Nuclear Astrophysics

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Lecture - 38

Kinematics - I

Welcome students to today's lecture. In the previous lecture if you remember I have discussed activation method. Sometimes in the literature you will also see it is also known as activity method. So, what was the advantage of activation method? When you deal with a compound nucleus formed in nuclear reaction having a complicated decay scheme of a gamma rays.

Then you need to use the large number of detectors around the target chamber or sometimes you may have to use large volume detector. And many times, you have to handle the beam induced background as well. To get rid of these problems one of the beautiful and convenient methods suggested was activation or activity method, in which how did we measure the cross section? It is a two-step process.

Number one, activate the target material. In general, this is used for neutron induced reactions but it is applicable to charged particle activation method also. In general, you know the elements beyond iron they are synthesized due to neutron induced reactions. To perform neutron induced reactions in the laboratory one of the methods suggested is this activation method.

The only condition is that the compound nucleus after reaching the ground state it has to be unstable having half-life of few seconds to few days. Sometimes, people have been for few months also but in general it is suggested for few minutes to few hours. With this in the step one you will activate the target using the projectile. Then you will switch off the beam in the control room of the particle accelerator.

Then you take out the activated target which is the decaying continuously and keep that target which is undergoing decay which is emitting particles in front of the detector. If you are interested in better resolution then use the high purity germanium detector in general when the activated target emits gamma rays of more than one energy. And if they are closely separated from each other then HPG is mandatory to use.

If the activated target emits 1 or maximum 2 gamma rays of energies which are widely separated from each other, then one can easily think of time-tested scintillation detectors like Sodium Iodide and nowadays people are using large volume advanced scintillation detector like lanthanum bromide and cerium bromide. So, this is what I have discussed in the previous class.

The mathematical derivation to calculate the cross section was also discussed in the previous class. So, in today's lecture I am going to cover following topics.

I have covered the recap just now and some salient features of activation method. Some conditions I will discuss then I will try to provide some useful information regarding the kinematics of a nuclear reaction, when collision is happening between projectile and target nucleus in the laboratory. How to understand the change in the energy of product nucleus and change in the energy of the product particles?

Sometimes, we use the word ejectiles with respect to the incident beam energy. So, the today's lecture mostly it is going to cover mathematical formulation and it is very important when we discuss the experimental aspects of nuclear astrophysics. So, though under the category of experimental nuclear astrophysics I am discussing these topics. Today's lecture will be mostly based on mathematical formulation to understand the kinematics of nuclear reaction.

And if time permits, I will discuss another direct measurement, the time of flight method which is very interesting to measure the resolution of neutron beams in an efficient manner. So, let us start today's lecture with remaining part of the activation method, then I will discuss the kinematics of the nuclear reaction.

So, in activation method, what should be the thickness of the target it should be thick or it should be thin? How to select the thickness? Remember, activation method deals with the activating the target with incident beam. Mostly it is neutron beam. So, how does the thickness of the target matters in this case? If it is too thin what will happen if it is too thick what will happen.

So, is there any criteria to decide the thickness of the target meant for activation method? In which the reaction yields a nucleus which is unstable having half-lives ranging from a few minutes to few months. So, there are three important considerations for the thickness of the target material. Number one if, it is too thick then what is the problem? There can be significant attenuation of incident neutrons.

Now, I am focusing on the neutron beams, I am considering the incident particles as neutrons how to produce neutrons. I have discussed in one of the previous lectures. So, assume that neutrons are used to activate the target and then in the step two we are taking the activated target to another room. So, for the target in the step one when reaction take place, if it is too thin what will happen the count rate will be less.

In general, we prefer the thin target in order to avoid the energy loss mechanism within the target material, but if it is too thin then the count rate is of course a concern. So, let me write another thing if it is too thin then count rate is of course a concern and less count rate means bad statistics. So, either you have to acquire for a long time which requires large amount of beam time.

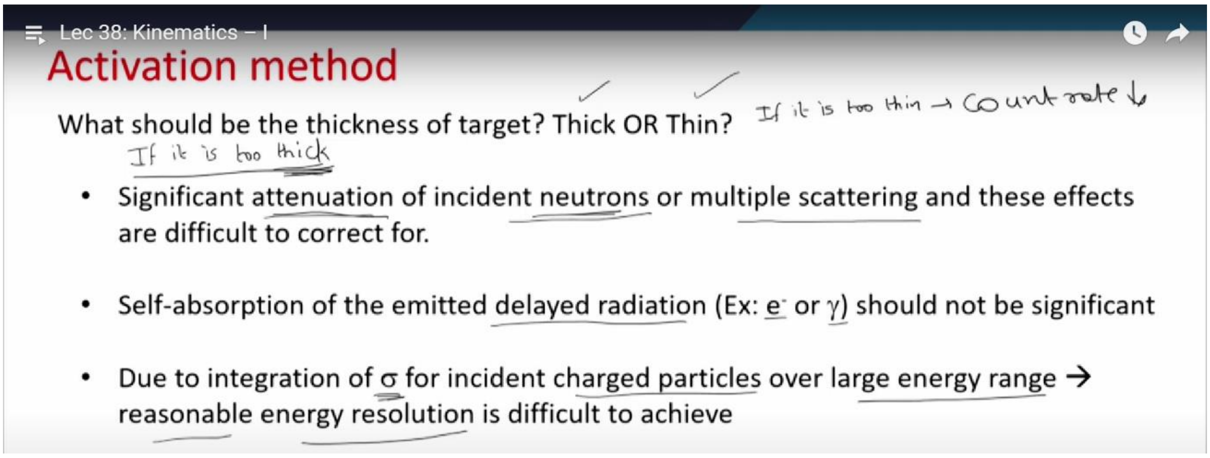
Then can we go for too thick material? It also has some problem like following. If you go for very thick target material the incident neutrons can undergo attenuation in a significant way, not only that multiple scattering can also take place. Do you remember how neutron interacts with matter? See what is the energy range of the neutrons? Which are of interest for the nuclear astrophysics?

Starting from few keV I mean starting from very small energy like thermal neutrons to a few keV because it is this energy range which is initiating the s and r process. Sometimes you will go for the fast neutrons like in the MeV also. Neutrons interact with matter via scattering and absorption that we have seen. It is having mass but there is no charge so there is no coulomb interaction.

So, whenever the situation demands, I will try to repeat the way neutron interacts with matter so that you can have a convincing information and you remember. So, that you can plan the experimental things and solve the numerical also accordingly. So, the scattering can be elastic scattering or inelastic scattering and absorption can lead to the fission or charged particles emission or again neutron emission more than one.

So, when we consider very thick material for activation technique multiple scattering can pose a problem. And which is not very easy to correct for. If the attenuation is huge and multiple scattering also huge then whatever effects are there which we do not want they have to be corrected when you measure the cross section. So, it is difficult to correct for the effects of multiple scattering and significant attenuation within the target material by the incident neutron beam.

So, this is one of the important concerns. Number 2 if the incident particle beam is charged particles. Then, coming back to these neutrons only whenever neutron interact with this target material if it is too thick then the emitted delayed radiation it could be electron or gamma undergo self-absorption. If the target is too thick so the self-absorption of the emitted delayed radiation should not be significant. So, that is why thickness also should not be too high.



Activation method

What should be the thickness of target? Thick OR Thin? If it is too thin → Count rate ↓

If it is too thick

- Significant attenuation of incident neutrons or multiple scattering and these effects are difficult to correct for.
- Self-absorption of the emitted delayed radiation (Ex: e^- or γ) should not be significant
- Due to integration of σ for incident charged particles over large energy range → reasonable energy resolution is difficult to achieve

Number 3 because when we; consider the charged particles as the projectile beam. And if the thickness is too large there is a need to integrate the cross section for incident particles over large energy range because of this it is difficult to get reasonable energy resolution. So, because of these 3 considerations one should not go for target material of high thickness. At the same time, one should not go for the very low thickness also.

Otherwise, it will give us very less count rate. So, one has to decide the thickness of the target materials by keeping in mind all these considerations. So, with this let me close the topic of activation technique or activity method which is strictly applicable when the reaction leads to an unstable nucleus only. And what is other advantage of this activation method? It can get rid of the beam induced background.

And also, when you come across the complicated decay scheme of the excited component nucleus formed in the nuclear reaction. No need to use huge infrastructure in terms of the experimental setup. So, with this I am closing the topic of activation technique and as I have said in the starting of my class let me discuss the kinematics of a nuclear reaction in which, I am going to give a good number of formulae for understanding the relation between various parameters that we come across in any nuclear reaction.

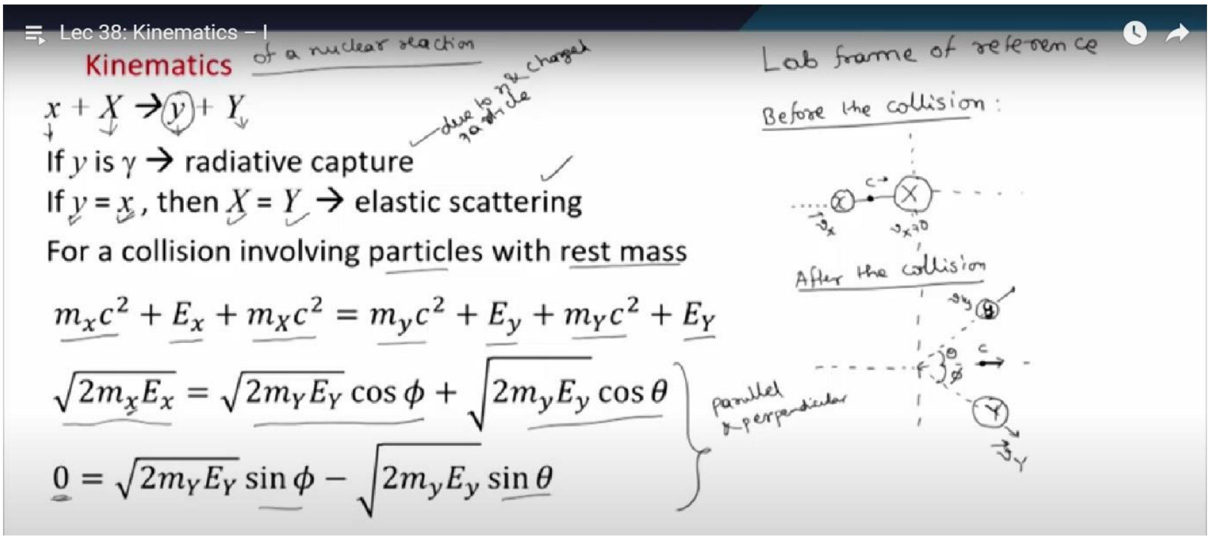
So, the kinematics of a nuclear reaction in any text book you can get. This is standard mathematics. So, I will discuss in brief. So consider this x as projectile and capital X as the target material small y as the ejectile and capital Y as product nucleus. Now, if this ejectile is gamma ray then we call this reaction as a radiative capture reaction. It could be due to neutrons or charged particles.

In any case if the ejectile is gamma ray then we say this reaction as radiative capture reaction. What if y and x both are different small y and small x? That means ejectile and projectile if both are same. Then the target nucleus and the production nucleus they are also same and then we say this as elastic scattering process this we know very well. So, consider a collision now which involves particle with rest mass strictly.

So, now to derive formulae for energies of the ejectiles and product nuclei in different frames of reference. Let us consider a collision process considering the particles which have rest masses. Now I am confining the discussion to only laboratory frame of reference. Lab frame of reference or lab system; where before the collision considering the projectile which is moving with velocity vx.

Because in the laboratory frame of reference the target nucleus is at rest. So, vx = 0. So, the centre of mass if you consider it is moving. The centre of mass is moving in the laboratory form of reference. This is before collision where target nucleus is at rest and projectile is moving with velocity vx and let us draw like these different directions. Now after the collision we can draw the same thing as to maintain the uniformity.

Let me draw like this, we have this species b is moving with velocity vy and this is the direction of incident beam and we have capital Y product nucleus which is moving with velocity capital y. The direction can be specified in terms of theta with respect to the incident beam direction. And the product nucleus is moving in a direction specified by say phi and here we can denote the motion of centre of mass like this.



Kinematics of a nuclear reaction

$x + X \rightarrow \boxed{y} + Y$

due to the charged particle

If y is γ → radiative capture

If $y = x$, then $X = Y$ → elastic scattering

For a collision involving particles with rest mass

$$m_x c^2 + E_x + m_X c^2 = m_y c^2 + E_y + m_Y c^2 + E_Y$$

$$\left.\begin{aligned}\sqrt{2m_x E_x} &= \sqrt{2m_Y E_Y}\cos\phi + \sqrt{2m_y E_y}\cos\theta \\ 0 &= \sqrt{2m_Y E_Y}\sin\phi - \sqrt{2m_y E_y}\sin\theta\end{aligned}\right\}$$

parallel & perpendicular

Lab frame of reference

Before the collision :

After the collision

So, when we consider a simple collision, you might have seen many such cases when you have been taught the mechanics collision between two particles two bodies in mechanics. So, the difference that we are seeing here is the nature of the reaction, like if it is radiative capture reaction then what are the changes, whether there will be any change in the energies of the products with respect to the incident beam.

So, that nature of nuclear processes comes into picture when you consider the collision between these two entities like small x and capital X. Let us continue, so for a collision involving particles with rest mass applying the conservation of energy this is the rest mass of the projectile this is the kinetic energy of the projectile. And because that kinetic energy of the target nucleus is 0, I am using only the rest mass of the target nucleus.

And after the collision, the conservation of energy gives the rest mass of product particle like ejectile and kinetic energy of the ejectile, rest mass of the product nucleus and the kinetic energy of the product nucleus. So, this is the expression we get when we apply the conservation of energy. Now let us apply the conservation of momentum, the momentum of target nucleus is 0. So, it is missing on the left-hand side.

So, you have the momentum of incident beam which is designated with small x and when we consider two components parallel component and perpendicular component you have cos phi and cos theta and perpendicular component is 0 for the left-hand side and after the collision you have sin phi and sin theta. So, these two presents the parallel and perpendicular components of the nuclear reactions, when we apply the conservation of energy and momentum.

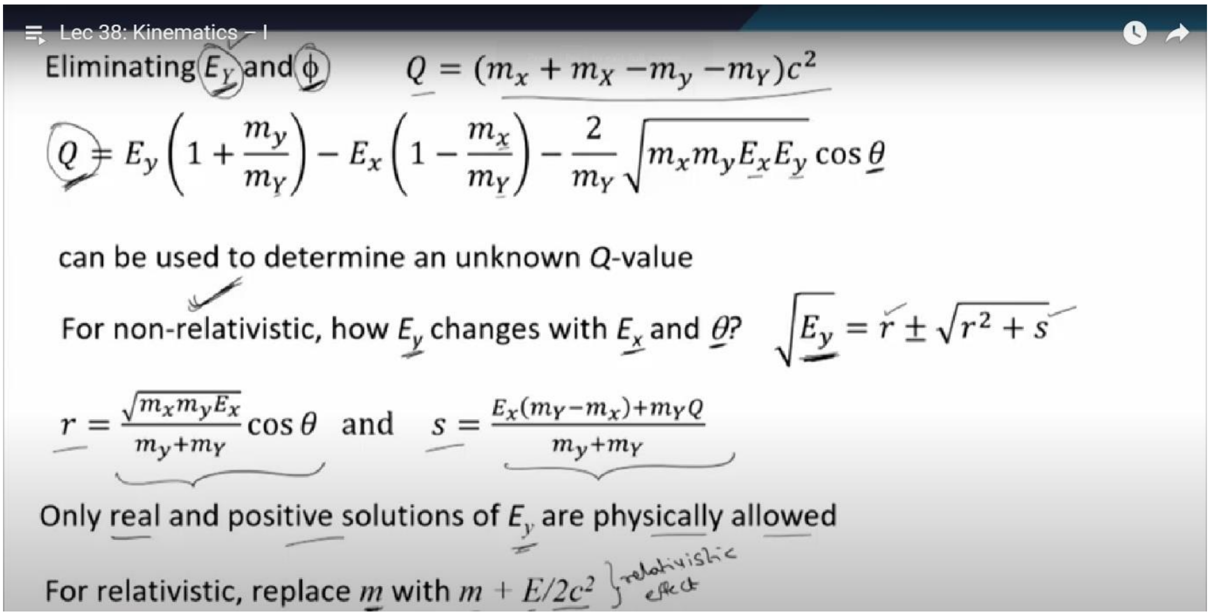
Let us continue, in general, in order to eliminate the kinetic energy of the product nucleus because in general they are heavy and the velocity and kinetic energy associated with is in general ignored. But we cannot ignore without any reason but because product nucleus is heavy when we compare with the ejectile mass we normally eliminate it and also the angle with which it is moving with respect to the incident beam direction.

So, when we eliminate these two quantities from the previous equations and using this well-known relation of the Q value of the nuclear reaction one can get expression for the Q value of the nuclear reaction in terms of masses of the species before and after collision in the nuclear reaction. And here, you do not see kinetic energy of the product nucleus and the angle with which it is moving.

And this equation can be used if the Q value of the reaction is not known. If we know the masses of all the entities in a nuclear reaction, one can easily get the value of Q value. But if we; do not know the mass of the product nucleus capital Y how one can determine the Q value of the nuclear reaction. So, this is a formula which helps us to calculate the Q value of the nuclear reactions.

What are the quantities required? Of course, you need energy of the projectile, energy of the ejectile and the angle with which ejectile is moving with respect to incident beam direction and masses of small x small y and capital X capital Y. Here, what is unknown is the energy of the product nucleus Ey if we do not know this, this equation can be helpful for calculating the, unknown Q value of nuclear reaction.

Now, confining the discussion with non-relativistic only then it is always interesting and in general we always look forward to have a relation between the energy of the ejectile and the angle with which it is emitted. How the energy of the ejectile is changing with respect to the incident particle beam energy and the angle with which this ejectile is moving with respect to the incident beam direction.



Eliminating E_Y and ϕ $\quad Q = (m_x + m_X - m_y - m_Y)c^2$

$$Q = E_y\left(1 + \frac{m_y}{m_Y}\right) - E_x\left(1 - \frac{m_x}{m_Y}\right) - \frac{2}{m_Y}\sqrt{m_x m_y E_x E_y}\cos\theta$$

can be used to determine an unknown Q-value

For non-relativistic, how E_y changes with E_x and θ? $\quad \sqrt{E_y} = r \pm \sqrt{r^2 + s}$

$$r = \frac{\sqrt{m_x m_y E_x}}{m_y + m_Y}\cos\theta \quad \text{and} \quad s = \frac{E_x(m_Y - m_x) + m_Y Q}{m_y + m_Y}$$

Only real and positive solutions of E_y are physically allowed

For relativistic, replace m with $m + E/2c^2$ $\Big\}$ relativistic effect

So, if you do simple mathematics one can come up with an expression for the energy of the ejectile in terms of two symbols that is small r and small s where this small r and small s can be given by these two terms. You can easily work out. Now you see we are interested in knowing the change in the ejectile energy with respect to the incident beam energy and the theta value.

Now, according to this equation we can see that only real and positive solutions of ejectile energy are physically allowed. Now, as I said these formulae represent when we consider the non-relativistic case. What about relativistic case? Easily can be written just by replacing the mass m with m + E by 2 c square to correct the relativistic effects.

Now, let us consider two cases when we are writing the change in ejectile energy with respect to the projector energy. Number one; in case if it is exothermic case and in general in the nuclear astrophysics we come across the projectile. Of course, projectile and target they are relative when reactions are happening within the stars but when we perform the nuclear reaction in the particle accelerator. So, we use the word projectile and target.

And in general, the projectile is having less mass compared to the product nucleus. Because in indirect methods this may not be the case. So, in the direct measurements and in general the reaction relevant for nuclear astrophysics they have mass of the projectile less than the mass of the product nucleus. Then in the previous slide if you see the value of small s we have written the expression for small r and small s.

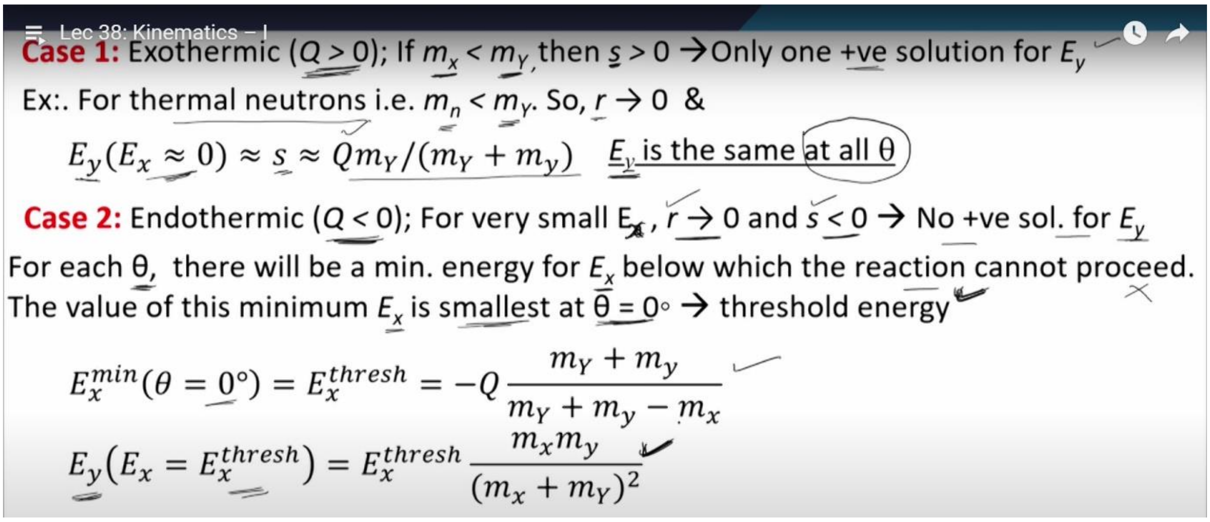
So, if you see small s is greater than 0, this implies that there is only one positive solution for the ejectile energy. See we are interested in the understanding of ejectile energy. Let us take an example for thermal neutrons of course the mass of the neutron is much less than the mass of the product nucleus and if you see the value of r it is tending to 0 and this gives rise to because the project energy is almost 0. Because, if you see the thermal neutron the energy is 0.025 you know electron volt not even one electron volt it is 25 milli electron volt. So, it is almost 0, so in that case the ejectile energy is nothing but s value and this gives rise to a formula involving masses of product entities small y and capital Y multiplied by Q value of the nuclear reaction. It also gives an interesting feature when incident particles have mass much lesser than the product nucleus mass.

Then at all values of theta the ejectile energy will be same it is not the case every time. If it is exothermic that means Q is positive and if the projectile mass is much less than the product nucleus mass. Then in this particular example of thermal neutrons Ey has the quantities Q value and the masses of the product entities that is small y and capital Y.

So, the energy of the ejectile is independent of the angle of emission. So, when you do some experiment this has to be kept in mind if you come across this kind of situation, you can measure the energy of the particles at any angle with respect to the incident beam direction. Now, let us go for the second case that is if, Q value is less than 0 that is endothermic.

And considering the same thing like it is a projectile energy is very less, Ex in that case also r is tending to 0 but s is less than 0 what is s and r, already have shown you in the previous slides. So, there are no positive solutions for Ey no positive solution for ejectile energy, what does it mean? It means we require some energy, some minimum energy for the projectile incident beam.

Then only reaction can happen then only we can have some energy for the ejectile. So, for each value of theta in this particular case there will be a minimum energy for projectile energy below which the reaction cannot proceed and the value of this minimum projectile energy is smallest at theta = 0 only and this is called as threshold energy.



Case 1: Exothermic ($Q > 0$); If $m_x < m_Y$, then $s > 0$ → Only one +ve solution for E_y

Ex:. For thermal neutrons i.e. $m_n < m_Y$. So, $r \to 0$ &

$E_y(E_x \approx 0) \approx s \approx Qm_Y/(m_Y + m_y)$ E_y is the same at all θ

Case 2: Endothermic ($Q < 0$); For very small E_x, $r \to 0$ and $s < 0$ → No +ve sol. for E_y

For each θ, there will be a min. energy for E_x below which the reaction cannot proceed.
The value of this minimum E_x is smallest at θ = 0° → threshold energy

$$E_x^{min}(\theta = 0°) = E_x^{thresh} = -Q\frac{m_Y + m_y}{m_Y + m_y - m_x}$$

$$E_y(E_x = E_x^{thresh}) = E_x^{thresh}\frac{m_x m_y}{(m_x + m_Y)^2}$$

Mathematically how to express this? Threshold energy at theta = 0 which is the minimum is given by this formula when we take the help of s and r values. So, accordingly Ey at the threshold energy can be expressed in this way. So, we will continue the discussion on this kinematics of the nuclear reaction. In the next lecture seeing the more aspects of the energies of the projectiles and ejectiles, what is the relation and in another frame of reference as well. Thank you very much.