Nuclear Astrophysics
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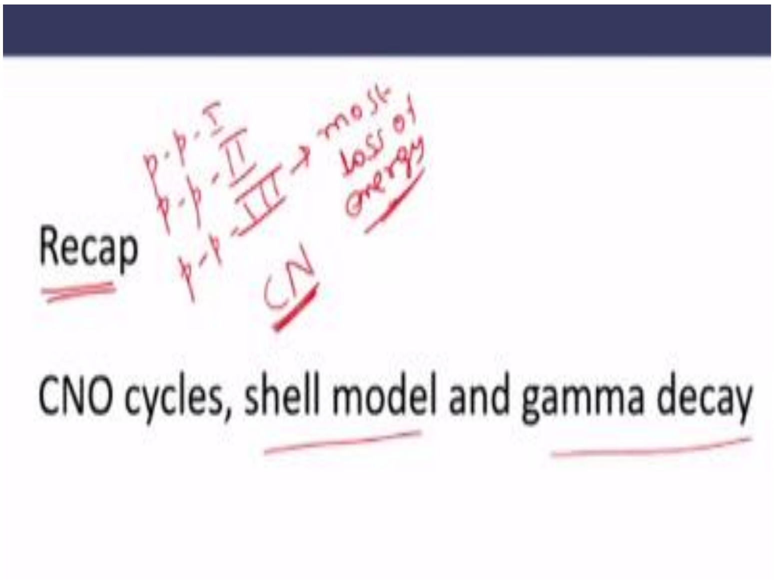
Module No. # 05

Lecture No. # 25

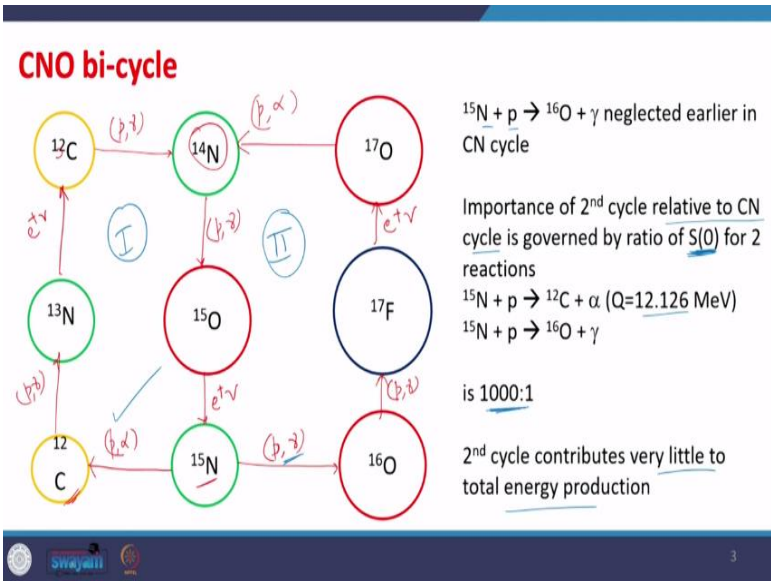
CNO Cycle, Shell Model and Gamma Decay

Welcome back to the discussion on different burning stages within the stars, which are responsible not only for the energy production, but also for the synthesis of different elements. I will have a quick review on what we have discussed in the previous lecture and then I will continue today's discussion.



In the previous lecture, I have discussed pp-1, pp-2 and pp-3 chains reactions. And, we have seen that most loss of the energy is within the pp-3 chain because of the escape of the neutrinos. And, I have discussed Carbon-Nitrogen (CN) cycle which are mainly happening within the Population-I stars where enough heavier elements, like carbon and nitrogen, are present.

In today's lecture, I will continue on how this CN cycles are happening in different ways; and how more cycles are happening within the stars, which are responsible for different types of chemical elements. I will also spend some time on the shell model and the energetics of gamma decay which are very important to understand different aspects of nuclear astrophysics.



In CNO bi-cycle, the Carbon-12 led to the production of Nitrogen-13 via (p,γ). And this Nitrogen-13 gave rise to Carbon-13 via positron decay. This Carbon-13 has given rise to the production of nitrogen 14, based on (p,γ) reaction. Nitrogen 14 produced the oxygen 15 via (p,γ). And, oxygen 15 has produced nitrogen 15 via positron decay.

This nitrogen 15 has produced carbon 12 via (p,α) reaction. We have said that carbon 12 acts as a catalyst and 4 protons are converted into helium-4. However, we have ignored a process in this cycle. Nitrogen 15, instead of (p,α), can also undergo (p,γ) reaction to produce oxygen 16.

Oxygen 16 undergoes (p,γ) reaction to produce F-17. Protons are able to induce so many charged particle reactions because of their abundant nature. Fluorine 17 is unstable, and they undergo positron decay. As the mass number is increasing, the circle size is also increasing. Just for the sake of convenience, I am using different sizes of circles for different nuclei.

Oxygen 17 undergoes reaction with proton producing alpha particle and giving rise to Nitrogen 14. So, we can say that these are two cycles in CNO. In +2, you might have heard of pp chain and CNO cycle. They are responsible for the energy production in the stars and the Sun. In terms of energy production, the pp chains play the major role. But, in terms of more number of synthesis of elements, the CNO cycle plays an important role.

Let us see some more features of the CNO cycle. As I said earlier in the previous slide, we have neglected the reaction p + ^{15}N. After including this reaction, which can always happen within the stars, we have 2 cycles. However, the importance of the 2nd cycle relative to the 1st cycle i.e., CN cycle, is governed by the ratio of S factor at zero energy. S at zero energy i.e., S(0) can be obtained by measuring the S factors at available energies and extrapolate to the zero energy.

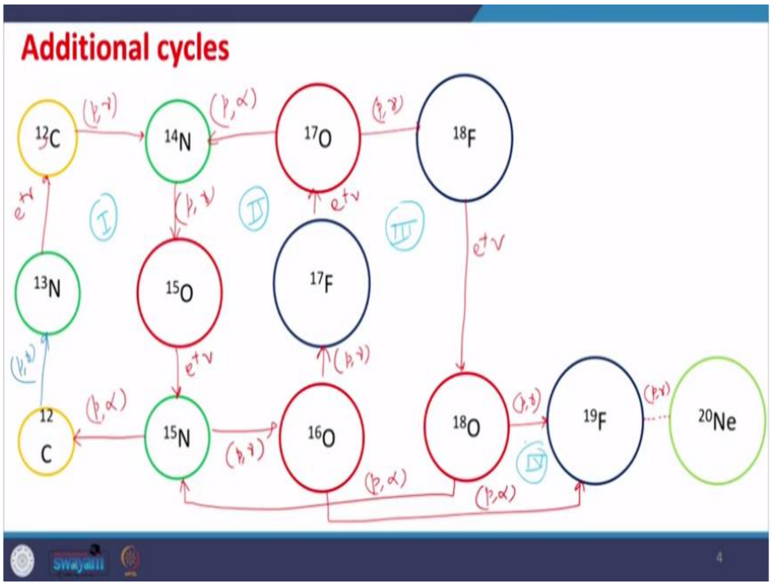
This is an important parameter in the nuclear astrophysics. Now, for these 2 reactions

$^{15}N + p \rightarrow {}^{12}C + \alpha$ (Q=12.126 MeV)

$^{15}N + p \rightarrow {}^{16}O + \gamma$

Q value is 12.126 MeV for the first reaction. And for these 2 reactions, the ratio of S(0) is 1000:1. So, $^{15}N + p \rightarrow {}^{12}C + \alpha$ is more probable than (p,γ) reactions. (p,α) is more probable compared to (p,γ). Nevertheless, it is possible that 1 in 1000 reactions can be through (p,γ), in the presence of large number of nitrogen 15 nuclei.

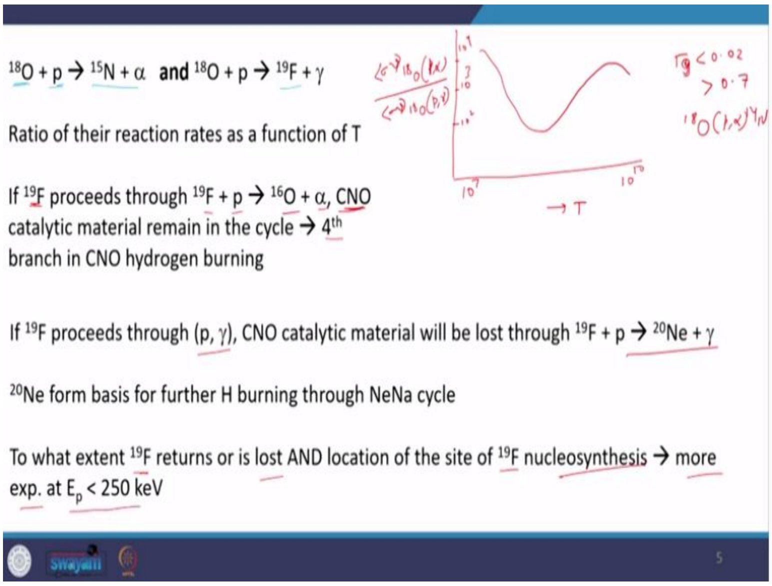
The second cycle contributes very little to the total energy production, as expected.



Now, let us see some additional cycles within the CNO cycle. Consider the same CNO cycle as I drew earlier. There is also a possibility that oxygen 17 can undergo reaction via (p,γ) to produce fluorine 18, which further produces oxygen 18 nucleus via positron decay. This oxygen 18 nucleus can undergo reaction with proton, giving rise to nitrogen 15 and emitting alpha particle. Not only that, oxygen 16 can also produce fluorine 19 via (p,α) reaction. Okay. Even oxygen 18 can produce fluorine 19 via (p,γ) reaction.

This cycle will end after formation of Neon-20. So, 4 different cycles can happen within the CNO cycle, depending on the availability of different nuclei. CNO cycles are important mainly when considering the synthesis of elements, not the energy produced from the stars.

The kind of reaction that any nucleus undergoes at any stage, has to be understood by seeing the mass numbers. And, to some extent, you need to remember them. But, this CNO cycles gives an overall idea about kind of isotopes of different elements produced. Now, let us continue the discussion.



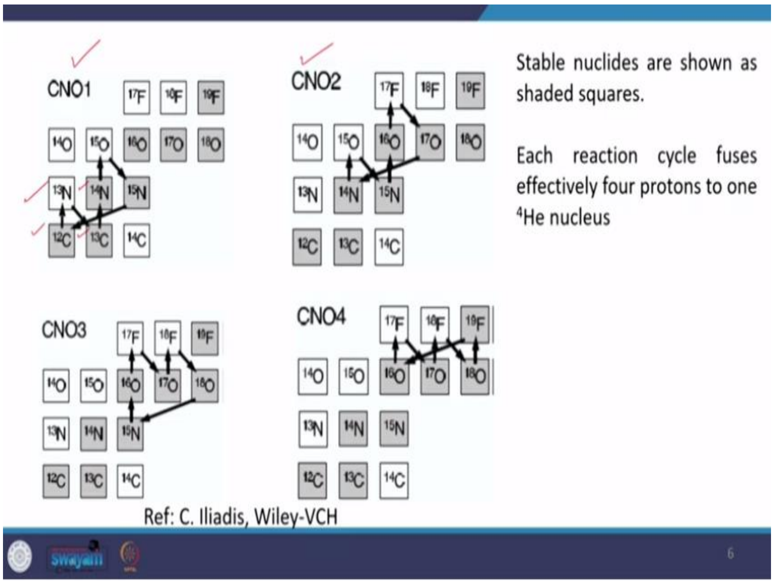
Consider $^{18}O + p \rightarrow {}^{15}N + \alpha$ and $^{18}O + p \rightarrow {}^{19}F + \gamma$. The ratio of the reaction rates as a function of T for these reactions is quite interesting. At T_9 less than 0.02 and greater than 0.7, the hydrogen burning takes place entirely through (p,alpha) reaction. So, at what temperature range by what kind of reaction oxygen 18 is undergoing destruction mechanism. That will be clear if you take the ratios of 2 different reactions in which entrance channel is same, but exit channel is different.

This diagram shows which reaction is dominant under what temperature range.

Now, we consider another nucleus, fluorine 19, which proceeds through $^{19}F + p \rightarrow {}^{16}O + \alpha$. Because of this, CNO cycle catalytic material remains in the cycle, which leads to the occurrence of 4th branch in CNO hydrogen burning.

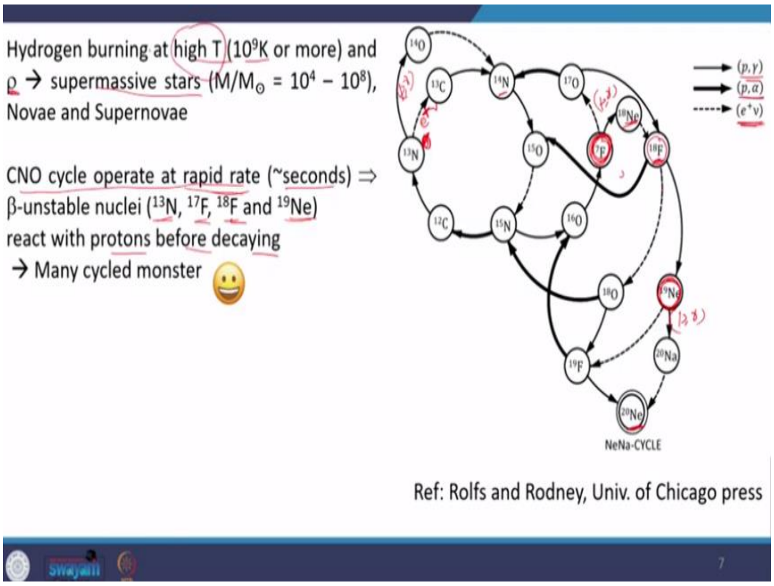
However, if fluorine 19 does not participate in (p,α), but it participates in (p,γ), then the catalytic material will be lost through $^{19}F + p \rightarrow {}^{20}Ne + \gamma$. Thus, this CNO cycle ends, forming neon 20. And, this neon 20 initiates some other cycles. This neon 20 forms basis for further hydrogen burning through neon sodium (NeNa) cycle, which is interesting to understand how different elements are formed.

To understand the location of the site of F-19 nucleosynthesis and the extent to which it returns or is lost, more experiments are needed to be carried out for proton energies less than 250 keV.



So, let me quickly show you in a different way. These 4 branches of the CNO cycle, CNO1, CNO2, CNO3 and CNO4. Initially, carbon 12 goes to nitrogen 13. It comes to Carbon 13, that goes to nitrogen 14. These diagrams help us in terms of mass numbers, proton numbers and neutron numbers. This is some representation of 4 branches in the CNO cycle which I have drawn in the previous slides.

Here, the stable nuclides are shown as shaded squares. Each reaction in CNO branches involves the conversion of 4 protons to one He-4 nucleus.



If you see the hydrogen burning at high temperatures, say 10^9 K or more, and at higher density, like supermassive stars such as nova and supernova, CNO cycle operates in a rapid way.

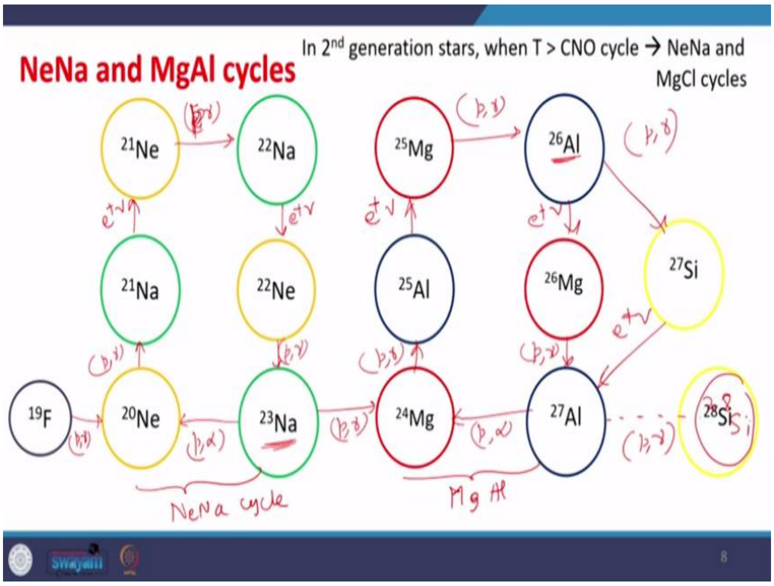
An unstable nucleus undergoes positron decay to give a stable nucleus, as we saw in CNO cycles. This may not happen always if the same unstable nuclei exist in an environment of very high temperature and concentration.

Before it undergoes positron decay, it can always react with proton leading to another reaction. See a beautiful diagram in this slide. When CNO cycle comes across very high temperature, it operates at a rapid rate. Then, unstable nuclei, like nitrogen 13, fluorine 18 and neon 19, react with protons before decaying.

So, you can expect a many cycled monster in the stars. Earlier, ^{13}N was undergoing positron decay. But, at high temperature and high density, this can undergo (p,γ) reaction, if enough number of protons are available, which is always the case. And, oxygen 14 will give rise to nitrogen 14.

Fluorine 17 can react with proton and give rise to neon 18, which produces fluorine 18 and which in turn produces oxygen 15. Earlier, F-18 was produced because of another reason, as discussed. Now, it goes to another cycle marked by the dotted line, because it is reacting with the proton.

And, if it is not going to oxygen 16 via (p,α), it can also go to oxygen 18. Neon 19 can also lead to the production of neon 20. By reacting with proton, it can produce sodium 20 which can undergo positron decay to produce neon 20. So, you have this (p,gamma), (p,alpha) and positron decay possible at high temperature.



In CNO cycle, fluorine 19 produces neon 20 via (p,γ). And, neon 20 can produce sodium 21 via (p,γ). And, this can undergo positron decay to produce neon 21. Neon 21 can undergo (p,γ) reaction to produce sodium 22. Sodium 22 can undergo positron decay to give neon 22. And, neon 22 can undergo (p,γ) reaction to produce sodium 23.

Sodium 23 can undergo (p,α) reaction to produce neon 20. This is called as neon sodium (NeNa) cycle.

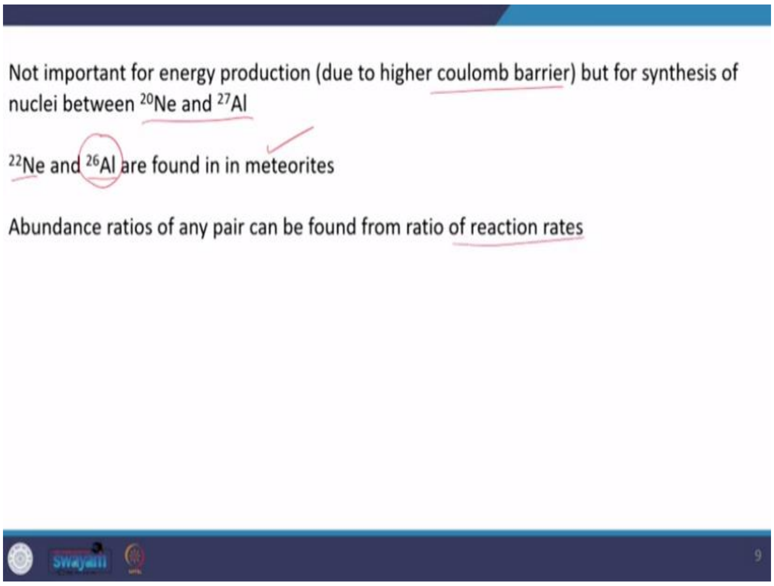
Now, sodium 23 can also undergo (p,γ) reaction to produce ^{24}Mg, which can produce aluminium 25 via (p,γ). ^{25}Al is unstable and can produce Mg 25 via positron decay. ^{25}Mg can undergo (p,γ) to produce ^{26}Al.

This ^{26}Al can produce Mg-26 via positron decay. Mg 26 can undergo (p,γ) reaction to produce ^{27}Al. Al-27 can undergo (p,α) reaction to produce ^{24}Mg. This cycle is called MgAl cycle.

Now, ^{26}Al can also undergo (p,γ) reaction to produce silicon 27 which can undergo positron decay to produce aluminium 27.

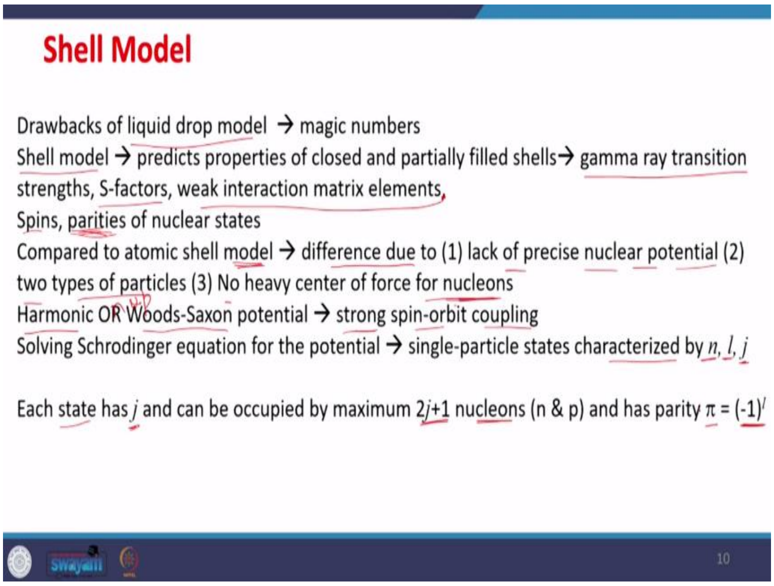
And, this aluminium 27, instead of undergoing (p,α), can undergo (p,γ) reaction to produce ^{28}Si. Depending on the availability of nuclei and the probability of undergoing reaction via (p,γ) or (p,α), different cycles can occur within the stars. In this neon sodium and magnesium aluminum cycle, there is a situation when it is coming out of the cycle by forming silicon 28.

Then, we have silicon burning and then different burning stages up to iron element. And beyond iron, we will see it is because of the neutron induced reactions. Here, you have not come across the neutron induced reactions. Here different elements are formed because of the charged particle induced reactions and the positron decays.



NeNa and MgAl cycles are not important for energy production like CNO cycle because of the very high Coulomb barrier, but for the synthesis of elements between neon 20 and aluminium 27. And interestingly, in these cycles, you can see neon 22 and aluminium 26. Aluminium 26 is found in meteorites.

This is how these elements are important. Abundance ratios of any pair can be found from the ratio of the relevant reaction rates. So, up to this, I have discussed pp chain, CNO cycles and additional cycles. There are 2 topics which are which seem to be different from the ongoing discussion, other than pp chain and CNO cycles. But, I thought it makes sense to throw some light on these 2 topics so that the concepts will be useful to you whenever gamma decay is discussed.



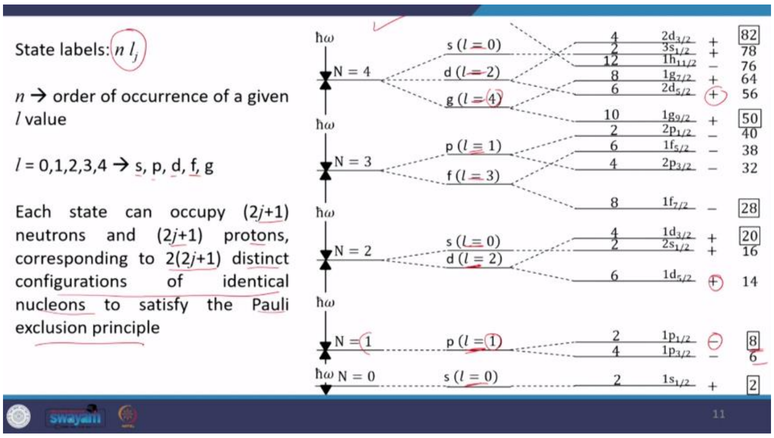
Let me quickly give you some insight about the shell model. I will just discuss the salient features. In +2, you might have been taught the liquid drop model, which could not explain the magic numbers and the existence of different energy states. The advantage of shell model is that it predicts the properties of not only closed filled shells, but also partially filled shells.

The importance of Shell model in this course can be understood as follows. Many a times, you need to calculate the gamma ray transition strengths. You have come across (p,γ) reactions numerous times in the lectures. Once the nucleus reacts with the proton, the compound nucleus is formed in the excited state. And, during de-excitation, gammas are emitted. When you try to model the stars while developing the stellar models, it is important to have the value of this gamma ray transition strength and spectroscopic factors and weak interaction matrix elements. Whenever proton is involved, normally, you come across weak interactions. This shell model plays important role there.

This helps us understand the spin and parity of the nuclear state. Parity is related to the symmetry, when wave function is reversed in order. Nuclear shell model is similar to the atomic shell model with some difference because of the lack of precise nuclear potential. We are aware of precise atomic potential. We do not have the proper information about nuclear potential. Also, here, we have 2 types of particles, like neutrons and protons, whereas, we have only electrons in the atomic shell model. Moreover, there is no heavy centre of force for nucleons, whereas there is a heavy centre of force in the form of nucleus in the case of atom. So, nuclear shell model cannot be understood in the same way.

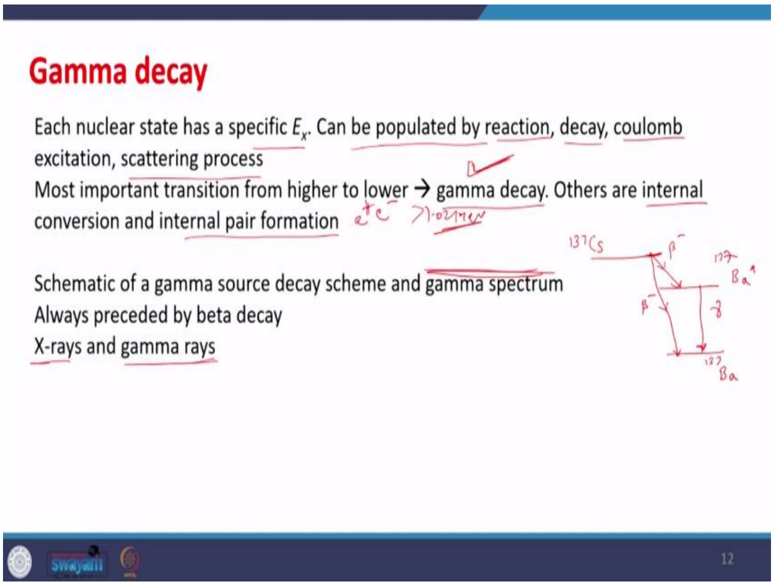
Nevertheless, we can get some idea by assuming simple harmonic potential; and by using a potential like Woods-Saxon potential, which tells us there is a strong coupling between the spin angular momentum and orbital angular momentum. Solving the Schrodinger equation for this harmonic or Woods-Saxon potential, we can see that single particle states are characterized by principal quantum number (n), orbital quantum number (l) and total angular momentum quantum number (j). Each energy level has total angular momentum j. A maximum of (2j + 1) nucleons, either neutrons and protons, can be occupied in each state.

Parity is denoted as π = $(-1)^l$. If l is equal to 0 or even, we say even parity. If l is odd, we say odd parity i.e., the wave function does not change.



Now, this is the shell model essence. We can see in the figure how parity is defined. If l is equal to 1, $(-1)^l$ is odd. So, parity is negative. If l is equal to 4, $(-1)^l$ is positive parity.

One can find out the parity, the j values and the total number of nucleons occupied in this manner. The stable levels are represented by the symbol nl_j where n denotes the order of occurrence of a given l value and l = 0, 1, 2, 3, 4 correspond to different orbitals s, p, d, f, g, etc. Each state can occupy (2j+1) neutrons and (2j+1) protons corresponding to 2(2j+1) distinct configurations of identical nucleons to satisfy the Pauli exclusion principle.



This is how the shell model tells us whether the spin and parity is plus or minus and some fraction. These values are important because the emission of gamma depends on the spin and parity according to the selection rule.

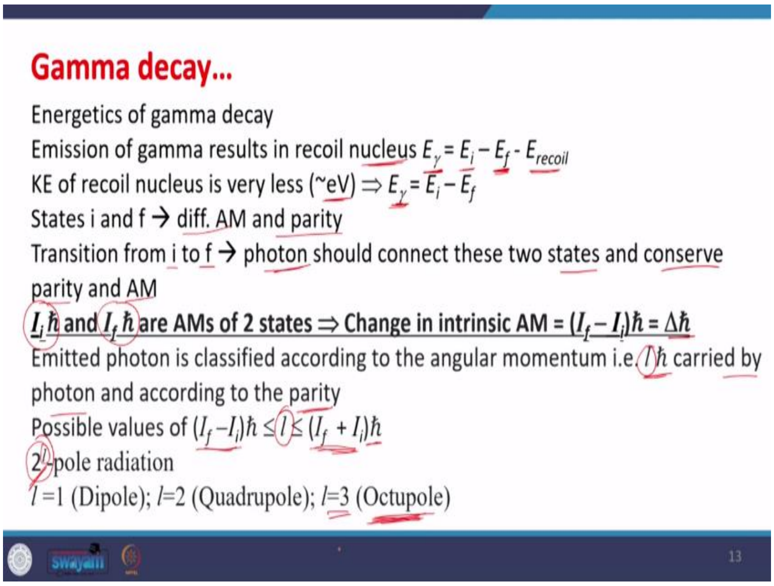
Let us move on to gamma decay, which is one of the important process in nuclear astrophysics. Each nuclear state has a specific excitation energy E_x. It is possible to populate the energy levels by a reaction like capture or decay or Coulomb excitation or scattering process.

Most important transition from higher to lower state is via gamma decay, though other processes are also possible like internal conversion and internal pair formation. In internal pair production, the energy difference is converted into positron and electron if it is greater than 1.02 MeV. But, gamma decay is the most preferred transition.

We can see that schematic of a gamma source decay scheme. The widely used gamma source in the laboratory is Cesium-137. It is unstable and decays to the excited state of barium 137 via β^- ,whose endpoint energy is up to around 0.5 MeV. The de-excitation leads to the emission of gamma ray.

^{137}Cs can rarely go directly to the ground state via beta minus. In due course, I will explain how a gamma spectrum looks like. This is not of our interest at this stage. Gamma emission, when we test the detectors in the laboratory, are always preceded by the beta decay.

The difference between X rays and gamma rays is basically their origin, though both are part of electromagnetic radiation. Gamma rays originate because of the nuclear processes whereas X rays are emitted because of the atomic processes, during rearrangement of the electrons in the shells.



Once the gamma is emitted, the nucleus will undergo recoil.

$$E_\gamma = E_i - E_f - E_{recoil}$$

Normally, E_f can be ground state. The kinetic energy of this recoil nucleus is normally very very less.

So, the energy of gamma can be considered as the difference in the energies of 2 levels which are involved in the transition.

$$E_\gamma = E_i - E_f$$

The two states have different angular momentum and parity. The transition from initial to final state is related to the emission of photon. This is the reason it should connect these two states and conserve the parity and angular momentum.

You are aware of conservation of energy and momentum when collision happens. In the same way, when gamma is emitted from the excited nucleus, the conservation of parity and angular momentum also plays very important role.

The angular momenta of initial and final states are in the integral multiple of ħ. And, the change in the intrinsic angular momentum is Δħ. So, the emitted gamma ray can be classified according to the angular momentum i.e. $l\hbar$ carried by photon according to the parity.

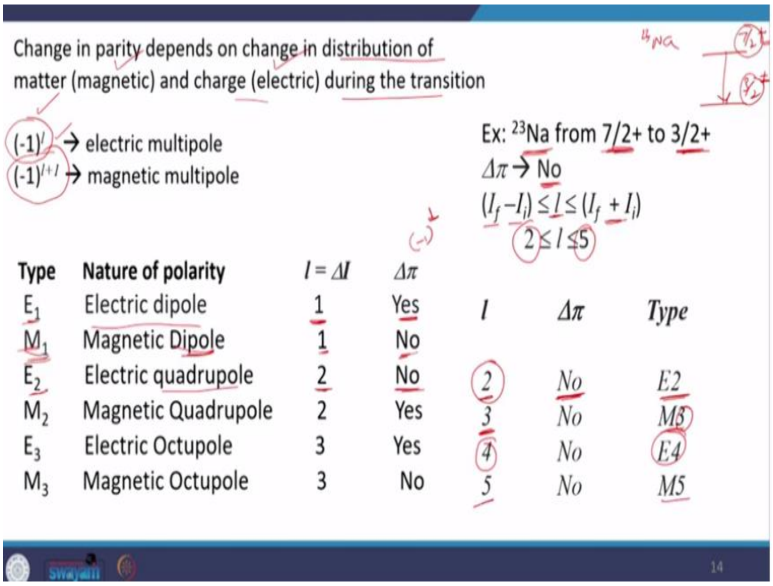
$I_i\hbar$ and $I_f\hbar$ are AMs of 2 states ⇒ Change in intrinsic AM = $(I_f - I_i)\hbar$ = Δħ

Possible values of l: $(I_f - I_i)\hbar \leq l \leq (I_f + I_i)\hbar$

The nature of this gamma ray could be dipole, quadrupole or octupole. It is decided by the expression 2^l. The derivation and theory behind this expression can be looked up in any textbook.

If l is equal to 1, 2^l is equal to 2. That is a quadrupole. If l is equal to 1, a dipole. If l is equal to 3, 2^3 is 8. That is an octupole in nature.

$l = 1\ (Dipole);\ l = 2\ (Quadrupole);\ l = 3\ (Octupole)$



This was regarding angular momentum.

The change in parity depends on change in distribution of matter (magnetic) and charge (electric) during transition. The change in the distribution of matter and charge will decide whether it is electric or magnetic in nature.

$(-1)^l \rightarrow electric\ multipole$

$(-1)^{l+1} \rightarrow magnetic\ multipole$

If l is equal to 1, it is electric in nature.

So, it is E1. And, the nature is electric dipole. There is a change in parity according to $(-1)^l$, because l is odd. Again, if l is equal to 1 for magnetic, the change in parity is not possible according to $(-1)^{l+1}$. Same goes for quadrupoles and octupoles.

Let me take one example for better understanding.

A ^{23}Na nucleus is undergoing transition from a state which characterized by $7/2^+$ to 3 to a state characterized by $3/2^+$.

You can clearly see that there is no change in the parity because both the states have positive parities. So, Δπ→No.

Possible values of L are: $(I_f - I_i) \leq l \leq (I_f + I_i)$ = $2 \leq l \leq 5$

Now, based on these l values, you can write down the electric or magnetic nature. For example, if l is equal to 2 i.e., it is even and there is no change in the parity anyway, then the type is E2. And, if l is equal to 3, the transition is magnetic in nature i.e., M3. Okay.

To summarize, I discussed additional cycles possible within the stars based on carbon and nitrogen and continued up to neon sodium and magnesium aluminium. Then, I discussed some salient features of the shell model which tells us the values of spin and parity of the states. Once you know how spins and parity are decided, you can say whether there is a change in the parity or not, whether the emitted gamma ray is a dipole, quadrupole or octupole; and whether it is electric or magnetic in nature.

These topics will be useful for us to understand the transition in different nuclei when they are in excited state. I hope you are able to follow this lecture. Continuing these different burning stages, I will take up some advanced burning stages in the next lecture. Thank you so much for your attention. See you!