_t} \frac{h}{\sqrt{2 m_p E_l}}$$

So, this is the relation for calculating the cross-section of a nuclear reaction based on quantum mechanical treatment. So, this is a general description of the quantum mechanical treatment to express the cross-section of a nuclear reaction. Now another important thing in understanding the cross-section is its dependence on the projectile's energy. Let us try to understand this in a better way.

$$^{15}N(p,\alpha)^{12}C \qquad \sigma \sim 0.5\ b \quad Strong\ force$$

$$^{3}He(\alpha,\gamma)^{7}Be \qquad \sigma \sim 10^{-6}\ b \quad EM\ force$$

$$p(p, ve^{+})d \qquad \sigma \sim 10^{-20}\ b \quad Weak\ force\ (yet\ to\ be\ measured)$$

Now you see if I consider the same energy for the projectile, say, 2 mega electron volt. It could be any radiation it is okay. If the strong force is involved in the nuclear reaction, for example, proton on nitrogen 15 then the cross-section is 0.5 barn. If electromagnetic interaction is involved, then you see alpha plus helium 3.

If electromagnetic interaction is involved for the same energy of the incident projectile, the cross-section is reducing into some micro barn. If you go for weak interaction like proton plus proton which is giving me deuterium, neutrino and positron. Then the cross-section comes down to 10^{-20} barn. As I said experimentally, it has not been measured yet. Then how do we know this? We have a strong theory supporting this cross-section value.

And why it is not possible to measure the cross-section of proton plus proton reaction. What are the experimental challenges? With beautiful numbers, I will discuss in due course. What is the essence of this slide? Cross-section of a nuclear reaction is not only dependent on the energy of the projectile, but also the nature of the interaction involved in the nuclear reaction.

Depending up on the nature of the interaction, out of these three forces, strong force, weak force and electromagnetic force the cross-section value dramatically changes. Remember, I have fixed the energy of the projectile constant in this case.

So, as part of our discussion on cross-section, I have shown you the classical treatment, and I have shown you how it depends on the nature of the interaction as well. So, it is important to keep it in mind because it is a cross-section of a nuclear reaction which is a fundamental quantity in this course. I hope you know the basics of quantum mechanics. By knowing about the wave function, you can get almost every piece of information about the system.

So, if you have wave function in hand, you can do many things in quantum mechanics like this in Nuclear Astrophysics by knowing the value of cross-section you can understand the role of nuclear physics in explaining the features of the universe like synthesis of elements in the universe and production of the energy from the stars. So, let us continue.

Let me do some mathematical calculations or derivation of the cross-section of nuclear reaction. For that, what are the quantity that comes into picture? Suppose you assume the projectile beam whose energy is A. The number of particles in this beam, say N. It is falling on some target material with number of target nuclei, say N_t.

And with respect to the incident direction, if the particles are undergoing some kind of scattering. Or if undergoes some kind of reaction. Number of product nuclei or ejectile are getting detected by detector. Detector's front region is defined by

$$dF = r^2 d\Omega$$

$d\Omega$ is a solid angle subtended by the detector at the reaction point. r is the distance between the detector and source. Now here sigma is the probability that an interaction takes place. N_b is particles per unit time t covering the target area A. N_t is total number of non-overlapping nuclei. N_R/t is number of interactions per unit time. This is also equivalent to say number of particles emitted per unit time.

What is this R? Number of reactions per unit time every reaction is giving some kind of product.

Sigma is defined mathematically

$$\sigma = \frac{N_R/_t}{N_b/_t \times N_t/_A}$$

as N_R divided by t it is divided by N_b/t into Nt / A. Now you have to pay attention to the dimensions on both sides units. So, what is the number of interactions taking place per unit time divided by the number of incident particles.

N_t corresponds to number of target nuclei within the beam.

Finally area is coming as a unit that is centimetre square and we will use this general definition to describe the reaction probabilities. We will use this concept to describe the probability of the nuclear reaction in astrophysical plasmas, also, in the measurement of the nuclear reactions in the laboratory. So, with this basic idea of the nuclear reaction cross-sections let me continue the discussion on nuclear cross-sections and some more parameters in the next lecture. Thank you so much for your attention. See you in the next lecture.