Nuclear Astrophysics
Prof. Anil Kumar Gourishetty
Department of Physics
Indian Institute of Technology- Roorkee
Module – 04
Lecture – 19
Resonant Reactions

Students, in today's lecture I am going to discuss another type of nuclear reaction which are very important to understand some of the important features of nuclear astrophysics. For example, when I have discussed the salient features of the universe, selected general properties of the universe if you remember I have discussed triple alpha reaction. The existence of carbon in the universe and what is the role of triple alpha reaction in the formation of ^{12}C nucleus?

While dealing this topic I have said it was a two-step process, initially $\alpha + \alpha$ gives rise to ^{8}Be then it is reacting with another α particle to produce ^{12}C. That was happening at a particular exited state of the ^{12}C which we called as Hoyle state. So, like this there are many important reactions which occur only at a specific value of the projectile energy and we call them as resonant reactions.

So, in today's lecture I am going to focus on the topic resonant reactions, nuclear reactions happening at some resonance energies. So, before that let me spend a couple of minutes on the content of previous lectures, so that I can maintain the flow.

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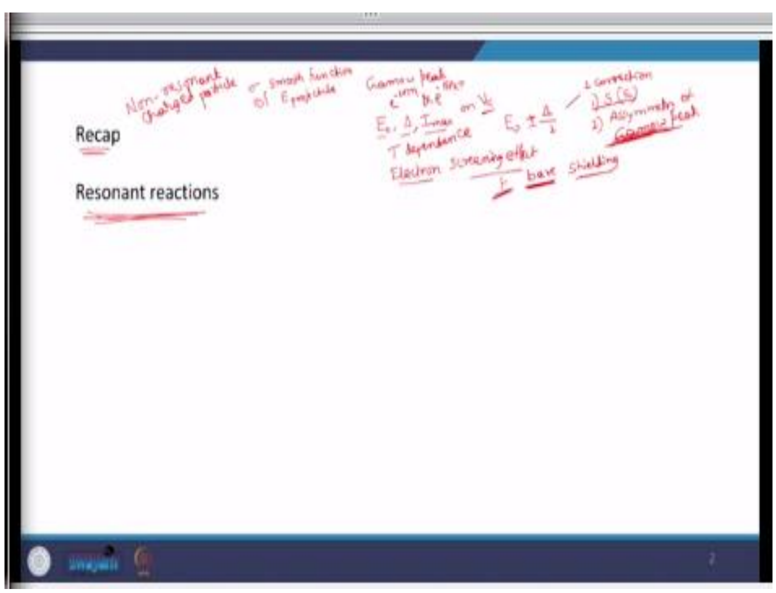

So, in the recap, please remember I have discussed non-resonant reactions induced by charged particles where sigma cross section is smooth function of energy of projectile. And we have seen the concept of Gamow peak which occurs because of the product of $e^{-2\pi\eta}\pi\eta$ and $e^{-E/kT}$.

Then I have discussed the importance of centroid of the Gamow peak E_0 that is most effective energy and width of the Gamow peak that is Δ and the maximum value corresponding to the centroid of the Gamow peak I_{max}. And what is the dependence of these parameters on Coulomb barrier? And finally, the temperature dependence I have discussed. And I have presented a lot of data which clearly states that the change in the Coulomb barrier dramatically changes the values of centroid of the Gamow peak.

And it is the range of $E_0 \pm \Delta/2$ over which the majority of the nuclear reactions take place in stars and they are the nuclear reactions which are playing important role in the energy production and also in the nuclear synthesis finally. After this I have discussed an important correction that is electron screening effect. But any way when I say two corrections in the reaction rate of non-resonant charged particle induced reactions, please remember they were the dependence of S-factor on energy and asymmetry of Gamow peak.

So, these two corrections have to be considered practically. After that there is one effect called as electron screening effect which is based on the fact that whatever discussion has done it is considered that the nucleus was bare, there are no electrons surrounding the nucleus. Within the stars also almost all reactions are taking place without the presence of electrons around the nucleus.

But in laboratory when projectiles react with the target nucleus, these target nuclei are surrounded by nucleus. So, in fact you have the atoms inside the target material, so whether the presence of electrons surrounding the nucleus will have any role in the measurement of cross section and the conclusion we have seen was negligible. Except at high stellar density one can ignore the electron screening factor f.

So, up to this we have discussed in the previous lectures. Now in today's lecture as I said I am going to discuss another type of nuclear reaction that is resonant reactions. So, how we are going to deal with this topic? In the same way as I discussed non-resonant reactions. The aim is to write down the expression for a reaction rate for resonant reactions, how one can calculate the rate of resonant reactions. Let us see how these resonant reactions are important and how to come up with an analytical expression for reaction rate when resonances are involved?

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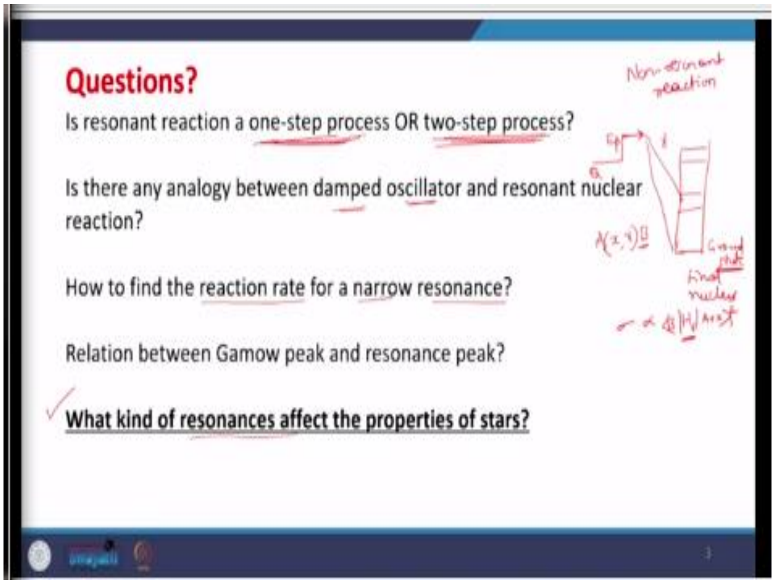

So, one of the best ways to understand any topic is to start with some questions. Like in the previous lecture I have asked a few questions regarding Gamow peak and also the importance of non-resonant reactions. Similarly, I am making an attempt to prepare a list of questions, so that we can understand the concept of resonant reactions in a better way. Is resonant reaction a one-step process like a non-resonant reaction or a 2-step process?

In one-step process what is happening in non-resonant reaction? The Q-value and the projectile, it is directly going into the final nucleus. For example, take a target nucleus A, x is a projectile and for the sake of convenience let us consider capture reactions say gamma is emitted and B is the final nucleus. So, here several nuclear energy levels are there within the final nucleus.

This can happen in any projectile energies these non-resonant reactions and this is directly transitioned to the ground state or the one of at any state above the ground state. It is basically direct capture that is why we call it as one-step process, this is a non-resonant reaction. And the cross section for this direct capture reaction can be expressed as matrix element given by one of matrix operator (refer to the above slide).

So, this is an electromagnetic operator, it is via electromagnetic interaction, it can occur at any projectile energy. Remember, the reactions in which cross section has a smooth dependence on the incident projectile energy; we are calling them as non-resonant reactions. And because it is a smooth dependence it is a one-step process and it can occur at any projectile energies. What about resonant reactions? Is it a one-step process like non-resonant reactions or is it a two-step process?

We will try to understand considering the same radioactive capture reaction. Can we understand the concept of resonant reactions with the help of the concept of damped oscillator? It is also a physical system; there are many physical systems which possess these resonance phenomena. I hope you remember the system of damped oscillator when external force is applied, it gives the response is maximum only at certain values of the incident force, the response is maximum and those responses we call as Eigen values.

Can we take analogy between this damped oscillator and the nuclear reaction involving resonance energy so that we can get a better idea and finally we can get expression for the cross section and then the reaction rate? We will see. Now as I said one of the common goals of these discussions is the expression for reaction rate when narrow resonances are involved. Now I am confining to the discussion on narrow resonance. The energy level density should be very small when it is a narrow resonance. Some more ways of defining this narrow resonance I will discuss very soon.

Another question, earlier we discussed Gamow peak as part of discussion on non-resonant reaction the product of $e^{-2\pi\eta,}$ that is Gamow factor and $e^{-E/kT}$, a factor coming while including the Maxwell -Boltzmann distribution. Then we have seen a beautiful Gamow peak almost close to the Gaussian function. What about the Gamow peak presence in resonant reactions, whether the resonance region and Gamow peak they are same or different? If yes how?

This question will address in today's lecture. Now one of the most important question we have to understand in the stars evolution and the role of resonant reaction is whether low level resonances, that means low energy resonances or high energy resonances play important role in the energy production and also in the nuclear synthesis and finally the stellar evolution. So, the properties of a star, do they depend on low energy resonances or high energy resonances?

If yes why? So, let us see how we can answer all these questions while trying to understand the concept of resonant nuclear reactions happening within the stars. I hope these questions are helpful to you.

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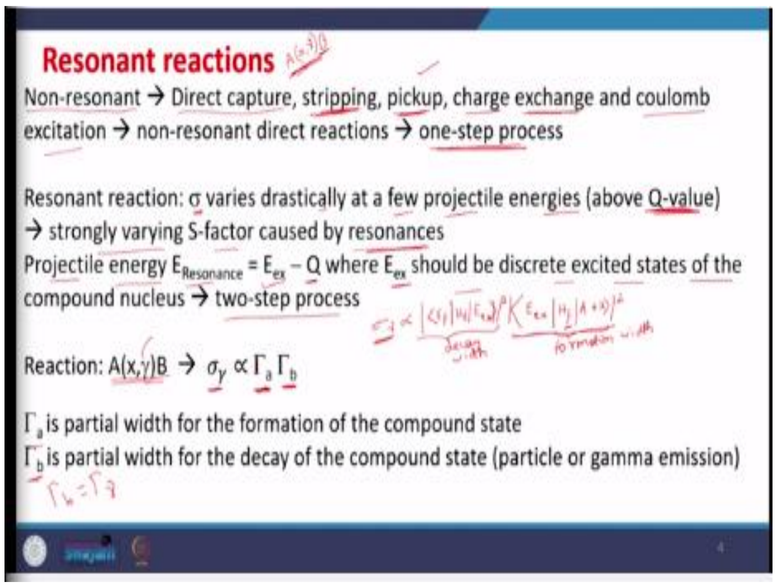

Let me give you some examples for non-resonant reactions. Like direct capture as I have said, A when it reacts with some projectile x and gives gamma ray and product nucleus B, this is one type of a direct capture that is a radiative capture reaction. Then stripping when incident nucleus takes out few parts of the target nucleus and then undergoes reaction, this is called as stripping reaction.

So, this is also an example for non-resonant reaction. Then pickup reaction, if some of the incidental nucleus are absorbed by the target nucleus then we call it as a pickup reaction. And then if there is no change in the mass number only the exchange of the charge is taking place then we call them as charging exchange process.

If the incident nucleus excites the target nucleus because of the Coulomb field while traveling near to it, then we call it as a Coulomb excitation and followed by the Coulomb dissociation. So, these are some of the reactions which come under the category of non-resonant reactions and they are basically one-step process. Now whereas in case of resonant reactions cross section it does not possess smooth dependence on the energy, it varies suddenly and drastically at a few projectile energies only.

And that to the energy state of the nucleus is above the Q-value. Incident projectile energy is not above Q-value but when incident projectile has some energy and if the incident projectile energy plus Q-value giving rise to some excitation energy matches with the one of the excited states in the nucleus then we can expect a sudden rise in the cross section of the reaction. And because cross section is not a smoothly dependent function anymore in resonant reaction, the S-factor also strongly varies because of the resonances.

Now as I said it occurs only when the incident projectile energy + Q-value is nothing but one of the excited states of the compound nucleus, this is what we mean by resonant reaction. Now this excited state in the compound nucleus should be one of the discrete excite states of the compound nucleus. That is the reason it is a two-step process, why it is two-step process? In the first step the entrance channel goes to the one of the excited states of the nucleus and in the second step this excited state decays to the final form.

So, in that sense it is a two-step process. So, consider the same radiative capture reaction how can we envisage as two-step process? In this case we can say that σ which is basically dealing with the gamma emission from the nuclear reaction is proportional to the two matrix elements. If you remember σ_γ can be written as 2 matrix factors (refer to slide 4).

And the first matrix element is compound nucleus formed in the excited state and formation operator f stands for formation and A + x, so this is the 2 matrix elements. So, here these 2 matrix elements represent the probabilities for the 2 steps to occur. Earlier also while discussing forward and reverse reaction I have explained the concept underlying this matrix elements representation.

They are connected through some operator decided by the interaction mechanism. So, one is the formation width and the other is partial width, that is probability for this formation to happen. So, the cross section of total reaction depends on the cross sections of 2 individual processes. Number 1, the probability for entrance channel to enter into the one of the excited states of the compound nucleus and then the probability for excited state to decay into a specific final state and we represent these 2 matrix elements with partial widths.

So, Γ_a is the partial width for the formation of the compound state and Γ_b is the partial width for the decay of the compound state, it could be particle emission or gamma emission. So, this is the way we are trying to understand the resonant reaction. So, in this slide I have explained resonant reaction is a two-step process, it can occur only at specific projectile energies, so that the projectile energy plus Q-value produce an energy which is just equivalent to one of the excited states of the compound nucleus. So, let us continue the discussion.

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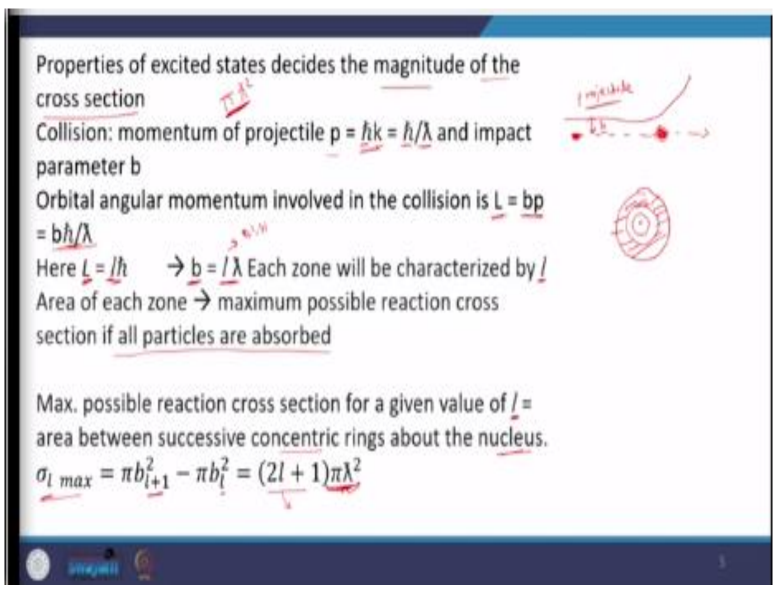

The properties of this excited state in the compound nucleus of course that will decide the magnitude of the cross section. So, when I say properties of this excited states of course it comes like a lifetime, parity, spin all those properties comes into picture. Now, here I can assume this formation width and decay width to understand in a better way.

See, what is the maximum cross section a nuclear reaction can attain? It is $\pi ƛ^2$, maximum cross section a nuclear reaction can attain. Now to express in a better way when resonance phenomenon is considered, I am assuming this process as collision between 2 parameters. Number 1, the momentum of the projectile reduced Planck's constant and the wave vector k which can be represented in terms of reduced Planck's constant by reduced de Broglie wavelength.

And the impact parameter, so how can we envisage this? So, this is the axis of the target nucleus along some direction, now a projectile is coming and it is going like this. And assume some distance between the original direction of the target nucleus and the projectile. The separation is nothing but impact parameter; you might have understood the concept of impact parameter when Rutherford alpha scattering experiment was taught to you. In this particular case I can assume the shadow of the nucleus seen by the incident projectile as regions whose area represents the cross section.

And in this collision if orbital angular momentum is involved then the product of impact parameter and the momentum gives you the orbital angular momentum value. So, that is equal to $b\hbar/ƛ$. Now in this particular case the orbital angular momentum also can be expressed as the orbital angular momentum quantum number. So, in the collision between projectile and target the orbital angular momentum is involved, that is what we are assuming in this particular case which is a normal case.

And here, L = bp and L = $l\hbar$. From these expressions we can get the impact parameter in terms of orbital angular momentum like, b = l ƛ. So, L is equal to 0, 1, 2, 3, 4 like that. Now each value of L corresponds to each zone which is characterized by the **s** value of orbital angular momentum quantum number.

And the area of each zone represents maximum possible reaction cross section if all the projectiles are absorbed. And what is the maximum possible reaction cross section for a given value of L nothing but the area between successive concentric rings about the nucleus. So, this is how we can relate the orbital angular momentum quantum number and the cross section of the nucleus, listen to me carefully.

Now the maximum cross section is nothing but area between the 2 successive concentric rings l and l + 1, corresponding areas what is the difference? It is nothing but $(2l + 1)\pi ƛ^2$. So, this is the maximum possible cross section for a given value of l, if l = 0, 1, 2, 3, 4 accordingly the sigma maximum, $\sigma_{l\ max} = \pi b_{l+1}^2 - \pi b_l^2 = (2l + 1)\pi ƛ^2$ also will be varying. Remember, this is the maximum, maximum means beyond which there is no possibility for the cross section to exist that is $\pi ƛ^2$.

But the cross section as I said in the resonant reactions it is changing dramatically at only specific value of the incident energy. And in that particular situation if orbital angular momentum is involved in the collision, then the value of l, orbital angular momentum decides the maximum possible cross section at that particular l value. I hope you are following me what I am trying to convey.

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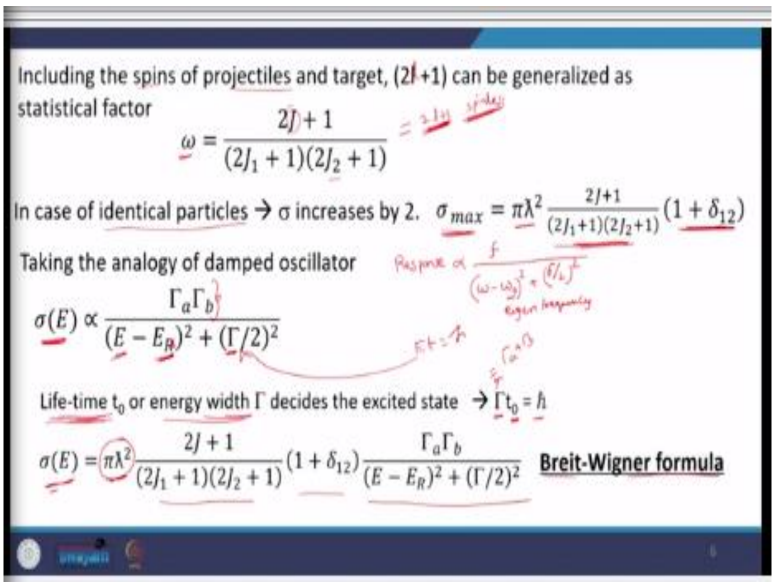

Now let us include the spins of the projectiles also and target. In that case, so not only orbital angular momentum but also involving the spin one can generalize the 2l + 1 as statistical factor, $\omega = \frac{2J+1}{(2J_1+1)(2J_2+1)}$. I mean this term is nothing new for you, while discussing forward and reverse reaction I have discussed the role of a statistical factor 2J + 1. This is the spin of the compound nucleus and J_1, J_2 are the spins of the projectile and target.

And if l = 0 of course it will finally reduce this to 2l + 1 only, if spin is 0 because J = l + s, if spin is 0 this statistical factor reduces to 2l + 1 as expected, for spinless particle it will reduce to 2l + 1. Now we also have to take into account the concept of identical properties. Then the cross section increases by 2, so let me express the maximum cross section as, $\sigma_{max} = \pi ƛ^2 \frac{2J+1}{(2J_1+1)(2J_2+1)}(1 + \delta_{12})$.

Now this is incomplete because we also need to know the concept of the cross section by taking analogy into with some kind of physical systems, why it is important? That will be clear soon to you. Now let me take the help of a damped oscillator, there are Eigen values for this damped oscillator, they correspond the maximum response when external force is given to this damped oscillator. And how do you write down the response of the damped oscillator?

Which is basically you can write like response of damped oscillator as oscillator strength f and frequency at which response is maximum, this is Eigen frequency. This you might be having idea about the damped oscillator and damping factor delta. So, this is how we represent the response of a damped oscillator when external force is involved. Only for certain values the response is maximum and that is characterized by omega 0 Eigen frequency.

By taking the analogy of damped oscillator one can write down the resonant phenomenon in the nuclear reaction as here the response can be replaced with the cross section because the cross section is maximum only at certain values of projectile energy. So, the response is replaced with the cross section and the oscillator strength is nothing but the product of the partial width of formation and decay width.

And the Eigen frequency is replaced with projectile energy E_R at which the Q-value plus E_R coincides with one of the excited states. Now whatever the energy state you are considering it has certain lifetime and it has an energy width also.

That will decide the excited state and using Heisenberg uncertainty principle like Et = $\hbar$. Here the corresponding value for energy is nothing but total partial width Γ, which is sum of Γ_a and Γ_b. And the product of these 2 is given by $\hbar$. By taking this one can write down the final expression for cross section of a resonant reaction.

$\sigma(E) = \pi ƛ^2 \frac{2J+1}{(2J_1+1)(2J_2+1)}(1+\delta_{12})\frac{\Gamma_a\Gamma_b}{(E-E_R)^2+(\Gamma/2)^2}$. This is called as the famous Breit-Wigner formula.

So, with this let me close today’s lecture. To summarize, I have provided the difference between resonant reaction and a non-resonant reaction. And step by step I am trying to come up with an expression for the cross section of a resonant nuclear reaction. So, in the next lecture I will continue the discussion on resonant reaction and I will also discuss neutron induced non-resonant reactions. Because in the previous lecture I have discussed non-resonant reactions induced by charged particles, we also should discuss the non-resonant reactions induced by neutrons. I hope you have enjoyed today's lecture, see you soon, thank you very much.