Nuclear Astrophysics
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Module – 04
Lecture – 20
Resonant Reactions

Welcome back students; today's lecture is last part of the 4th unit that is non resonant reactions and resonant reactions. In the syllabus if you see there are 8 parts and today is the last lecture of the 4th part. So, in today's lecture I am going to complete the discussion on resonant reaction and I will be discussing neutron induced non resonant reactions as I have said in the previous lecture.

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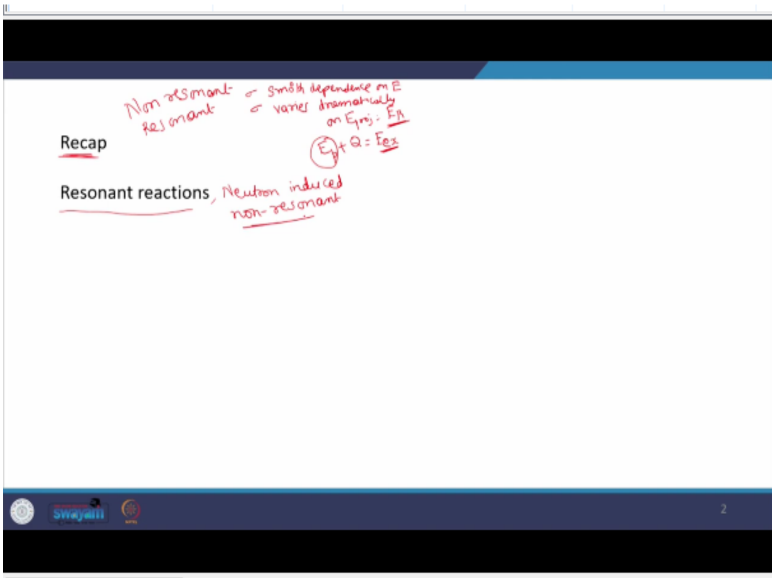

So, let us quickly go through the content of the previous lecture where I have spent some time on the non resonant type of nuclear reaction where σ is smooth dependent function of energy and resonant reaction which is contrary to this reaction, σ varies dramatically at a few energies of projectiles which we call as resonant energies. So, those energies of projectiles, the Q-value and the projectile energy gives rise to excited state, like projectile energy it will become resonant energy when this value by adding with Q-value gives rise to an exited state which is the characteristic of the nucleus.

And I have written expression for the cross section of a resonant reaction considering it as two-step process and I have also assumed the collision between the projectile and the target nucleus as the collision between the momentum of the projectile and the impact parameter. And then for different values of orbital angular momentum we have seen the maximum possible cross section for each value of l is nothing but (2l + 1) $\pi ƛ^2$ and if we include the spin we can come up with a general representation like statistical factor $\frac{2J+1}{(2J_1+1)(2J_2+1)}$ and then for identical particles I have included one Kronecker symbol.

Then by taking the analogy between damped oscillator and resonant reactions because in both systems the response is maximum at specific incident values, specific input values. So, by comparing the strength of the oscillator with the product of the partial width, what is partial width? It represents the probability for each step to occur and the Eigen frequency is compared with the resonant energy and the damped factor f is equivalent to the total partial width of the nuclear reaction fine.

That was famous Breit-Wigner formula. So, continuing the salient features of this Breit-Wigner formula and some examples, I would like to spend some time on resonant reactions. So, in today's lecture I will be spending some time on resonant reactions and also neutron induced non-resonant reactions.

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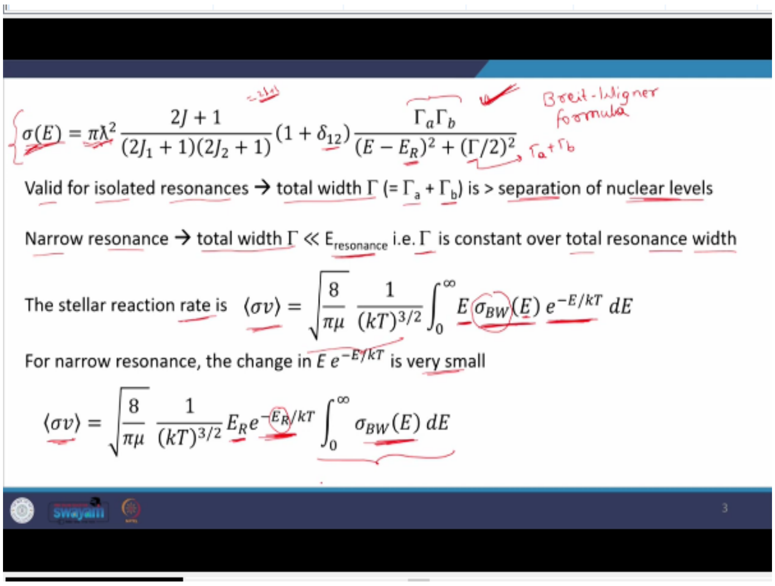

So, let me start with the formula with which I have ended the previous class that is Breit-Wigner formula. $\boldsymbol{\sigma(E) = \pi ƛ^2 \frac{2J+1}{(2J_1+1)(2J_2+1)}(1+\delta_{12})\frac{\Gamma_a\Gamma_b}{(E-E_R)^2+(\Gamma/2)^2}}$. So, $\boldsymbol{\Gamma}$ is the total partial width that is formation width $\boldsymbol{\Gamma_a}$ plus decay width $\boldsymbol{\Gamma_b}$ and $\boldsymbol{E_R}$ is the resonant energy and $\boldsymbol{\delta_{12}}$ is the Kronecker symbol and $\boldsymbol{\frac{2J+1}{(2J_1+1)(2J_2+1)}}$ will reduce to (2l + 1) if spins are not included, but for general expression we have included both orbital angular momentum and also the spin of the entities in entrance channel.

You know very well that πr^2 is the geometrical cross section that is area seen by the projectile, the area seen by the target when projectile falls on it and this term is replaced by $\pi \lambda\!\!\!^{-2}$. So, this is the equation which we have derived in the previous lecture. What to do with this? We need some more discussion to answer a question which I have posed in the previous lecture. What kind of resonances play important role, to decide the properties of the stars, low energy resonances or higher energy resonances.

To understand this we need to take some interesting approach,   it follows like this. See this Breit-Wigner formula is valid for isolated resonances. So, basically when I am saying resonant reactions it has 2 properties that is isolate resonances and narrow resonances. So, when I say isolated, it means in terms of total partial width which is the sum of the formation width and decay width. This is greater than the separation between the excited states of compound nucleus.

The width of the state which was populated because of the nuclear reaction, should be greater than the separation of the nuclear levels. That is what we mean by isolated resonances and when I say narrow resonance it means, the total width which is some of the formation and decay width, should be much less than the projectile energy at which this resonance is happening.

That means the total width is constant over total resonance width, whatever resonance region is there throughout the resonance energy region total width should be constant. That is what we mean by isolated and narrow resonance. Considering this the stellar reaction rate which we are waiting for to write down based on this cross section formula looks like this, before that let me write down the general expression for the stellar reaction rate. $\langle \sigma v \rangle = \sqrt{\frac{8}{\pi\mu}}\ \frac{1}{(kT)^{3/2}} \int_0^\infty E\ \sigma_{BW}(E)\ e^{-E/kT}\ dE$. So, E is a well-known quantity, center of mass energy.

And here the cross section is replaced with Breit-Wigner formula which depends on the energy slightly, though it is a resonance energy region, so we have taken a general representation that is in σ as a function of energy and anyway the particles are distributed according to Maxwell

Boltzmann form. So, this is a well known reaction rate formula. Now replace this cross section with the Breit-Wigner formula this one.

So, for narrow resonance if you consider then the change in this Maxwell Boltzmann distribution is very small, because the residence is happening at a very sharp value of the energy around that if you consider the distribution of the velocities the change is very small. So, $e^{-E/kT}$ will be very small. So, by saying this I am taking this quantity outside the integral fine.

Then the reaction rate can be represented by this equation, $\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu}}\ \frac{1}{(kT)^{3/2}} E_R e^{-E_R/kT} \int_0^\infty\ \sigma_{BW}(E)\,dE$, where the energy is also taken out because we are dealing with the resonant reaction and $e^{-E_R/kT}$ is constant, it can be taken outside and the Breit-Wigner formula is inside this integral. Now the task is to calculate the value of this integral. So, I will not go into the details you can do easily using the expression in the very first place which I have written. You need to calculate the value of this integral. So, the final result I am showing, $\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu}}\ \frac{1}{(kT)^{3/2}} E_R e^{-E_R/kT} \int_0^\infty\ \sigma_{BW}(E)\,dE.$

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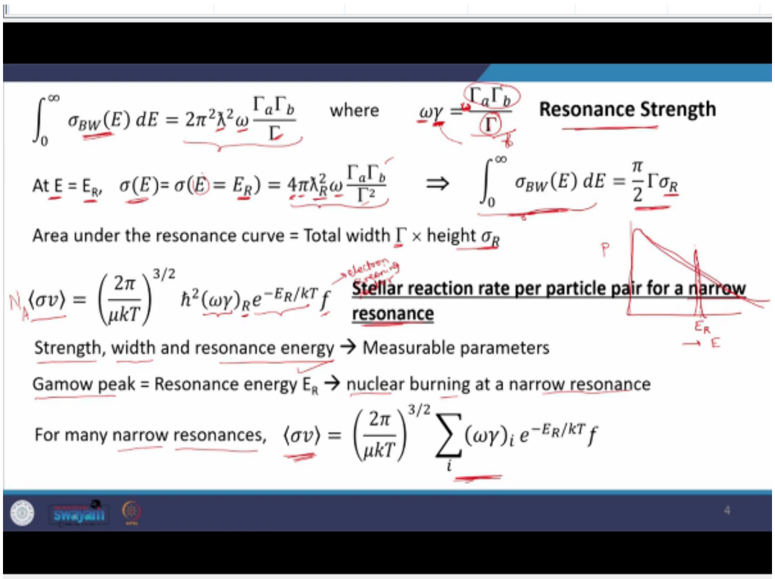

So, the integral of this cross section of the nuclear resonant reaction given by the Breit-Wigner, can be derived as like this, $\int_0^\infty\ \sigma_{BW}(E)\,dE = 2\pi^2 ƛ^2 \omega \frac{\Gamma_a\Gamma_b}{\Gamma}$, where, $ƛ$ is the reduced de Broglie wavelength, ω is statistical factor, Γ_a is formation partial width, Γ_b is decay partial width and Γ is the total width. So, let me introduce one more parameter, $\omega\gamma$, this is called as resonance strength, where ω is a statistical factor which involves J, J_1, J_2 and gamma $\gamma = \frac{\Gamma_a \Gamma_b}{\Gamma}$. Why am I emphasizing this word, because this is one of the experimentally measurable parameter to understand the cross section of the resonant reactions.

So, resonance strength is represented by the statistical factor and the ratio of product of partial width corresponding to formation and decay width and the total partial width that is sum of these partial widths. This gives us information about the strength of the resonance. So, use this for representing the reaction rate in future. At incident projectile energy equivalent to resonant energy, the cross section exactly at that particular energy I can write down by replacing E as exactly E_R then by taking the help of this integral and remaining things I can come up with this kind of expression.

At exactly resonant energy the cross section of the resonant nuclear reaction can be represented as $\sigma(E)=\sigma(E = E_R) = 4\pi\lambda\!\!\!^{-2}_R\omega\frac{\Gamma_a\Gamma_b}{\Gamma^2}$. So, this leads to the fact that I can always write down the integral value like $\int_0^\infty \sigma_{BW}(E)\,dE = \frac{\pi}{2}\Gamma\sigma_R$. So, this is just rewriting the expression of cross section at E_R and the integral symbol. There is no much difference. Now the area under the resonance curve is given by the total width and the height σ.

So, the reason for expressing this integral in terms of sigma R is following. Now earlier when you have taken the plot of the probability and the energy and initially when you take this Maxwell Boltzmann distribution $e^{-E/kT}$, there is a sharp peak at resonant energy. So, the gamma peak at which majority of the nuclear reactions used to take place for non resonant reaction is just replaced with a resonance peak.

At this particular value cross section is very high, at this value only majority of the reaction take place, at remaining places no. So, that is how we have initially defined the nuclear reactions when cross section changes suddenly at particular value of energy. So, the area of this resonance curve is nothing but the total partial width and the height is nothing but this cross section σ_R. So, taking this let me write down the final expression for reaction rate when resonant reactions are discussed. $\langle\sigma v\rangle = \left(\frac{2\pi}{\mu kT}\right)^{3/2} \hbar^2(\omega\gamma)_R e^{-E_R/kT} f$.

So, $(\omega\gamma)_R$ is the strength of resonance exactly at the resonant energy and $e^{-E_R/kT}$ is the Maxwell Boltzmann distribution term and f is the electron screening factor. Though it is negligible just for the sake of representation I am including this f also in reaction rate. If I tell the stellar density is very high one has to take into account this electron screening factor. So, this is nothing but stellar reaction rate per particle pair because $\langle\sigma v\rangle$ is there and if you multiply with Avogadro number then no need to write down the particle pair, but for narrow resonance. This reaction rate expression is for narrow resonance not broad resonances.

So, to measure the nuclear reaction rate what one should find out? The strength of the resonance width of the resonance and the resonance energy. These three are the parameters one has to measure experimentally. So, that one can come up with reaction rate directly, you see here in this expression of reaction rate there is no cross section formula. There is no cross section term involved in this reaction rate for narrow and isolated resonances.

Just measure the width of the resonance, just measure the strength of the resonance and anyway incident projectile energy is known, measure it accurately, you are done with the reaction rate measurement for narrow resident reactions. So, as I said the gamma peak is replaced with resonance energy E_R.

So, the nuclear burning take place at a narrow resonance, the burning happens at these narrow resonances only. If more narrow resonances are involved in the reactions, how do you express this reaction rate? Simply take the summation of strengths of the resonances. That will take care of this reaction rate when many narrow resonances are involved.

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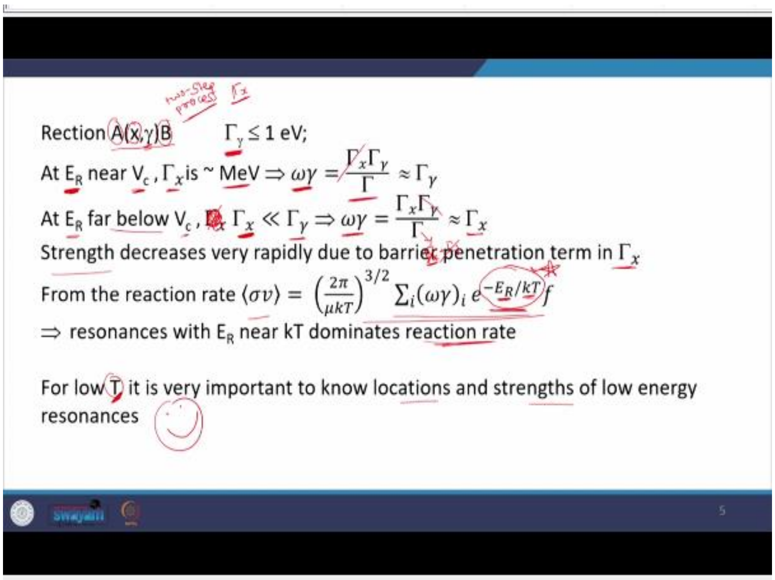

Now I am going to discuss one of the very important feature of this nuclear astrophysics and that is the importance of low energy resonances, for the sake of simplicity let us consider this reaction where projectile, x is reacting with target nucleus A giving rise to product nucleus B and gamma ray. In two step process remember if it is one step process then it is basically non resonant reaction, there is no point in discussing this topic.

If this radiative capture reaction happens in two steps formation and decay width term. Then in general that is gamma decay width because we are assuming this as a ready to capture reaction, it is very, very less sometimes you know of the order of 1 eV or less than 1 eV, very less. This very less value of the gamma decay partial width is going to play very important role.

Please pay attention to this topic. I want your special attention to this particular discussion. So, I am trying to discuss the importance of a low energy resonances. For that I have considered a reaction, where x is reacting with A and giving rise to gamma ray and B product nucleus. So, there are 2 partial widths. Γ_x is the formation width and Γ_γ is the decay width.

In general the decay width is always less or equal to 1 eV. Now I am bringing this statement if the resonance energy is near the Coulomb barrier, please listen to this carefully. If at all the resonance energy is near to Coulomb barrier remember in any case it is happening only above the Q-value of the reaction, but if resonance energy is near the Coulomb barrier then the formation width is of the order of MeV.

And already have said Γ_x is very large, so Γ_γ can be neglected and Γ_x and Γ will get cancelled. Finally, you are left with Γ_γ. The strength of the resonance is decided by the very low value of the gamma decay width. If the resonant energy is far below Coulomb barrier then please ignore this then this formation width is no more of the order of MeV, it is much less than the decay width. So, what does it mean? The resonance strength decreases very rapidly due to barrier penetration term in formation width why because formation partial width considers the barrier penetration phenomenon not the decay width.

This you have to understand and from the reaction rate we can write down this equation which I have shown in the previous slide I am continuing with that. From this reaction that you can see this term especially, please pay attention here. This is very important resonances with values of energies near kT dominate the reaction rate. See nowhere you have energy and kT in this expression except here. So, if E_R and kT are comparable to each other then this reaction rate is maximum.

So, basically the resonances with E_R and near kT only will dominate the reaction rate. So, for low temperatures that means at low values of kT, that means at low values of resonant energies it is very important to know the locations and the strength of the low energy resonances, not high energy resistances are important. So, I hope you are following what I am trying to convey.

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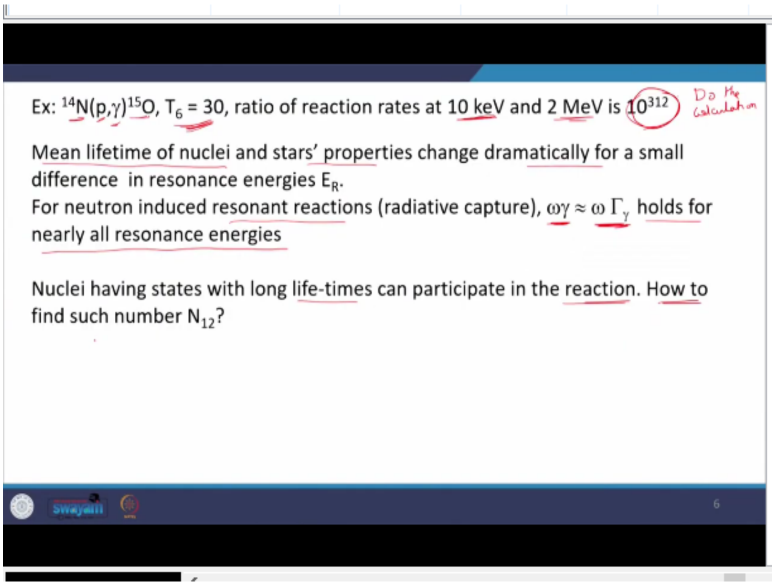

For example let me take a reaction $^{14}N(p,\gamma)^{15}O$. Assume $T_6 = 30$. If you see the ratio of reaction rates at low energy resonance say 10 keV and at 2 MeV it comes out to be 10^{312}. Please do the calculation.   For more details please look into the textbook Rhodes and Rodney otherwise, we can assume some values and see how it is very important.

So, this tells us the mean lifetime of the nuclei and the properties of the star change dramatically for a small difference in the resonant energies. Now if you consider the resonant reactions induced by neutrons then the strength of the resonance is nothing but this $\omega \Gamma_\gamma$ which holds for all resonance energies nearly because in the formation width there is no Coulomb barrier for neutrons.

So, it always depends on the partial width of the decay process second step. Now the interesting question which I am posing here is nuclei having states with long lifetimes can take part in the reaction. I am talking about the metastable states or sometimes isomers. That means states with longer lifetime, you might have went out the concept of lasers there you might be getting the idea of you know this metastable states where the levels are having longer lifetimes.

In stars when nuclear reactions are happening, sometimes the compound nucleus exerted state may possess longer lifetime. So, it will have enough time to react with other nuclei before coming to the ground state. I am talking about this reaction of the reaction between the other nuclei and the compound nuclei in exerted state not after reaching the ground state. Then how to find such number?

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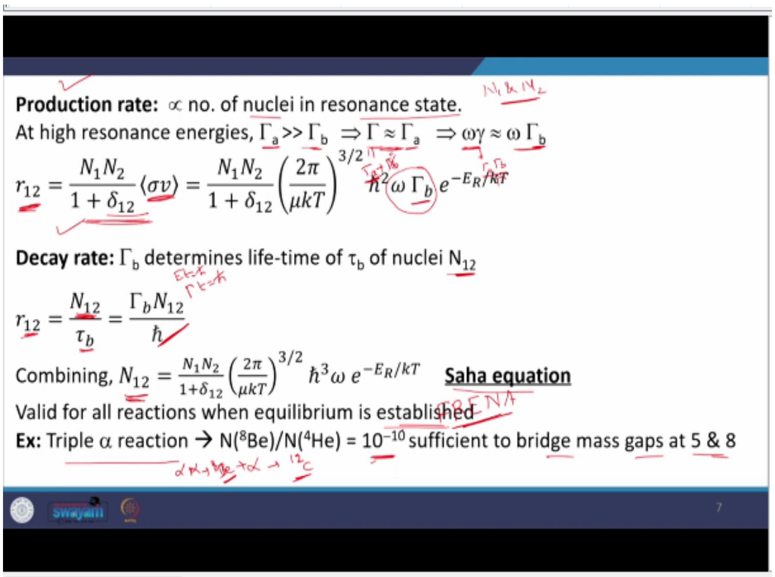

Let me take the production rate and the decay rate. So, the production rate is always proportional to number of nuclei in resonant state. That means N_1 and N_2 and at higher resonance energies if you consider then the formation which is always greater than decay width of course not at the low energies. Then this is equal to nothing but total partial width is nothing but which is the sum of formation width and decay width and this is negligible.

So, total partial width is nothing but formation width. Accordingly the strength of the residence is nothing but decay width will dominate as I have discussed earlier, because of this γ is nothing but $\Gamma_a\Gamma_b/\Gamma$. In that case one can write down the production rate as this is a well-known expression for the production rate. Here I am writing the expression for reaction rate. $r_{12} = \frac{N_1 N_2}{1+\delta_{12}}\langle\sigma v\rangle = \frac{N_1 N_2}{1+\delta_{12}}\left(\frac{2\pi}{\mu kT}\right)^{3/2} \hbar^2\omega\,\Gamma_b\, e^{-E_R/kT}$

And there the strength of the residency is replaced with this Γ_b decay width. Now coming to the decay rate; this decay rate determines the lifetime of the nuclei in the reactions. So, the decay rate is nothing but the ratio of number of nuclei N_{12} and the lifetime of the N_{12} nucleus and using this concept of Et = $\hbar$, here $\Gamma_b\tau_b = \hbar$. I can write down the expression for decay rate, $r_{12} = \frac{N_{12}}{\tau_b} = \frac{\Gamma_b N_{12}}{\hbar}$.

Combining these two terms decay rate and production rate I can find out the expression for total number of nuclei presenting in the reaction, $N_{12} = \frac{N_1 N_2}{1+\delta_{12}}\left(\frac{2\pi}{\mu kT}\right)^{3/2} \hbar^3\omega\, e^{-E_R/kT}$. This is called as famous Meghnad Saha equation. So, this is the contribution of Meghnad Saha in finding out the number of nuclei participating in the reactions within the stars and the institute of nuclear physics named after Meghnad Saha which is located in Kolkata, Saha institute of nuclear physics and it is a great coincidence that it is hosting the facility of FRENA.

Facility for Research in Experimental Nuclear Astrophysics, FRENA. So, the contribution of Saha we can remember in this particular context it gives us the number of nuclei participating in the collision. So, this equation is valid for all reactions when equilibrium is established, because production rate is there and decay rate is there. What is the famous example? In the last lecture initially I have hinted this point. So, what is the famous example for this number of nuclei participating in the exited state of the compound nucleus?

Triple alpha reaction, $\alpha + \alpha$ gives rise to ^{8}Be, this ^{8}Be lifetime is very less even then if the transit time between the α particles is small then the lifetime of the ^{8}Be, it can again react with the α giving rise to ^{12}C. For that there has to be equilibrium between the production rate of ^{8}Be and decay rate of ^{8}Be. Remember ^{8}Be is highly unstable.

So, the ratio of ^{8}Be nuclei and alpha nuclei is found to be 10^{-10}, it is very small however this is sufficient the ratio of ^{8}Be and α particles is not above 1, it is much less 10^{-10}. However, it is sufficient to bridge the mass gaps at 5 and 8. How these mass gaps came into picture? In the initial stage of this course I have discussed one of the fascinating aspects of nuclear astrophysics.

I mean without bridging the gap between at mass numbers 5 and 8 how ^{12}C is formed? Triple alpha reaction and this is where this formula for nuclei participating in the nuclear reactions comes into picture. So, please remember the contribution of Meghnad Saha in this particular context. So, in the next lecture I will discuss neutron induced reactions today because of limited time I could not discuss it.

So, let me summarize today's lecture. In the last lecture I have derived the expression for cross section of a resonant reaction, taking it forward, earlier I have taken the help of this damped oscillator and then the importance of low energy resonances. If the resonant energies are near Coulomb barrier or much below the Coulomb barrier how low energy resonances are playing very important role?

The turning point is that the reaction rate corresponding to the low energy and high energy resonances is about 10^{312}. So, that is low energy resonances play very important role in understanding the properties of the stars and then to know the number of nuclei present when compound nucleus in excited state participates in nuclear reaction with other nucleus Meghnad Saha equation really helps us. So, with this let me complete today's lecture, see you soon thank you very much.