Nuclear Astrophysics

Prof. Anil Kumar Gourishetty

Department of Physics

Indian Institute of Technology- Roorkee

Lecture-16

Non-Resonant Charged Particle Induced Reactions

Dear students, welcome to 4th week of this course. So, till now we had 15 half an hour duration lectures and in these 3 weeks I could spend time on presenting some of the relevant aspects of astronomy and astrophysics for nuclear astrophysics. And some selected general properties of the universe like elemental abundance curve, H-R diagram mass luminosity relation, Hubble's law, remnants of 2.7 Kelvins.

And then I started discussing thermonuclear reactions because under the presence of high temperature starting from 10 to the power of 6 Kelvins to 10 to the power of 10 Kelvins in that normal range. Reactions are taking place within the stars which are responsible for the energy production from the stars and also these nuclear reactions are responsible for the origin of the elements in the universe.

And as part of thermonuclear reactions I have discussed various aspects like the velocity distribution of the particles, energy production from the stars, principle of time reversal invariance and mean lifetime of a nuclear reaction. In the last lecture I have discussed abundance evolution of a particular nucleus, for example I have considered aluminium whose mass number is 25.

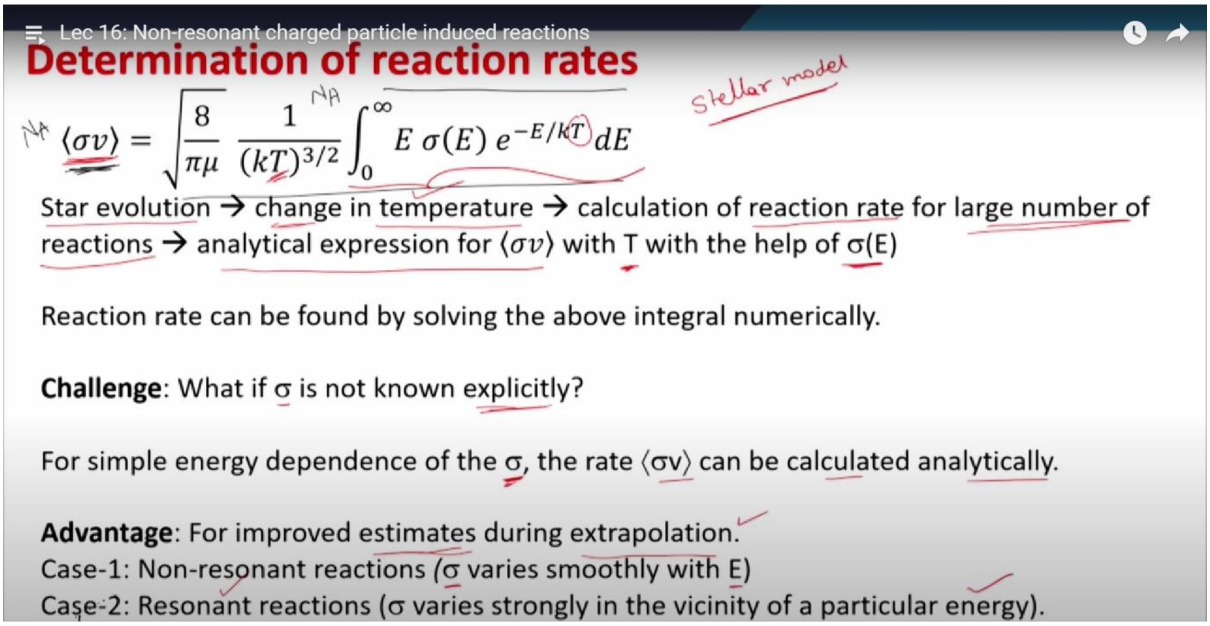
You need to consider various processes responsible for the production of a particular nucleus and various processes responsible for the destruction of the nucleus. If you remember I had considered charged particle induced reactions like proton and alpha is projectile reacting with some nuclei is giving rise to the nucleus aluminium 25. At the same time there are a few nuclei which can undergo decay to give the same nucleus aluminium 25 and then photo disintegration process.

Same set of processes are responsible for the destruction of the same nucleus. So, considering these aspects I have written a general expression for the time dependent abundance evolution of aluminium 25 nucleus. And I had asked you to write down a general expression for the abundance evolution of any nucleus. In this 4th week, in particular in today's lecture at least to me this is the most important lecture of this course.

We are going to discuss the actual nuclear astrophysics part in this lecture and of course in next couple of lectures as well. But in this 4th week I am going to use various parameters related to nuclear reactions which are very important for understanding the astrophysics aspects. It gives me immense pleasure to share with you a large number of questions. I hope these questions will inculcate interest regarding the subject.

As I said to me personally this lecture is the actual nuclear physics part of the course. The topics which I am going to cover in today's lecture are as follows. So, I have already covered the recap that means abundance evolution of the nucleus in the previous lecture I have covered. Astrophysical S-factor that is a spectroscopic factor, S stands for spectroscopic factor I will discuss it is role in nuclear astrophysics. I am confining to nuclear reactions induced by charged particles. It could be mainly protons and alpha, but of course it can cover any other charged particle as well.

I am going to discuss the non resonant type of reactions. I am going to introduce the most important concept of the nuclear astrophysics course that is Gamow peak when the reactions are induced by charged particles. I hope you will enjoy today's lecture as much I am enjoying.



Determination of reaction rates

$$N_A \langle \sigma v \rangle = \sqrt{\frac{8}{\pi\mu}}\,\frac{\overset{N_A}{1}}{(kT)^{3/2}}\int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE$$

Stellar model

Star evolution → change in temperature → calculation of reaction rate for large number of reactions → analytical expression for $\langle \sigma v \rangle$ with T with the help of σ(E)

Reaction rate can be found by solving the above integral numerically.

Challenge: What if σ is not known explicitly?

For simple energy dependence of the σ, the rate ⟨σv⟩ can be calculated analytically.

Advantage: For improved estimates during extrapolation.

Case-1: Non-resonant reactions (σ varies smoothly with E)

Case-2: Resonant reactions (σ varies strongly in the vicinity of a particular energy).

Determination of reaction rates and this is nothing new for you, this is the reaction rate per particle pair of course you can always multiply with any Avogadro number on the left hand side and you can always get Avogadro number on the right hand side also. This is the expression we came across in one of the previous lectures. Now as star evolves as star evolves there is a change in the temperature regularly. Why this temperature changes when star starts evolving?

Whatever matter is present within the star because of the gravitational effect contraction of this nuclear matter takes place. And the gravitational contraction proceeds until a stage is reached when this is converted into thermal pressure. And in the presence of thermal pressure the temperature is sufficient to provide the particle’s energies to undergo nuclear reactions. And these nuclear reactions are emitting energy, so this energy emitted by the nuclear reactions tries to stabilize the star.

This will go on until the consumption of a specific set of nuclei completes. So, that change in temperature is a very common phenomenon in the star evolution. At each temperature one has to find out the reaction rate because temperature is changing.

So, in order to model a star evolution that means in stellar model calculations reaction rate has to be calculated regularly which is time dependent and it has to be done for large number of nuclear reactions. Because at a time whatever nuclei are present they are undergoing reactions through various ways and in the previous lecture I have discussed aluminum 25, if I take one example it could be produced because of some decay of 1 nucleus or photo disintegration process, proton induced reaction or alpha induced reaction.

In all these processes one has to find out the reaction rate. Like that you need to consider reaction rate of different combinations of nuclides. Now it is a very tedious process, because of the involvement of large number of nuclear reactions the calculation of reaction rate is a very tedious process. That is the reason one can go for analytical expressions for the reaction rate which is depending on the temperature.

This analytical expression for the reaction rate depend which is depending on the temperature can be arrived with the help of the calculation of cross section. Because reaction rate involves cross section, so the cross section of nuclear reaction is a fundamental parameter and without cross section how to find out the reaction rate, it is not possible. The cross section which depends on the energy of the nuclear entities involved in the reaction, the calculation of cross section is very important.

So, based on this let us go forward in defining different types of nuclear reactions. Now, this reaction that can be found by solving the integral, this integral by numerically also. But the challenge is that if the cross section is not known explicitly then what can be done? So, this requires the explicitly cross section information has to be provided. So, considering the simple energy dependence of cross section one can find out the reaction rate analytically.

Of course this is valid only when the simple energy dependence is happening with the cross section. Because the energies at which nuclear reactions are taking place they are too low for direct measurements to happen I mean to carry out in earth's laboratory.

So, at those energies when it is not possible to carry out the nuclear reaction but we want the cross section at those nuclear reactions, what we will do? Whatever energy range available, we will measure cross section for that range and then we will do the extrapolation. But one has to be careful that this extrapolation should not include a high level of uncertainty. Now, it has been established that the estimation quality is improved when we take the analytical expression.

So, beyond this there are other advantages also but this is one of the major advantages by going to analytical method for calculating the reaction rate. Now if the cross section varies smoothly with energy then those reactions are called as non-resonant reactions. Whereas, if the cross section varies suddenly and strongly in the vicinity of a specific energy then those reactions are called as resonant reactions.

So, nuclear reactions in which cross section varies smoothly, there is no sudden change in the cross section we can term them as non-resonant nuclear reactions. Whereas, depending on the combination of target and projectile and the energy values available, there is a possibility that cross section can vary drastically in the vicinity of a specific energy value. This kind of nuclear reactions are called as resonant reactions.

I am confining the discussion to non-resonant nuclear reaction which takes into account the smooth dependence of cross section on the energy of the system. We can consider centre of mass energy when 2 different types of nuclei are involved. So, I hope this background helps you in understanding the most important nuclear astrophysics aspect.

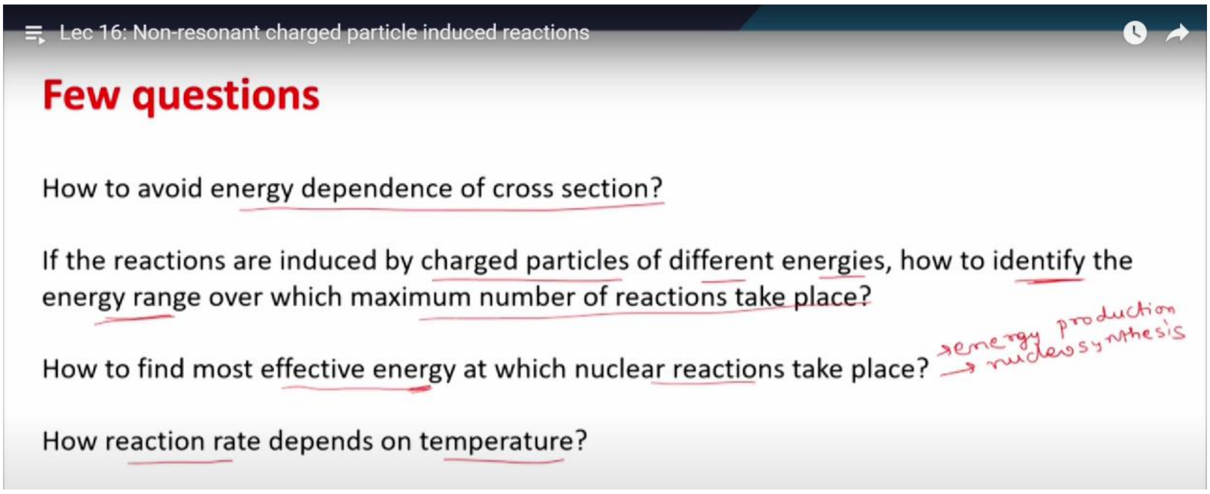
Let us continue I would love to pose few questions before introducing some new parameters in this course. Is there any way to avoid the energy dependence of the cross section. Because if there is a huge energy dependence then it would be difficult to have accurate values after extrapolation. If there is no way to get the cross sections other than extrapolation but we need accurate values of the cross section then the energy dependence can it be avoided when we deal with the cross section calculation?

If yes, how it is possible? So, listen to the questions carefully. If the reactions are induced by charged particles of different energies. How to know the energy range over which maximum number of reactions take place? So, with these questions it must be clear to you, when the values of energy of the nuclei are spread over a wide range.

Then naturally an important question should come to our mind. Whether the probability for a reaction to happen is same, over entire energy range available? If not how to identify the energy range, the energy window over which maximum number of reactions take place. It is very important, why because in earth's laboratory when we try to measure the nuclear reaction assume that energy is available, fortunately we have a few underground facilities using which we can go to the energies which are existing in the stars and at which reactions are taking place.

But before that we need to know at what energy or during what energy window the maximum reactions are taking place, how to know that? That we are going to answer in today's lecture. I hope you are realizing the importance of today's lecture with the help of these questions. There are few more questions. How to find the most effective energy at which nuclear reactions take place? Earlier I was talking about the energy window, now I am talking about the most effective energy value at which nuclear reactions take place which are responsible for the energy production in stars and also nucleosynthesis.

Is there any relation between reaction and temperature? Yes, in the previously I have said reaction rate has to be calculated at different values of temperature, this is a qualitative statement. Can we come up with some mathematical statement, some kind of analytical expression, so that we can get some good numbers which can give us feeling that oh! reaction rate is dependent on temperature like this.



Few questions

How to avoid energy dependence of cross section?

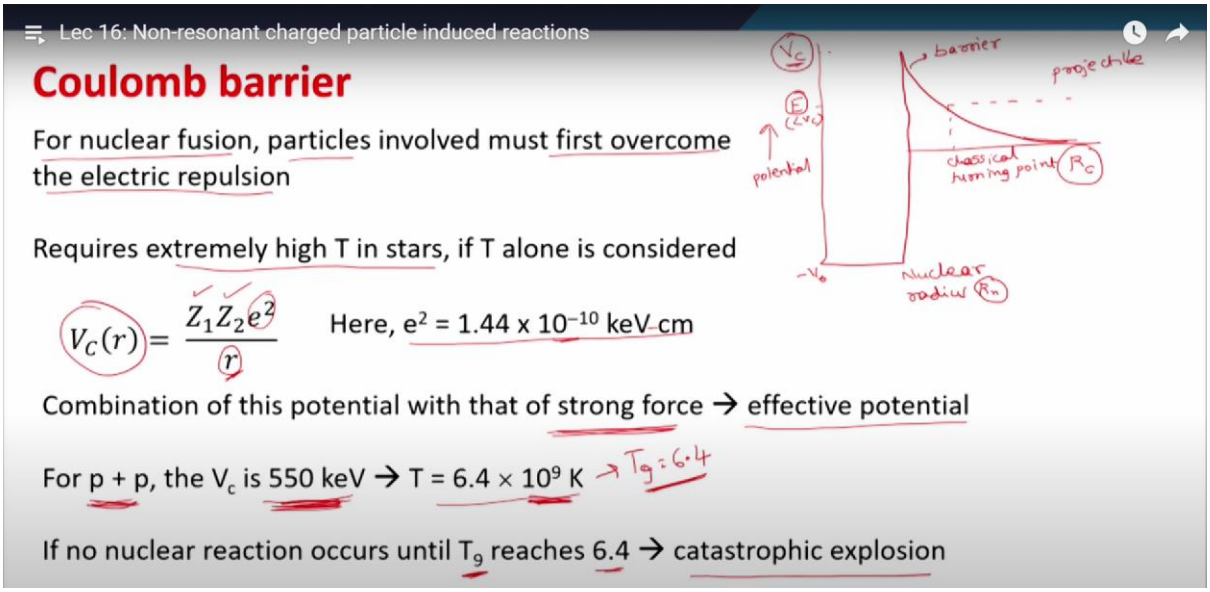
If the reactions are induced by charged particles of different energies, how to identify the energy range over which maximum number of reactions take place?

How to find most effective energy at which nuclear reactions take place? → energy production → nucleosynthesis

How reaction rate depends on temperature?

For 1% of change in the temperature reaction rate will change like this. For this particular type of nuclear reaction. To know all these things we need to come up with mathematical relation which can relate reaction rate and temperature. So, this is the one of the final goals in this lectures, the relation between reaction rate and temperature.

Now for nuclear fusion which is the basic nuclear reaction in stars mainly, the particles involved must first overcome the electric repulsion. See, initially after the big bang when quarks and gluons were formed then protons were formed. So, when protons were formed fusion of these protons gave rise to helium. But before that proton and proton being charged entities, they have repulsion between each other.



Coulomb barrier

For nuclear fusion, particles involved must first overcome the electric repulsion

Requires extremely high T in stars, if T alone is considered

$$V_C(r) = \frac{Z_1 Z_2 e^2}{r}$$

Here, $e^2 = 1.44 \times 10^{-10}$ keV cm

Combination of this potential with that of strong force → effective potential

For p + p, the V_c is 550 keV → T = 6.4×10^9 K → T_9 : 6.4

If no nuclear reaction occurs until T_9 reaches 6.4 → catastrophic explosion

To overcome that repulsion, so that nuclear fusion can happen and energy can be liberated and a new nuclear can be formed, what is required? Sufficient energy should be available for these protons to overcome the coulomb repulsion. Normally how do we draw simple nuclear potential? Let me draw this, so it looks like this. This is a potential well and this is the nuclear radius Rn. For example this is the coulomb barrier Vc and on y axis you have potential and it is almost touching the x axis.

If the incident particle has energy say E which is less than Vc, then this is the classical turning point in terms of radius. So, we can designate with Rc, this is the projectile. And what is this curve? This is the barrier which has to be crossed by the projectile, so that it can come into the nuclear domain, is not it? So, you have Rn and you have Rc, you have Coulomb barrier denoted by Vc and incident energy is E.

You are aware that the potential because of this coulomb repulsion is given by Z1 that means atomic number of projectile and atomic number of the target nucleus e square by r, where this r can be like interaction radius. e square numerically can be represented as 10 to the power of -10 keV centimetre. Now this coulomb potential to overcome high temperature is required in stars.

If only temperature is considered within the stars for reaction to happen we need very high temperature. How to calculate the high temperature to overcome on this barrier? It is very simple calculation. But before that let me also continue by saying that this Coulomb potential combining with the strong interaction, assuming that interaction is via strong interaction, strong force then you can come up with some effective potential, which looks like this as shown in the diagram.

Now let me consider the most important nuclear reaction in the universe. To me this is the most important nuclear reaction in the universe which is a responsible for the major part of the evolution of universe. For proton + proton what is the coulomb barrier? The energy required to undergo fusion, if you do the calculation, it comes out to be 550 keV.

This corresponds to an energy of 10 to the power of 9 Kelvins. The temperature corresponding to the Coulomb barrier is 10 to the power of 9 Kelvins. If 10 to the power of 9 Kelvin is sufficient for protons to undergo fusion classically, then due to some reason if this temperature is created, maybe due to gravitational contraction it is very much possible.

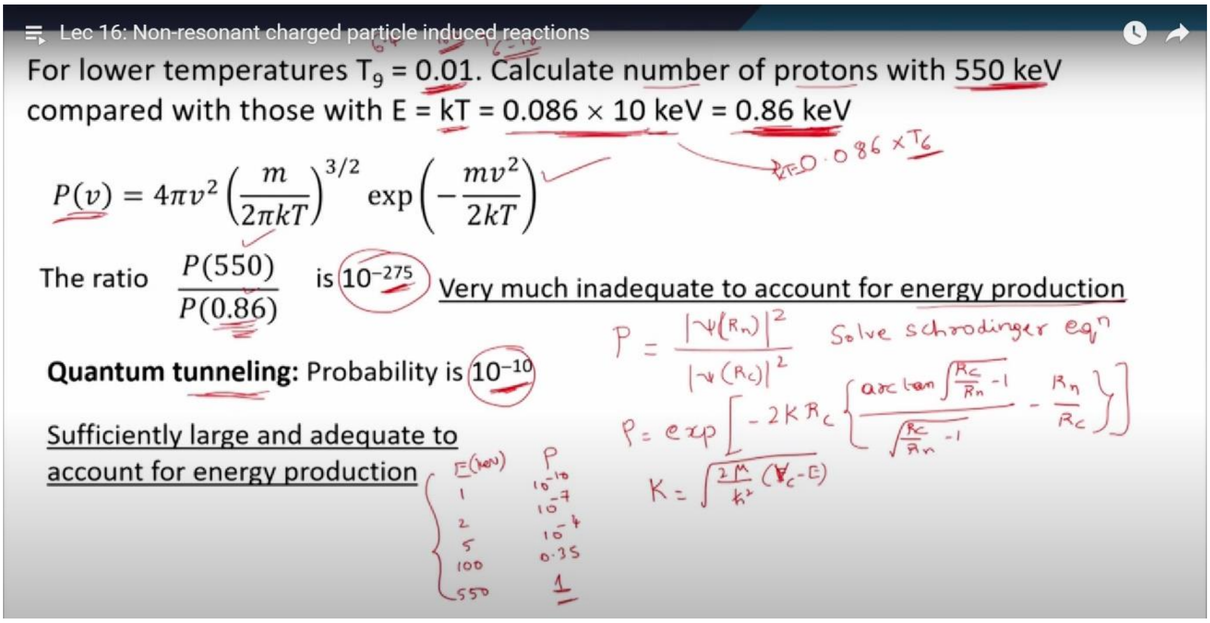
So, the moment 10 to the power of 9 Kelvin temperature is happening within the star due to some reasons like gravitational contraction. Then whatever protons are present they should undergo fusion instantaneously and then there should be a catastrophic explosion. So, if no nuclear reaction occurs T9, that means temperature in terms of gigakelvin reaches 6.4 because this can always be written as T9 = 6.4.

So, there will be explosion of the star catastrophically. So, whatever stars we are seeing now we cannot see them anymore, but we are able to see and these explosions are not taking place catastrophically and simultaneous or instantaneous reaction between protons is not happening. What does it mean? It means before 10 to the power of 9 Kelvin temperature is reached protons are undergoing reaction with each other which is not possibly classically.

So, of course the conclusion which I am going to draw is not something new for you. But I am trying to provide some interesting numbers while leading to the conclusion. So, the conclusion of this slide is as follows. If alone temperature is considered then 10 to the power of 9 Kelvin temperature is required for protons to undergo fusion. If that is the case all protons will simultaneously react with each other and explosion take place, which is not happening practically.

So, that means before this temperature is created protons are undergoing fusion. This was very much I mean confusing for the researchers at that time when a subject did not reach it is mature state. So, for lower temperature if we go instead of 6.4 Gigakelvin if I go for 0.01. Please remember earlier I was considering T9 as 6.4, now I am considering 0.01. That means 10 to the power of 7 Kelvins which is 100 times less than the temperature required for fusion to happen classically. Classically for nuclear fusion to happen the incident particle should have an energy above the coulomb barrier. That leads 10 to the power of 9 Kelvin.

Our discussion clearly stated that it is not possible that fusion reaction should happen only when 10 to the power of 9 Kelvin is reached. So, to understand in a better way let me consider a temperature which is much less than the temperature corresponding to coulomb barrier. That is 10 to the power of 7 Kelvins. Now let us calculate the number of protons having energy 550 kilo electron volt compared with those with k into T, where T is here not 10 to the power of 9 Gigakelvins.



For lower temperatures $T_9 = 0.01$. Calculate number of protons with 550 keV compared with those with E = kT = 0.086 × 10 keV = 0.86 keV

$kT = 0.086 \times T_6$

$$P(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right)$$

The ratio $\frac{P(550)}{P(0.86)}$ is 10^{-275} Very much inadequate to account for energy production

$P = \frac{|\psi(R_n)|^2}{|\psi(R_c)|^2}$ Solve schrodinger eqn

Quantum tunneling: Probability is 10^{-10}

$P = \exp\left[-2KR_c\left\{\frac{\arctan\sqrt{\frac{R_c}{R_n}-1}}{\sqrt{\frac{R_c}{R_n}-1}} - \frac{R_n}{R_c}\right\}\right]$

Sufficiently large and adequate to account for energy production

$K = \sqrt{\frac{2M}{\hbar^2}(V_c - E)}$

E(keV)	P
1	10^{-10}
2	10^{-7}
5	10^{-4}
100	0.35
550	1

0.86 kilo electron volts. Using this formula probability distribution of the particles, what will happen here? See, I hope this is clear to you T9 = 0.01 means T6 = 10. Earlier if you remember I have discussed one formula 0.086 into T6 K into T value and at T 6 0.01 I mean Gigakelvin. That means 10 kilo electron volt, the energy available is 0.86 kilo electron volt.

But remember the fusion can happen classically only at 550 kilo electron volts and we are considering the situation at low temperature at which kT gives rise to only 0.86 kilo electron volt less than 1 keV, Now you calculate the number of protons with 550 keV when compared to with 0.86 keV using this distribution function. The ratio of probability for 550 keV particles to probability for 0.86 keV particles comes out to be 10 to the power -275.

I strongly suggest you to please do this calculation yourself, 10 to the power of -275, why this calculation became important? Because people have argued at that time that you know Maxwell-Boltzmann distribution is going to the higher end may be at a particular temperature which is less than the 10 to the power of 9 Kelvins.

At lower temperature fusion is happening that was clear, because if it is happening at higher temperature catastrophic explosion should have taken place which is not happening practically. So, at lower temperature only reactions have to happen. At lower temperature like 0.01 Gigakelvin for reactions to happen people have argued that there must be good number of particles which are having energy at the higher end of the Maxwell-Boltzmann distribution whose maximum is kT. MB distribution maximum is kT.

But at the higher end if you go energy will be higher, at the same temperature. At the same temperature particles can possess energy not only kT but also higher energy also. Maybe these higher energy particles are inducing fusion classically to know that this calculation is important. But this came out to be 10 to the power of -275 which is very much inadequate to account for the energy production in stars. So, the only way out as suggested by Gamow at that time is during alpha decay because of the quantum tunnelling alpha can come out.

In the same way maybe protons can undergo fusion quantum mechanically based on tunnelling process. The probability is not -275 but it is -10 which is sufficiently large which is sufficiently large and adequate to account for the energy production. Here I am willing to give some interesting numbers. See the probability in terms of quantum mechanics at nuclear radius the square of the wave function you know the probability for finding the particle at the nuclear radius.

The probability for finding the particle at classical radius that is Rc, if you solve Schrodinger equation then you can come up with an expression like exponential - 2 K R c arc tan R c by R n ok square root divided by R c by R n - square root ok - R n by R c. There is no need to understand the whole derivation. Just I am trying to highlight one important thing here. That is here the value of K is 2 mu divided by h square coulomb barrier V c – e, this is square root.

Now if I go for energy and the probability for 1 keV, 2 keV, 5 keV, 100 keV and 550 keV. The probability is 10 to the power of -10 for 1 keV, 10 to the power of -7 for 2 keV and 10 to the power of -4 for 5 keV. 0.35 for 100 keV and as you expect 1 is the probability for tunnelling to take place for fusion to happen is 1 at 550 keV because this is the coulomb barrier.

What these values tell us? Even at 1 keV energy the probability of tunnelling is 10 to the power of -10, this one can calculate using quantum mechanical treatment. And as expected the probability should be 1 at the coulomb barrier energy. So, this tells us that 10 to the power of -10 is not very low when compared 10 to the power -275 and this is sufficiently large when you go for the ratio of P 555 to P 0.86.

Adequate to account for energy production in the stars. So, to summarize today's lecture what I have discussed? I have posed a few questions to understand various aspects of charged particle induced reactions and the definition of non-resonant reaction and resonant reaction.

I have discussed one of the most important nuclear reaction which is responsible for the evolution of this universe that is proton + proton whose Coulomb barrier is 550 keV.

It is not possible for the reaction to happen via nuclear fusion based on classical physics concepts and thanks to the discovery of Gamow. Based on tunnelling process we could arrive at some number that is 10 to the power of -10 which is the probability for tunnelling to happen is adequate for accounting the energy production in stars. So, in next lecture I will continue few more important aspects of this charged particle induced reactions, thank you so much.