Nuclear Astrophysics
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Lecture – 26
Formation of ^{12}C

Namaste students. In today's lecture we are going to discuss one of the most important nuclear reactions in the field of nuclear astrophysics. As I have been telling many times in the lectures, nuclear reaction is the reason for evolution of universe and synthesis of elements since the big bang, say 14.7 billion years ago. As part of recap, let me quickly inform you about what I have discussed in the previous lectures.

In PP Chain and then CNO cycle, I have shown a few nuclear reactions which are responsible for the synthesis of various elements, isotopes, isobars and isotones. We have seen various nuclides in the synthesis of elements, the role of PP chain and CNO cycle and burning of hydrogen which plays very important role. What will happen after hydrogen is exhausted?

We are going to discuss in today's lecture about what is the ash of hydrogen burning and how it plays important role in the creation of Carbon-12 nucleus. Please recollect from HR diagram, the location of white dwarfs, sequence stars, red giants, super red giants. I am sure you will remember that super red giants have not yet been discussed.

In today's lecture, I am going to correlate the synthesis of Carbon-12 with super red giants and how helium burning is playing role in the synthesis of this Carbon-12 and also in the formation and evolution of super red giants, and then the nuclear reaction which is responsible for the creation of Carbon-12, called as triple alpha process. This is one of the beautiful nuclear reactions which take place inside the star.

This triple alpha process is going to answer one of the important questions which I had posed in the initial lectures, that is stability gaps. What do you mean by stability gap?

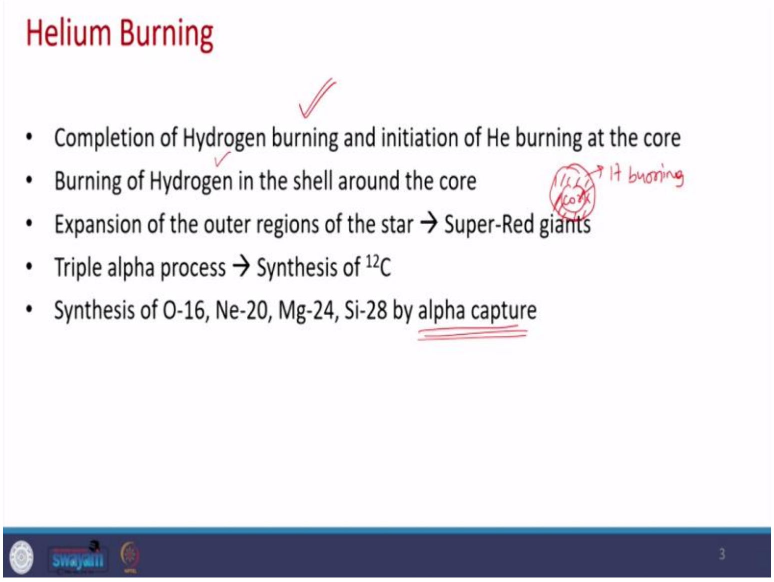
Mass number 5 and mass number 8: these are termed as stability gaps. How are elements formed by maintaining these gaps? It was a big question for the researchers. On one hand, we see the elements in the universe, while on the other hand, we see the gaps at A = 5 and A = 8.

What is the answer for this stability gaps? Who is bridging the stability gaps? We are going to understand these in today’s lecture. So, you can imagine the importance of today’s lecture when compared to all other lectures.

It is not only informative, but also a classical example of nuclear physics fundamentals and how they have played role in understanding the synthesis of elements and the evolution of universe finally. So, it requires more attention from your side. Now, can Carbon-12 be formed due to helium burning in normal state, like many nuclides?

Can it be formed in the ground state or not? I have discussed the concept of non-resonant reaction induced by charged particles and resonant reactions induced by charged particles.

In today’s lecture I am going to take the help of charged particle induced resonant reaction in understanding the formation of Carbon-12. So, overall lecture is centered upon the creation of Carbon-12. Whether the nuclear reactions leading to the formation of Carbon-12 can be done in the laboratory using the direct techniques or indirect techniques? If we cannot do it using direct technique, why? And what are the indirect measurements by which we can perform any reaction for the synthesis of Carbon-12. So, let us see how these points can be covered and what are the answers for some of the questions present in the slide.



Helium Burning

- Completion of Hydrogen burning and initiation of He burning at the core
- Burning of Hydrogen in the shell around the core
- Expansion of the outer regions of the star → Super-Red giants
- Triple alpha process → Synthesis of ^{12}C
- Synthesis of O-16, Ne-20, Mg-24, Si-28 by alpha capture

In helium burning you have to understand what happens once hydrogen gets completed. What is the ash of the hydrogen burning, once hydrogen is exhausted in the star? It is helium. So it is quite natural that once hydrogen is exhausted, the contraction of the helium core starts. Helium starts burning when hydrogen is exhausted and helium starts contracting at the core. There will be an increase in the density. There will be an increase in the temperature. It will continue. But, interestingly, as the helium core contraction happens, hydrogen starts burning in the shell around the core.

The expansion of the outer regions of the stars happens in such a manner that, many a times, the radius of the shell will be around 50 times larger than the original radius.

Burning of hydrogen in the shell is causing the expansion of the outer shell whereas helium burning is taking place in the central core. These two processes happen simultaneously.

So, the size is becoming bigger and bigger. Because of this, the star is considered as giant star. Now where is red color coming from? The contraction of the helium core and the expansion is happening in such a way that the surface starts cooling after a certain time. The temperature of the surface falls till the wavelength enters into the red region. So, because of the shifting of the wavelength into the red color and the outer expansion of the core to a big size, we call this star as super red giant star.

Remember hydrogen is exhausted at the core and it is helium which is burning. So, it is quite natural to expect so. What will happen because of this burning of helium, what kind of elements are formed and how it helps us to understand the creation of Carbon-12? We are going to discuss these in today's lecture.

So, it is α+α leading to the formation of beryllium 8. It is happening inside the core and there is always a very less probability that third alpha can also participate in the reaction. If this triple alpha process happens at all, we can expect the formation of Carbon-12. This is how synthesis of Carbon-12 can be understood. But things are not so easy. We will discuss how it happens, is it a single, double or triple step.

Why should only Carbon-12 formation happen? Once Carbon-12 is formed, we have plenty of helium in the core. So, α+C should lead to the formation of oxygen 16 and α+O should lead to the formation of neon 20, α+Ne should give rise to Mg 24, α+Mg should give rise to silicon 28. All these elements are expected to be synthesized just because of alpha capture.

Why are we discussing alpha capture? Because helium is an ash of hydrogen burning and we have 98% to 99% helium in the core. So, because the majority of the star's core is helium, we can always expect alpha capture reactions to take place continuously. Are all these elements formed in reality? No.

The formation of the elements stops at oxygen 16 only. It will not go further. There is a beautiful physics reason behind it that we will discuss later. Before that, we need to understand nuclear reactions and physics behind it, for the formation of Carbon-12. So, let us see how Carbon-12 is formed with this background.

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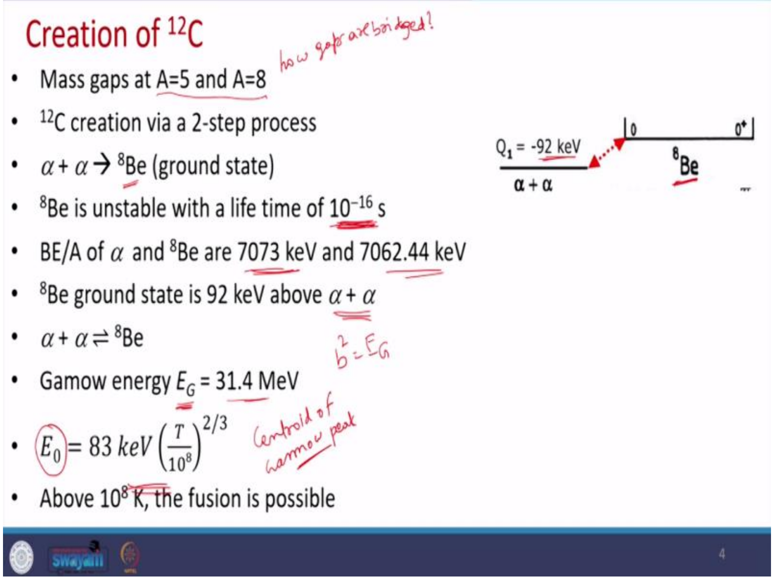

Creation of ^{12}C

how gaps are bridged!

- Mass gaps at A=5 and A=8
- ^{12}C creation via a 2-step process
- $\alpha + \alpha \rightarrow {}^{8}Be$ (ground state)
- ^{8}Be is unstable with a life time of 10^{-16} s
- BE/A of α and ^{8}Be are 7073 keV and 7062.44 keV
- ^{8}Be ground state is 92 keV above $\alpha + \alpha$
- $\alpha + \alpha \rightleftharpoons {}^{8}Be$
- Gamow energy $E_G = 31.4$ MeV
- $\boxed{E_0} = 83\ keV \left(\frac{T}{10^8}\right)^{2/3}$
- Above 10^8 K, the fusion is possible

$b^2 = E_G$

Centroid of Gamow peak

People have given a lot of thought on how to bridge the stability gaps, mass gaps. When it was clear that once hydrogen is exhausted, it is a helium at the core. May be α+α is giving rise to beryllium 8 and α + ^{8}Be is giving rise to the formation of Carbon-12.

For this, no element or nucleus with mass number 5 or 8 is needed. Salpeter was the researcher who proposed that the creation of carbon-12 as a two-step process. First α+α gives rise to beryllium 8 in the ground state. But the problem is that it's lifetime is of the order of 10^{-16} seconds.

So, what can you expect? Once α and α undergoes fusion with each other, beryllium 8 is formed and it quickly goes back to two alpha particles. So, we can expect some kind of equilibrium between two alpha particles and beryllium 8. On one hand, I am saying Carbon-12 can be formed because of this process. But on the other hand, I am saying α+α is giving rise to beryllium 8 and it is quickly coming back to two alpha particles. So, where is the question of Carbon-12 formation. That is where the interesting aspect lies. Binding energy per nucleon for alpha is 7073 keV and for beryllium 8, 7062.44 keV. You can see in this level scheme that ground state of ^{8}Be is 92 keV above the α+α threshold. It is very less.

Now, if you calculate the Gamow energy (E_G) for this α+α, it is basically b^2. If you have Gamow energy, it becomes easy to calculate the centroid of the Gamow peak.

I am using the word Gamow peak because these two are charged particles. Had it been induced by some non charged particle, like neutrons, there would be no point in discussing about the Gamow peak. For α+α reaction, Gamow energy is about 31.4 MeV. Please remember the value. Substitute this Gamow energy into the formula of centroid of the Gamow peak.

What is the meaning of this centroid of Gamow peak? It means, if the temperature is above 10^8 K, there is a possibility that fusion can happen.

I am highlighting this because this is the temperature region in which α+α reaction is taking place. There are regions where synthesis of many elements can take place, like 10^7 K, 10^8 K or much below that. And sometimes you may have to go for 10^{10} K. As part of this course, you need to know the temperature region during in which a certain type of nucleus is formed.

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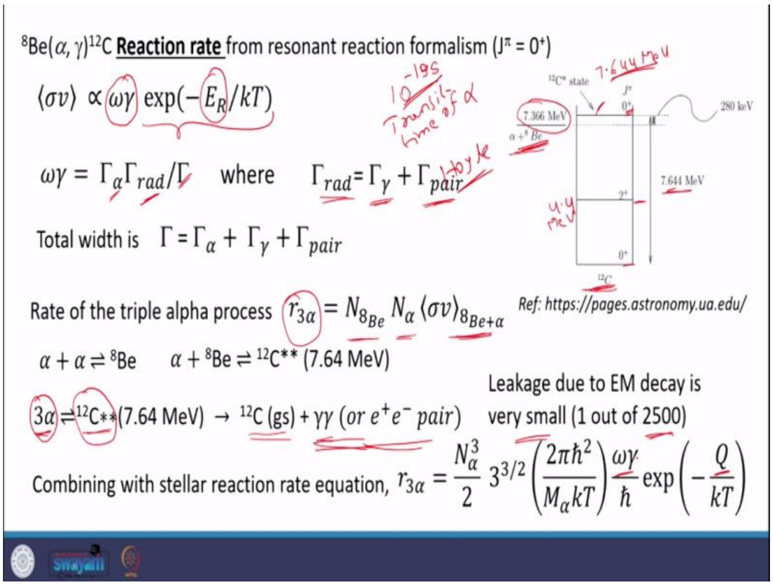

^{8}Be$(\alpha, \gamma)^{12}$C **Reaction rate** from resonant reaction formalism ($J^\pi = 0^+$)

$$\langle \sigma v \rangle \propto \omega\gamma \exp(-E_R/kT)$$

$$\omega\gamma = \Gamma_\alpha \Gamma_{rad}/\Gamma \quad \text{where} \quad \Gamma_{rad} = \Gamma_\gamma + \Gamma_{pair}$$

Total width is $\Gamma = \Gamma_\alpha + \Gamma_\gamma + \Gamma_{pair}$

Rate of the triple alpha process $r_{3\alpha} = N_{^8Be} N_\alpha \langle \sigma v \rangle_{^8Be+\alpha}$

$\alpha + \alpha \rightleftharpoons {}^8$Be $\quad \alpha + {}^8$Be $\rightleftharpoons {}^{12}$C** (7.64 MeV)

$3\alpha \rightleftharpoons {}^{12}$C** (7.64 MeV) → ^{12}C (gs) + $\gamma\gamma$ (or e^+e^- pair)

Leakage due to EM decay is very small (1 out of 2500)

Combining with stellar reaction rate equation, $r_{3\alpha} = \frac{N_\alpha^3}{2} 3^{3/2} \left(\frac{2\pi\hbar^2}{M_\alpha kT}\right) \frac{\omega\gamma}{\hbar} \exp\left(-\frac{Q}{kT}\right)$

Ref: https://pages.astronomy.ua.edu/

10^{-19}s Transit time of α

Hoyle

7.644 MeV

4.4 MeV

In that temperature range, the transit times between the alpha particles is about 10^{-19} seconds. Before beryllium 8 goes back to the two alpha particles, there is a probability that beryllium 8 can capture alpha again. This is the turning point in the formation of Carbon-12

You can see the level scheme here. Ground state of ^{12}C, first excited state at about 4.4 MeV and second excited state at 7.644 MeV.

Second excited state is not very easy to measure. What is the reason? Q value of α + ^{8}Be reaction is about 7.366 MeV. What is so special about it? When people tried to understand the abundance of elements from the observations, there was no agreement with the theoretical predictions if the formation of excited state Carbon-12 happens in a normal way.

Hoyle gave a beautiful theoretical suggestion for which he won the noble prize. Hoyle suggested that we can always get agreement between the theoretically calculated abundance of Carbon-12 and experimentally measured abundance, provided the Carbon-12 nucleus is formed in the excited state through a resonance mechanism.

Spin and parity of ground state of ^{12}C is 0^+. Therefore, the excitation to or de-excitation from second excited state is forbidden (0^+ to 0^+). Hoyle predicted that the centroid of the Gamow peak for α + ^{8}Be is somewhere around 230 keV.

If you add α + ^{8}Be threshold, i.e., 7.366 keV and Gamow peak centroid, i.e., around 230 to 280 keV, there should be an excited state in Carbon-12 at that energy. You can easily calculate and verify that it is about 280 keV.

Now once it is known that the reaction is charged particle induced resonant reaction, let us go ahead with the reaction rate formalism. The reaction rate for the resonant reaction is given by:

$$\langle \sigma v \rangle \propto \omega\gamma \exp\left(-\frac{E_R}{kT}\right)$$

where E_R is Hoyle state, i.e., 7.644 MeV.

And the strength of the resonance is the product of the partial widths divided by total partial width

$$\omega\gamma = \Gamma_\alpha \Gamma_{rad} / \Gamma$$

Where $\Gamma_{rad} = \Gamma_\gamma + \Gamma_{pair}$

From this 7.644 MeV Hoyle state to the ground state, gammas are emitted in cascade.

From 7.644 to 4.4 MeV and then from 4.4 MeV to ground state, two gammas are emitted, as direct transition is forbidden. Second possibility is that 7.644 MeV energy can be converted into electron-positron pair as well. So, we need to consider the partial width corresponding to these two processes.

So, total width is:

$$\Gamma = \Gamma_\alpha + \Gamma_\gamma + \Gamma_{pair}$$

Rate of the triple alpha process $r_{3\alpha} = N_{^8Be} N_\alpha \langle \sigma v \rangle_{^8Be+\alpha}$

Coming back to the equilibrium phenomena we can express as below:

$\alpha + \alpha \rightleftharpoons {}^8\mathrm{Be}$; $\alpha + {}^8\mathrm{Be} \rightleftharpoons {}^{12}\mathrm{C}^{**}$ (7.64 MeV)

So, in a single notation we can do like below.

$3\alpha \rightleftharpoons {}^{12}\mathrm{C}^{**}$ (7.64 MeV) $\rightarrow {}^{12}\mathrm{C}$(gs) + γγ (or e^+e^- pair)

What will happen if Carbon-12 in the Hoyle state does not go back to three alpha? It goes to the Carbon-12 ground state either by emitting two gammas; from Hoyle state to first excited state and then first excited state to the ground state or it can emit electron-positron pair. Here, in principle, there is a leakage in this equilibrium, very small leakage. What is that leakage? As I said, electromagnetic decay is happening when Carbon-12 excited state and three alpha are trying to achieve the equilibrium.

This electromagnetic decay is the reason for the leakage. Fortunately, this is extremely small ,like 1 out of 2,500. So, we can always confidently say that the equilibrium can be reached. Combining this with the stellar rate equation and Saha equation, we can find out the expression for triple alpha reaction rate:

$$r_{3\alpha} = \frac{N_\alpha^3}{2} 3^{3/2} \left(\frac{2\pi\hbar^2}{M_\alpha kT}\right) \frac{\omega\gamma}{\hbar} \exp\left(-\frac{Q}{kT}\right)$$

What are the measurable quantities here? We have to find out Q value and the strength of the resonance.

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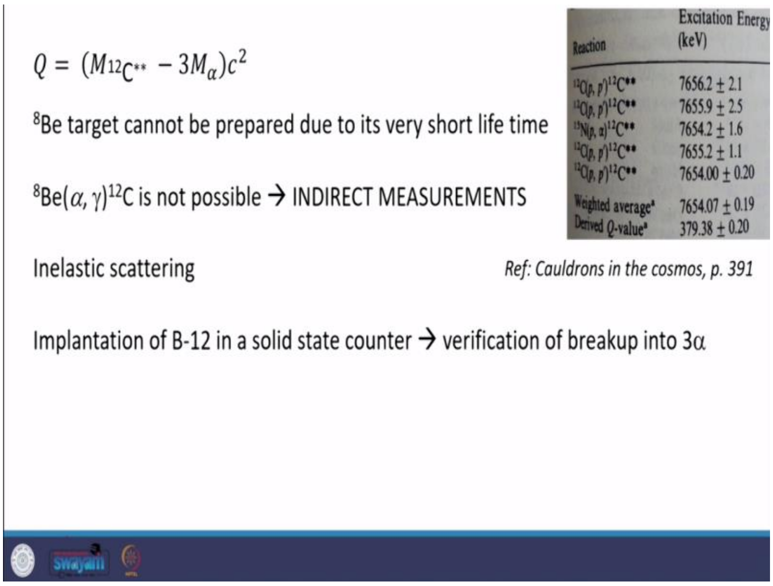

$Q = (M_{^{12}C^{**}} - 3M_\alpha)c^2$

^{8}Be target cannot be prepared due to its very short life time

$^8Be(\alpha,\gamma)^{12}C$ is not possible → INDIRECT MEASUREMENTS

Inelastic scattering

Reaction	Excitation Energy (keV)
$^{12}C(p,p')^{12}C^{**}$	7656.2 ± 2.1
$^{12}C(p,p')^{12}C^{**}$	7655.9 ± 2.5
$^{15}N(p,\alpha)^{12}C^{**}$	7654.2 ± 1.6
$^{12}C(p,p')^{12}C^{**}$	7655.2 ± 1.1
$^{12}C(p,p')^{12}C^{**}$	7654.00 ± 0.20
Weighted average^a	7654.07 ± 0.19
Derived Q-value^a	379.38 ± 0.20

Ref: Cauldrons in the cosmos, p. 391

Implantation of B-12 in a solid state counter → verification of breakup into 3α

How to find out Q value? Take the mass of the Carbon-12 in the excited state and take the masses of three alpha particles, take the difference and multiply with c^2, you will get Q value. Now there is a challenging part. One question that I had posed in the first slide. Is it possible to perform this reaction in the laboratory? How can you perform the reaction between alpha and beryllium 8? Maybe you can form α+α, but what about $\alpha + {}^8$Be ? What is the problem.

I told you that lifetime of ^{8}Be is 10^{-16}s. Because of this extremely small lifetime, it is impossible to prepare the target of beryllium 8. Not only this, but there is also another challenge.

Even when we are able to carry out the $\alpha + {}^{8}\text{Be}$ reaction, the transition is forbidden ($0^+ \rightarrow 0^+$). Because of these experimental challenges, it is not possible to carry out $\alpha + {}^{8}\text{Be}$ reaction. So, we have to go for indirect measurements. One example is inelastic scattering. So, like shown here, you can take a Carbon-12 target and take the proton beam of energy of your interest. Consider the inelastic scattering of this proton, it is p'. Because of the inelastic scattering, Carbon-12 gets excited to various states including the Hoyle state, that is, second excited state. Hoyle state will disintegrate into three alpha particles. You detect those three alpha particles using charged particle detectors like surface barrier detectors or cesium iodide charged particle detectors.

Take a set of charged particle detectors and measure three alpha particles in coincidence. Their count rate will help you in finding out the cross section of this triple alpha process. The table shows measurements performed by various researchers across the world on how the excitation energy can be measured. Finally, the Q value comes out to be like 379.38 MeV.

There is another technique to understand this triple alpha process and to measure the cross section of triple alpha process and that is implantation of Boron 12 within a solid state counter. When you implant Boron 12 within the solid state counter, there will be a breakup of Boron 12 into three alpha particles and these three alpha particles can be measured using detectors and one can find out the cross section of triple alpha process.

And to conclude today's lecture let me give you some number regarding the relation between energy produced in triple alpha process and the temperature of the star. The relation goes like this: it is proportional to T^{41}. So you can imagine the sensitivity of energy production with respect to the temperature.

We have seen how super red giants stars are formed. Because Carbon-12 cannot be formed in the early universe. If it was formed in the early universe, we could not explain the observed abundance. So it has to happen within the star. And if it has to happen within the star, you have to address how stability gaps are bridged.

The only way for this to happen, as suggested by Salpeter, is that, it is a two step process. Assume that alpha plus alpha give rise to beryllium 8 and beryllium 8 is reacting with alpha, but the problem is beryllium 8 is extremely short lived and it goes back to two alpha particles.

But there is a possibility for beryllium 8 to capture the alpha particle to form the Carbon-12. This is the contribution of Salpeter.

The contribution of Hoyle is suggesting that it is a resonant mechanism through which the excited state of Carbon-12 is just higher than the alpha plus beryllium threshold (around 280 keV) and because of that , there is a formation of Carbon-12. Only if we assume that this is happening through the resonance mechanism, the observed abundance can be explained. Later, it was confirmed by many people across the world experimentally. Experimentally, it is very challenging to do this experiment because of the impossibility to prepare ^{8}Be target and the forbidden transition between Hoyle state to ground state, direct transition by gamma rays.

So, one has to go for the indirect measurements like inelastic scattering induced by low energy protons.

In today’s lecture we have seen creation of Carbon-12. In next lecture we will understand how Carbon-12 survives in the helium burning phase. Thank you very much.