Nuclear Astrophysics
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Module – 05
Lecture – 24
pp Chain… and CN Cycle

Welcome students to the lectures on different burning stages within the stars. So, before going to the today's lecture, let me take a quick overview on the previous lecture and continue the discussion on different burning stages within the stars.

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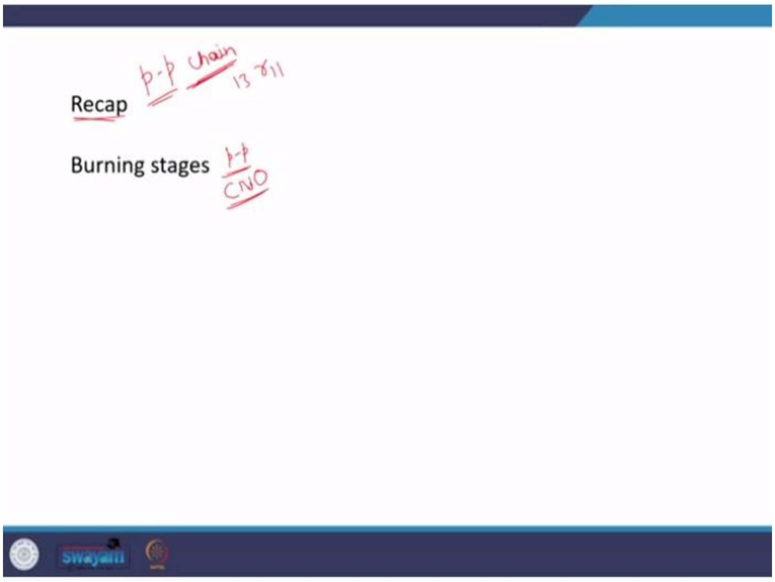

Recap p-p chain 13 γ_{11}

Burning stages p-p CNO

In the last class, we have seen that pp chain. Remember, it is not a cycle. It is a chain reaction. What is the energy production happens in stars because of the pp reaction? We have seen that it is about 13 times total reaction rate between proton and proton. 1 and 1 denotes the atomic number for proton and proton. We also have seen how the burning of deuterium takes place and how the burning of ^{3}He takes place.

So, initially the hydrogen burning then deuterium burning, and then ^{3}He burning because this deuterium and ^{3}He are the elements which are formed as part of pp chain. This is not the complete one regarding pp chain. There are many more fascinating things to know to understand and to experience the way reactions are happening within the stars.

In today's lecture, I am going to discuss more aspects of pp chain reactions and carbon nitrogen oxygen CNO cycle. So, in today's lecture, I am going to continue the topic pp chain and CNO cycle. Please remember, why this kind of burning stages are important to understand a nuclear astrophysics course. So, we have been very clear regarding the objectives of the course.

We need to understand the nuclear physics role to understand the energy produced from the stars, how it helps and what kind of chemical elements are synthesized with the help of what type of nuclear reactions. And, when we discussed non-resonant and resonant kind of reaction, it was for a particular reaction and seeing the temperature dependence of various parameters.

Practically, within the star at a particular time, large number of nuclear reactions are happening. And, those nuclear reactions are happening and the chemical composition is changing with time and also with respect to the temperature. The existence of different burning stages itself is a proof that there are ranges within the temperatures over which different nuclear reactions are taking place.

And we have started understanding the pp chain reaction whose main objective is conversion of 4 protons to 1 ^{4}He nucleus. And in the last class, I have shown you there are 4 steps by which protons + proton fusion leads to the production of ^{4}He nucleus. Is that the only one set of reactions which leads to the production of ^{4}He? No, there are set of reactions which leads to the production of ^{4}He.

Interestingly, not only the direct fusion of proton + proton followed by few more reactions leads to the production of ^{4}He, but also there are few sets of reactions in which 4 protons are converted into ^{4}He. So, that also I will discuss in today's lecture. So, let us see some more things about pp chain reaction.

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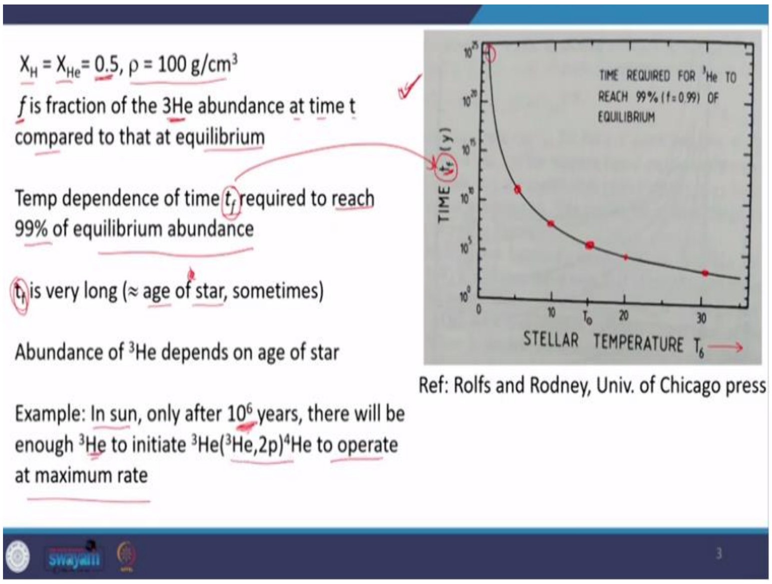

$X_H = X_{He} = 0.5$, $\rho = 100$ g/cm^3

f is fraction of the 3He abundance at time t compared to that at equilibrium

Temp dependence of time t_f required to reach 99% of equilibrium abundance

t_f is very long (≈ age of star, sometimes)

Abundance of ^{3}He depends on age of star

Example: In sun, only after 10^6 years, there will be enough ^{3}He to initiate $^3He(^3He,2p)^4He$ to operate at maximum rate

Ref: Rolfs and Rodney, Univ. of Chicago press

Consider this typical values mass fraction of hydrogen and helium. Assume that it is 0.5 and the stellar density as 100 gm/cm^3. So, this is one of the typical set of conditions for a star. And, I will introduce one parameter that is small f which denotes the fraction of ^{3}He abundance at a time t and when compared to that at equilibrium. Considering this parameter, now, if we are interested to understand the time required the time is denoted as t with f as subscript.

The time required to reach 99% of the equilibrium abundance. Then, this figure helps us. You see temperature in MK on x axis, time that is t_f in years. What is t_f my dear? This one, the time required to reach 99% of equilibrium abundance at 5 MK, 10 MK, 15 MK. At the core of the Sun, T_6 is equal to 15, 20 MK. Like that you take different values of temperatures in MK up to say 30 MK.

You can see that this time required to reach 99% of equilibrium abundance is very large starting from 10^{25} for very small values of the temperature for T_6 like one kind of thing to around 10^5 up to T_6 is equal to 30 MK. T_6 is equal to 30. So, we can see that this time is actually very long. Sometimes it is almost equal to the age of the star.

And, because the energy produced within the star during pp chain depends on whether the p + p reaction came into equilibrium with ^{3}He. This figure plays very important role. I repeat if we are interested to understand the energy produced from the stars because of pp chain, it depends on whether it has come to into equilibrium with ^{3}He or not. And, what is ^{3}He? When proton + proton undergoes fusion, you have deuterium.

And, when deuterium + proton undergoes fusion, you have ^{3}He. The equilibrium position of this ^{3}He decides the energy produced within the stars whether it has come into equilibrium or not. This is the reason I have discussed in this slide. The order of time required in years for ^{3}He to come into equilibrium. And, because of the large value of t_f that is time required to reach 99% of equilibrium abundance.

Because of this large value, the abundance of ^{3}He depends on age of the star. For example, if you take the Sun, only after say 10^6 years, there will be enough ^{3}He to initiate this to operate at a maximum rate. So, we can you can imagine the rate with which ^{3}He is produced within a star regarding the reaction rates ratios all those things I have discussed in the previous lecture. Please go through it again.

If you want to have a better understanding of this slide in which I have shown you the conditions under which ^{3}He comes into equilibrium and what is the order of the time taken for ^{3}He to reach 99% of equilibrium abundance. So, this is one of the important points to understand when we discuss pp chain as a responsible reaction which is responsible for energy production within the stars.

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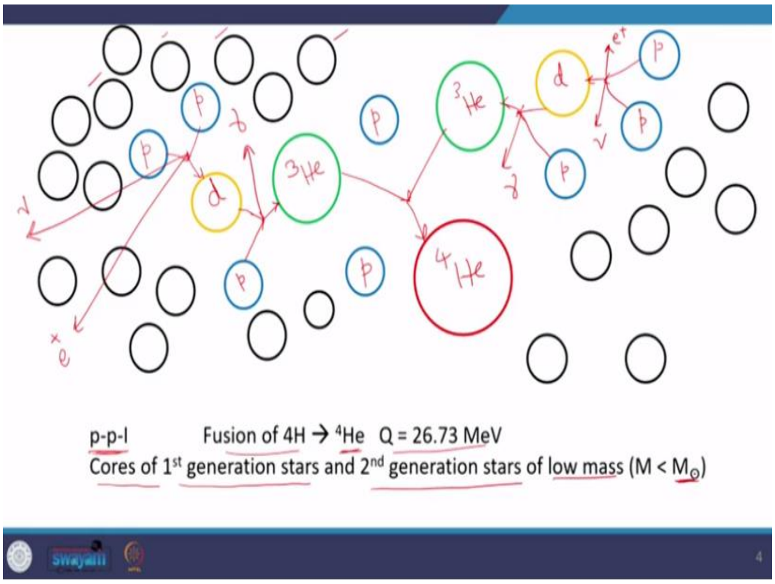

p-p-I Fusion of 4H → ^{4}He Q = 26.73 MeV

Cores of 1st generation stars and 2nd generation stars of low mass ($M < M_\odot$)

After this, let me show you a schematic representation within the stellar plasma. All these circles, they denote different nuclides. And for the sake of convenience, let me use the different colors for different nuclei. For example, this blue one denotes proton. Proton + proton gives rise to deuterium. See, the diameter of this yellow circle is bit high when compared to the proton because of its mass increase in the mass.

Not only deuterium, but you know that it also produces 1 positron. It also produces 1 neutrino. I have already discussed the energy loss of the neutrinos in pp chain reaction. Once your deuterium is formed deuterium and proton again reacts to produce ^{3}He. Similarly, in another set of reactions proton + proton produces deuterium in addition to positron and neutron in different directions.

And, this deuterium by reacting with proton produces again ^{3}He. That is why green color is used for ^{3}He. Orange is used for deuterium. Blue is used for proton for better understanding. And, this p + d not only gives ^{3}He but also gives gamma ray. Here also, p + d gives ^{3}He and also produces gamma. Now, this ^{3}He and ^{3}He they undergo fusion to produce ^{4}He. Is not it? Not only that, it also produces 2 protons.

So, overall, in this process, you have 4 protons converted into overall 4 protons are involved in the production of ^{4}He. So, this is called as pp-I chain where the fusion of 4 protons leads to the production of ^{4}He with the Q-value of 26.73 MeV including the neutrinos energy. This pp-I chain happens in the course of all 1st generation stars and low mass 2nd generation stars when the mass is less than that of mass of Sun.

Now, naturally, one question may come into your mind. What will happen if mass is greater than the mass of the Sun? Then, instead of pp-I, some other reactions will take place which I will discuss very soon. So, this diagram helps us to understand the reaction between protons, deuterium, ^{3}He to produce ^{4}He which is called as pp-I chain.

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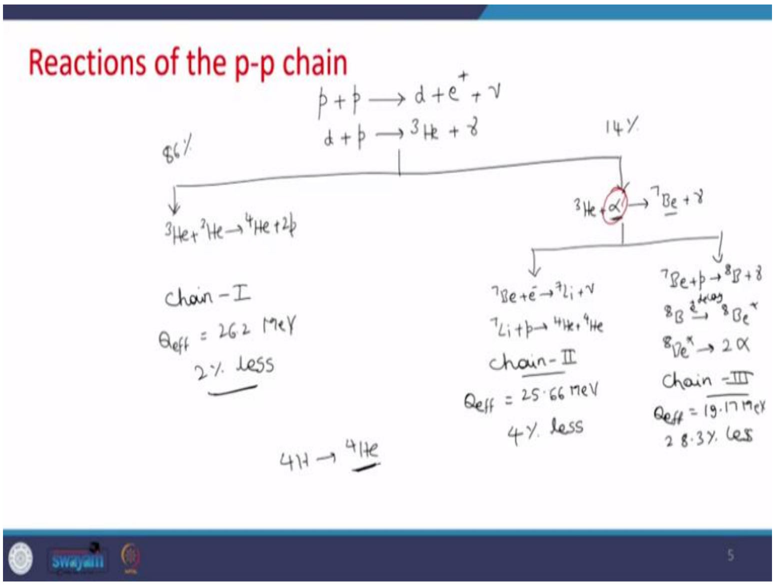

Reactions of the p-p chain

$p + p \longrightarrow d + e^{+} + \nu$

$d + p \longrightarrow {}^{3}He + \gamma$

86%

14%

${}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p$

Chain - I

$Q_{eff} = 26.2$ MeV

2% less

$4H \rightarrow {}^{4}He$

${}^{3}He + \alpha \rightarrow {}^{7}Be + \gamma$

${}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu$

${}^{7}Li + p \rightarrow {}^{4}He + {}^{4}He$

Chain - II

$Q_{eff} = 25.66$ MeV

4% less

${}^{7}Be + p \rightarrow {}^{8}B + \gamma$

${}^{8}B \xrightarrow{e^{+}\ decay} {}^{8}Be^{*}$

${}^{8}Be^{*} \rightarrow 2\alpha$

Chain - III

$Q_{eff} = 19.17$ MeV

28.3% les

Now, in addition to this pp-I chain, we have few more reactions which come under pp chain reaction. And those reactions I am going to write in this slide. So, p + p is producing deuterium + positron + neutrino. And, this deuterium is again reacting with proton giving rise to ^{3}He + gamma. Now, this can proceed in 2 different ways, 86% proceeds through the reaction which already we have discussed.

That is ^{3}He and ^{3}He produced in another reaction produces ^{4}He + 2 protons this is called as pp-I chain reaction. And 2nd possibility that is 14% probability is there for ^{3}He to not only react with another ^{3}He but alpha particle. That is ^{4}He giving rise to ^{7}Be + gamma. So, ^{3}He by reacting with ^{4}He produces ^{7}Be and gamma. Now, further, this reaction can proceed in 2 different ways because it is unstable. Of course, ^{7}Be can capture the electron. Electron capture may take place to produce ^{7}Li and neutrino. And, this ^{7}Li can react with proton and gives rise to ^{4}He + ^{4}He. That is 2 alpha particles are produced. There is another possibility for this ^{7}Be. ^{7}Be + proton may give rise to ^{8}B + gamma.

And, this ^{8}B can undergo positron decay to produce ^{8}Be in excited state. This positron decay happens which produces you know neutrino. And, ^{8}Be it decays into 2 alpha particles. That means 2 ^{4}He nucleotides. Now, this one in particular is called as chain one in pp. And, the effective Q-value is equal to 26.2 MeV. The calculation already I have done in the previous lecture.

So, the loss is only 2% because of the escape of the neutrinos. And, this is called as pp chain II where the effective Q-value is 25.66 MeV and the energy of the neutrinos escaping from here is bit more. That is why the effective Q-value is 4% less when compared to the total Q-value if we include the neutrino energies. And, the 3rd one is called as pp-III chain. Here, effective Q is equal to 99.17 MeV. You can see the loss is about 28.3%.

So, like this, we can observe 3 different types of pp chain reactions within the stars. So, it all depends on how ^{3}He is proceeding, once it is produced whether it is reacting with another ^{3}He or it is reacting with alpha. So, this is what it decides the occurrence of different sets of reaction in pp chain. Overall, the conversion of 4 protons to the formation of ^{4}He is taking place. This is what we need to understand.

Whereas, in chain II and chain III, you can see that alpha is acting as a catalyst. Is not it? So, this also we have to understand the role of ^{4}He as a catalyst in pp-II and pp-III chain.

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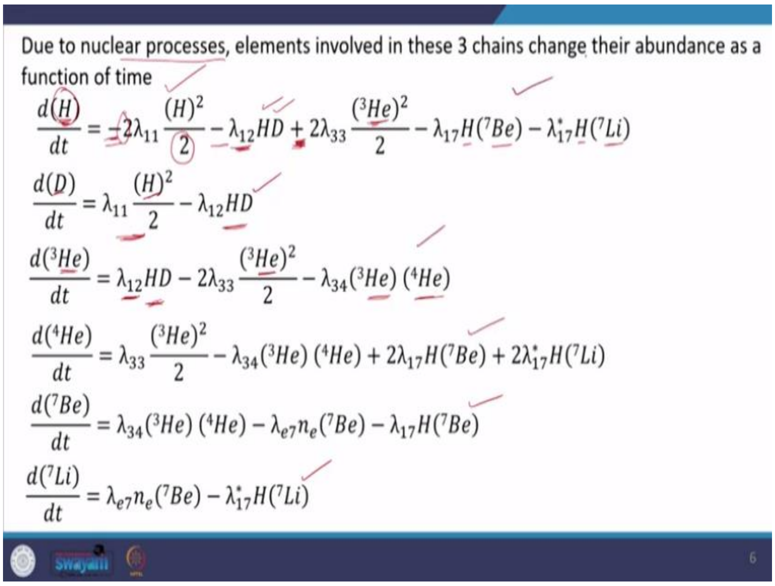

Due to nuclear processes, elements involved in these 3 chains change their abundance as a function of time

$$\frac{d(H)}{dt} = -2\lambda_{11}\frac{(H)^2}{2} - \lambda_{12}HD + 2\lambda_{33}\frac{(^3He)^2}{2} - \lambda_{17}H(^7Be) - \lambda^*_{17}H(^7Li)$$

$$\frac{d(D)}{dt} = \lambda_{11}\frac{(H)^2}{2} - \lambda_{12}HD$$

$$\frac{d(^3He)}{dt} = \lambda_{12}HD - 2\lambda_{33}\frac{(^3He)^2}{2} - \lambda_{34}(^3He)(^4He)$$

$$\frac{d(^4He)}{dt} = \lambda_{33}\frac{(^3He)^2}{2} - \lambda_{34}(^3He)(^4He) + 2\lambda_{17}H(^7Be) + 2\lambda^*_{17}H(^7Li)$$

$$\frac{d(^7Be)}{dt} = \lambda_{34}(^3He)(^4He) - \lambda_{e7}n_e(^7Be) - \lambda_{17}H(^7Be)$$

$$\frac{d(^7Li)}{dt} = \lambda_{e7}n_e(^7Be) - \lambda^*_{17}H(^7Li)$$

Now, because, due to the nuclear processes elements involved in these 3 chains changes their abundance as a function of time. So, once again I am showing you this slide. You have ^{3}He. You have ^{4}He. You have deuterium. You have ^{7}Be. You have ^{7}Li. You have ^{8}B. You have ^{7}Be. Already I have discussed. So, all these elements, the time dependence of the evolution of these elements decides the energy production within the stars.

So, let me quickly show you the equations to understand the time dependence of evolution of all these elements. So, these equations are nothing new to you in terms of representation, but here the reactions which are responsible for the production of a particular element and reaction responsible for the destruction of a particular element have to be combined to write down differential equations. So, let me show you the equations.

For example, if you take the proton with the time how the concentration of proton is changing. It is very simple. You see, minus denotes the destruction, + denotes the production. So, already I have explained in the previous lecture the importance of number 2 in the numerator and denominator. So, this is one of the ways by which proton undergoes destruction. Not only that, by reacting with deuterium, proton undergoes destruction.

And, what is the corresponding decay time constant? Now, here, there is a +. What does it mean? ^{3}He and ^{3}He, they are undergoing fusion to produce 2 protons. So, this is a production rate. This is a destruction rate. And, proton in another pp chain I mean it is basically pp-II chain reaction. So, here, when I am writing the time dependence of hydrogen, I am considering all 3 chains in pp process.

So, proton by reacting with ^{7}Be, proton by reacting with the ^{7}Li which is giving rise to boron in excited state. So, these are the ways by which proton can be produced and can be destroyed. Similarly, deuterium, this is how it is produced. Proton + proton and this is how it can be destroyed. And ^{3}He, if you see, this is how it is produced, proton + deuterium and corresponding decay constant.

And, the fusion of ^{3}He with another ^{3}He is a cause for the destruction of ^{3}He. Not only that, in pp-II, we have seen ^{3}He by reacting with alpha also undergoes destruction. Similarly, for ^{4}He, in how many ways it can be produced and destroyed? Similarly, for ^{7}Be, you can see here and ^{7}Li. So, these are the set of equations by solving which one can understand the evolution of the elements.

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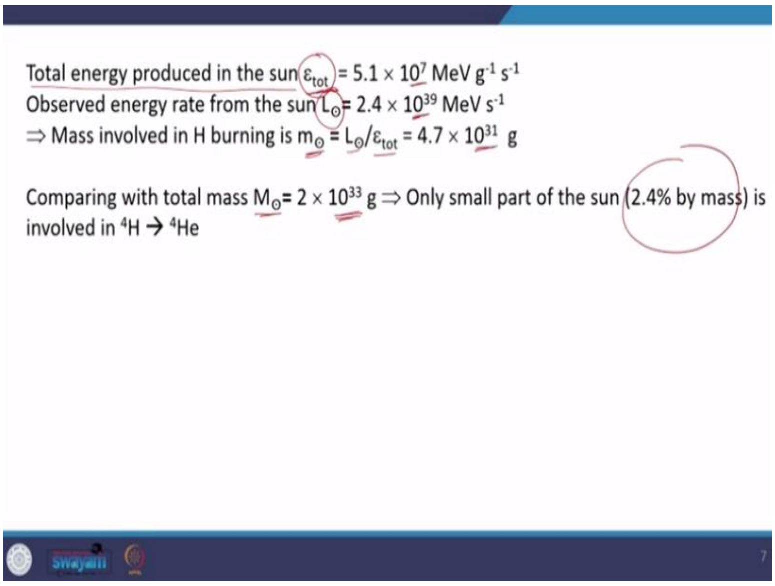

Total energy produced in the sun $\varepsilon_{tot} = 5.1 \times 10^{7}$ MeV g^{-1} s^{-1}

Observed energy rate from the sun $L_{\odot} = 2.4 \times 10^{39}$ MeV s^{-1}

$\Rightarrow$ Mass involved in H burning is $m_{\odot} = L_{\odot}/\varepsilon_{tot} = 4.7 \times 10^{31}$ g

Comparing with total mass $M_{\odot} = 2 \times 10^{33}$ g $\Rightarrow$ Only small part of the sun (2.4% by mass) is involved in $^{4}H \rightarrow {}^{4}He$

And, the solution of all these equations helps us to calculate the total energy produced in the Sun. And, one can see that the total energy produced in the Sun is of the order of 10^{7}. And, we know that from the surface of the Sun observed energy rate is of the order of 10^{39}. So, by having the total energy and the luminosity, one can have the mass of hydrogen involved in production of these 2 values, luminosity and total energy.

And, that mass is nothing but 10^{31} grams. Comparing with the total mass of the Sun that is 2×10^{33} grams, we can see that only 2.4% by mass hydrogen is involved in the form of pp chain within the Sun. I hope this information is very interesting for you. We know that abundant hydrogen and helium is present within the Sun.

And, because of the fusion, we are getting the energy from the Sun. But, to get in terms of numbers, it is 2.4% by mass is involved in the energy produced from the Sun based on pp chain reaction.

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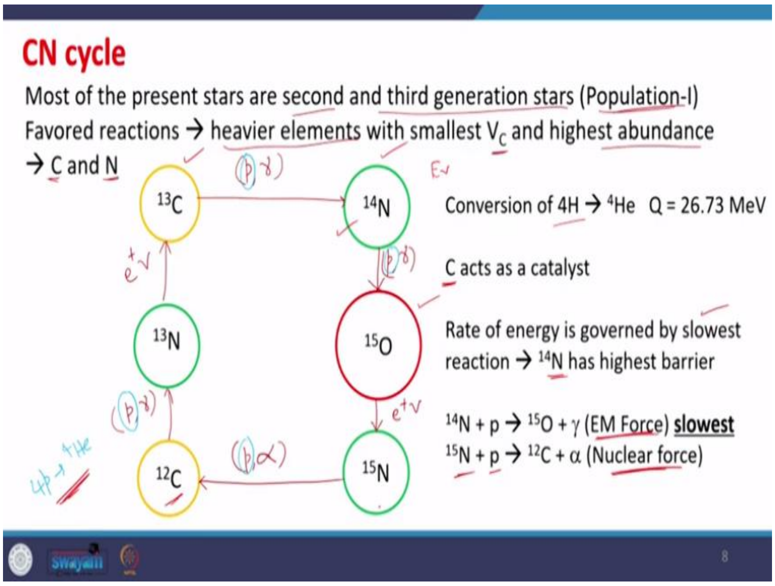

CN cycle

Most of the present stars are second and third generation stars (Population-I)

Favored reactions → heavier elements with smallest V_C and highest abundance

→ C and N

Conversion of 4H → ^{4}He Q = 26.73 MeV

C acts as a catalyst

Rate of energy is governed by slowest reaction → ^{14}N has highest barrier

$^{14}N + p \rightarrow {}^{15}O + \gamma$ (EM Force) **slowest**

$^{15}N + p \rightarrow {}^{12}C + \alpha$ (Nuclear force)

Now, coming to the another topic. That is CN cycle. Now, you see earlier we have discussed Population-II stars which are 1st generation where hydrogen burning is the dominant mechanism which are very old. However, most of the present stars they are not old, but they are categorized into 2nd and 3rd generation stars which comes into Population-I. And, in these stars, 2nd and 3rd generation, the favourable reactions are basically because of the heavier elements.

Not the hydrogen but the heavier elements, of course, which heavier element? The one which has least Coulomb barrier and highest abundance and when we consider this kind of criteria, the elements which comes into our mind is carbon and nitrogen, not the lithium, beryllium and boron. Just remember the elemental abundance curve, because of this, there extremely low abundance of lithium, beryllium and boron.

They are not preferred for the next set of reactions in first population. That is 2nd and 3rd generation stars. Whereas, compared to lithium, beryllium and boron, carbon and nitrogen they are abundant. And also, in that, the least Coulomb barrier comes to carbon. So, keeping this in mind let me give you a reaction which starts with carbon 12. Now, once carbon 12 is formed because of the earlier reactions, based on proton and gamma.

That means, p + ^{12}C gives rise to ^{13}N + γ. And, this ^{13}N undergoes positron decay to produce ^{13}C. And, this undergoes (p,γ) reaction producing ^{14}N. Once again this ^{14}N undergoes (p, γ) reaction to produce ^{15}O. Being unstable, it undergoes positron decay to give ^{15}N.

And, this ^{15}N by reacting with proton which are highly abundant via (p,α) reaction produces ^{12}C. So, what does it mean my dear? Here, in this cycle, overall, how many protons are involved? Just you can count, 1 proton, 2 protons, 3 protons and 4 protons. So, overall 4 protons are consumed to produce 1 ^{4}He. So, this is how a set of reaction takes place where 4 protons are consumed to produce ^{4}He.

And, the net result is this is the reaction. And, after hydrogen, helium, we have, as I said, lithium, beryllium, boron. But, their abundance is very low. So, these carbon and nitrogen, they fulfill these 2 conditions. Remember, energy of the neutrino is quite less and most of the energy is retained. Now, what is the interest in this reaction? If cycle begins with ^{12}C, it also ends with the ^{12}C.

So, ^{12}C can be used again and again. Of course, there is not much ^{12}C even though it is repeated that helps to make the cycle an effective energy source. Same is true if the cycle begins with the ^{14}N or ^{13}C or ^{15}O. So, the rate of energy is basically governed by the slowest reaction. And, what is that slowest reaction? The slowest reaction is the one which has highest Coulomb barrier within the available set of reactions.

And, which reaction has highest Coulomb barrier leads to the slowest reaction? Let us have a look. So, in this reaction, if you see, p + ^{14}N, it has the highest coulomb barrier. Please remember, nuclear interaction, electromagnetic interaction, weak interaction, they give different types of different values of cross section. So, it is important to understand energy production in massive stars.

And, it is also important to understand synthesis of isotopes of carbon and nitrogen. So, as I said conversion of 4 protons. And here, carbon acts as a catalyst. And, ^{14}N when it reacts with proton, it has the highest Coulomb barrier. So, the rate of energy production is governed by this reaction. And, p + ^{14}N as it gives via electromagnetic interaction it is the slowest.

We also have another reaction where p + ^{15}N is reacting. But, here, the nuclear force is involved. So, this is not the slowest reaction whereas electromagnetic force is the slowest reaction.

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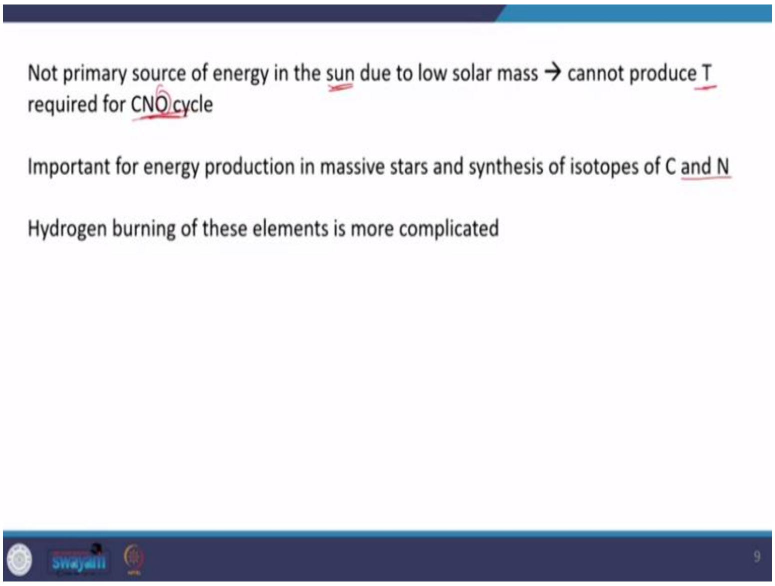

Not primary source of energy in the sun due to low solar mass → cannot produce T required for CNO cycle

Important for energy production in massive stars and synthesis of isotopes of C and N

Hydrogen burning of these elements is more complicated

Now, as I said this is not the primary source of energy in the Sun. This is CN cycle due to low solar mass. So, this cannot produce temperature required for CNO cycle. What is this oxygen? I will discuss in the next lecture. But, this is important for energy production in the stars and also the synthesis of isotopes of carbon and nitrogen. Now, the hydrogen burning of these elements is not very simple, it is quite complicated.

So, to summarize, in today's lecture, we have seen how ^{3}He abundance changes with respect to the temperature. And, the time required for the ^{3}He to reach the equilibrium is of the order of 10^{5}, 10^{6}, 10^{10} years. And, when we come to the Sun about, $T_6 = 15$. It is of the order of 10^{5} years. So, sometimes, it is equal to the age of the star.

And after that, I have discussed 3 sets of the reaction in pp chain, pp-I, pp-II, pp-III chain. And, in which chain the energy loss is more? It is pp-III chain. And after that, I have shown you a cycle which is important for the conversion of 4 protons into ^{4}He in Population-I stars. That is 2nd and 3rd generation stars. And, the slowest reaction is decided by the highest Coulomb barrier and that was p + ^{14}N via electromagnetic interaction.

Though in terms of energy production, this is not very important. But, in terms of the synthesis of isotopes of carbon and nitrogen, this CN cycle is very important. More features of carbon nitrogen cycle and their extensions, I will discuss in the next lecture. Till then, see you. Thank you so much for your attention, bye.