Nuclear Astrophysics

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Lecture – 15

Mean Life Time of a Nuclei… and Time Dependent Abundance Evolution

Welcome students to the discussion on nuclear reactions. How they play important role in understanding the abundance evolution of elements. Let me take a quick recap of the previous lecture. I have discussed for photon induced reactions. How, mathematically one can write down expressions for cross sections, connecting the forward reaction cross section to the reverse reaction cross sections. The underlying main concept of this inverse reaction is following.

So, that you can get the actual point. At low temperatures nuclear reactions with positive Q values play important role in the energy production of stars. Now at the low temperatures because energy is not sufficient to the nuclei involved in the reaction to participate in a reaction with negative Q value, at high temperatures where the energy of the nuclei will be equal to or more than Q value of the nuclear reaction. We can expect negative Q value nuclear reaction to also proceed significantly. That is one reason. Number two, practically many times it is not possible to measure the cross section of a nuclear reaction of our interest. Reason could be anything. Maybe we wanted to come out with the energy liberated because of that particular reaction. What is its contribution to the overall energy production from the stars or synthesis of new element.

It could be any reason but due to some technical difficulties it may not be possible to carry out that reaction. In this kind of scenarios researcher's look for the possibility to carry out inverse reaction. One of the beautiful examples I have discussed in terms of gamma ray induced reactions. When a nuclear reaction happens if one of the reaction products is gamma ray then that reaction is called as radiative capture reaction.

If it is induced by a proton, then we call it as proton capture reaction. If it is induced by neutron we call it as neutron induced reaction. If it is induced by alpha we call it as alpha radiative capture reaction. What about reverse process. If a gamma induces the reaction then we call it as photo disintegration process. What is photo disintegration? Process a nucleus absorbs the gamma of certain energy then it decays into different products gamma + type 3, nucleus gives rise to 1 and 2 types of nuclei this is photo disintegration reaction.

And in the last class I have discussed how the cross sections of forward and inverse reactions can be connected to each other taking the help of principle of time reversal invariance. One more thing you have to remember is that till now there is no experimental data which says that this time reversal invariant invariance is violated. So, at least for electromagnetic interaction and a strong interaction it has been established that by taking the ratio of the forward and reaction cross section data one can play the game.

And after writing the reaction cross sections, I have written the ratio of reaction rates. From the ratio of reaction rates we have seen the importance of one exponential term. I have discussed the dependence of the ratio of the reaction rates on the Q value and also the temperature. I have given few numbers which has clearly said that the ratio of inverse and forward reaction rates is very much sensitive to Q value of the reaction.

And it is more important at high temperatures that means at high temperatures inverse reactions will proceed significantly. So, this I have extended to photon induced reactions. After that I have discussed energy production in stars. While discussing energy production in stars I have highlighted the importance of non-deposition of energy by neutrinos.

Because the neutrinos are extremely weakly interacting, they pass through the stars. They come outside of the stars and reach the earth and different places in the universe as if nothing is stopping them. So, that is why it is quite challenging to detect the neutrinos. So, if you want to measure the energy produced by a star and if you have the information of all the reactions which I will discuss in detail in today's lecture then one should ignore the contribution of neutrinos. You can always say what about positron. Positron will immediately find electron in the surroundings and it will undergo pair annihilation. Which gives 511 keV gamma rays. Those gamma rays like other gamma rays produced by other reactions, will be absorbed within the star. And after that I discussed the importance of lifetime of the nucleus. Mean lifetime of the nucleus when it undergoes reaction with another nucleus.

So, what was the expression I have discussed in the last lecture. The mean lifetime of the nucleus 1 when it reacts with type 2 nucleus is equal to 1 by number density of nucleus of type 2 and the reaction rate. So, what this equation tells us. The mean lifetime of nucleus designated by number 1 depends on the number density of the same type because the lifetime of a particular nucleus depends on the number density of the second type nucleus with which it is interacting and it is losing its existence.

That is why the number density of type 2 decides the lifetime of the nucleus type 1 and the reaction rate per pair also decides the lifetime of the nucleus of type 1. This is what I have discussed in the previous lecture. So, based on this knowledge in today's lecture please let me focus on one of the fascinating aspects of the nuclear astrophysics. Coming back to the main objectives of the course. To understand the energy produced from the stars and also to understand the nucleosynthesis, the process of formation of new nuclei these two are the main objectives of the course.

Now let me focus on one particular nucleus say x. How mathematically one can represent the time dependent abundance evolution of this particular nucleus. So, in today's lecture we are going to get the answer for this question. How to represent time dependent abundance evolution of a particular nucleus within a star. Number 2 a nucleus can be destroyed in more than a way.

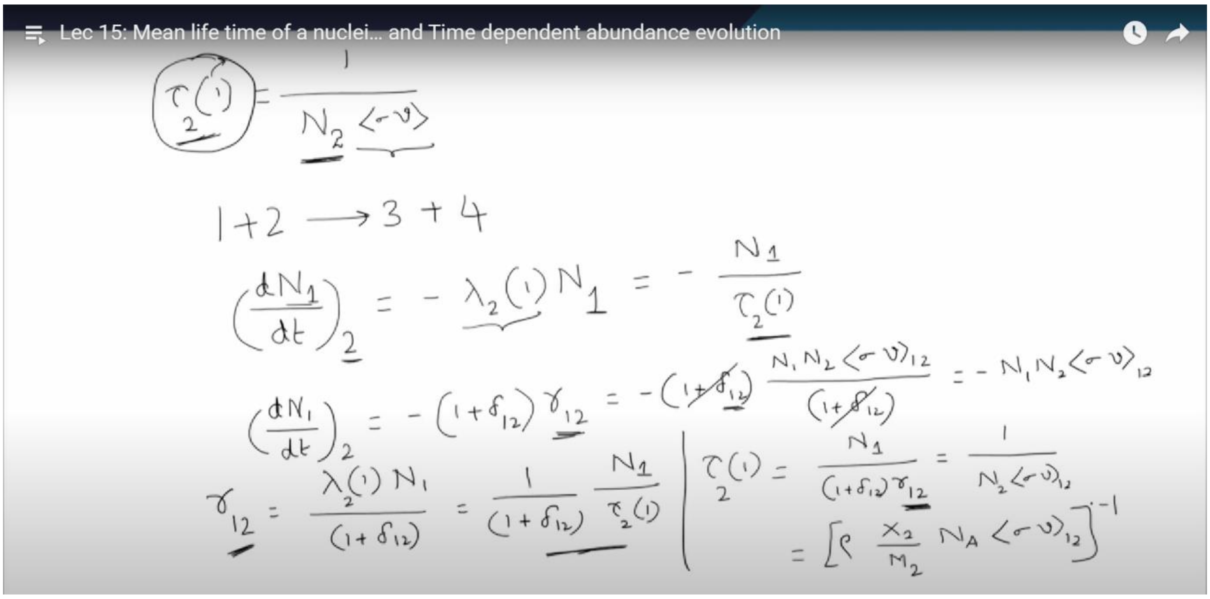
So, if nucleus can be destroyed by interacting with nucleus of type 2 or 3 the question is which one is more dominant in the destruction process. For example if you consider a nucleus type of type 1 it can interact with type 2 it can interact with type 3. For example if it interacts with say some proton, it can undergo destruction. If it can interact with say alpha particle it can undergo destruction which one is more dominant.

Sometimes you can come across a nucleus which is radioactive in nature. So, it can undergo decay. So, it has lost its existence but before it undergoes a decay if it reacts with another particle then also it undergoes destruction which process is dominant. On what parameters the dominant process of destruction can be understood. Why it is important?

The understand the abundance evolution of any nucleus, you need to understand in how many ways the nucleus is produced and in how many ways nucleus is destroyed. That combination of processes decides the evaluation of a particular nucleus. But before answering these two questions number one. How mathematically one can represent the abundance evolution of a nucleus. Number 2 how to understand the dominant process of destruction.

Before answering these two questions let me introduce a few parameters and a few important equations with the help of which we can answer these two questions. So, for that let me continue the discussion with which I entered the previous lecture that is mean lifetime of the nucleus. So, let us continue the discussion. For this I am considering the same type of the nuclear reaction that means 1 + 2 gives rise to 3 + 4 like a general reaction type.

Now the rate with which nucleus of type 1 undergoes destruction by interacting with a nucleus of type 2 dN 1 by dt because of the presence of 2 is equal to - lambda 2 that means the decay constant of nucleus of type 1 and original density of type 1 nucleus original number density of type 1 nucleus. This I can express in terms of lambda. That is decay constant in terms of mean lifetime can be represented as mean lifetime of nucleus 1 when it is interacting with type 2 nucleus.



$$\boxed{\tau_2(1)} = \frac{1}{N_2 \underbrace{\langle \sigma v \rangle}}$$

$$1+2 \longrightarrow 3+4$$

$$\left(\frac{dN_1}{dt}\right)_2 = -\underbrace{\lambda_2(1)} N_1 = -\frac{N_1}{\tau_2(1)}$$

$$\left(\frac{dN_1}{dt}\right)_2 = -(1+\delta_{12})\, r_{12} = -\cancel{(1+\delta_{12})}\frac{N_1 N_2 \langle \sigma v \rangle_{12}}{\cancel{(1+\delta_{12})}} = -N_1 N_2 \langle \sigma v \rangle_{12}$$

$$r_{12} = \frac{\lambda_2(1) N_1}{(1+\delta_{12})} = \frac{1}{(1+\delta_{12})}\frac{N_1}{\tau_2(1)}$$

$$\tau_2(1) = \frac{N_1}{(1+\delta_{12}) r_{12}} = \frac{1}{N_2 \langle \sigma v \rangle_{12}} = \left[\rho \frac{X_2}{M_2} N_A \langle \sigma v \rangle_{12}\right]^{-1}$$

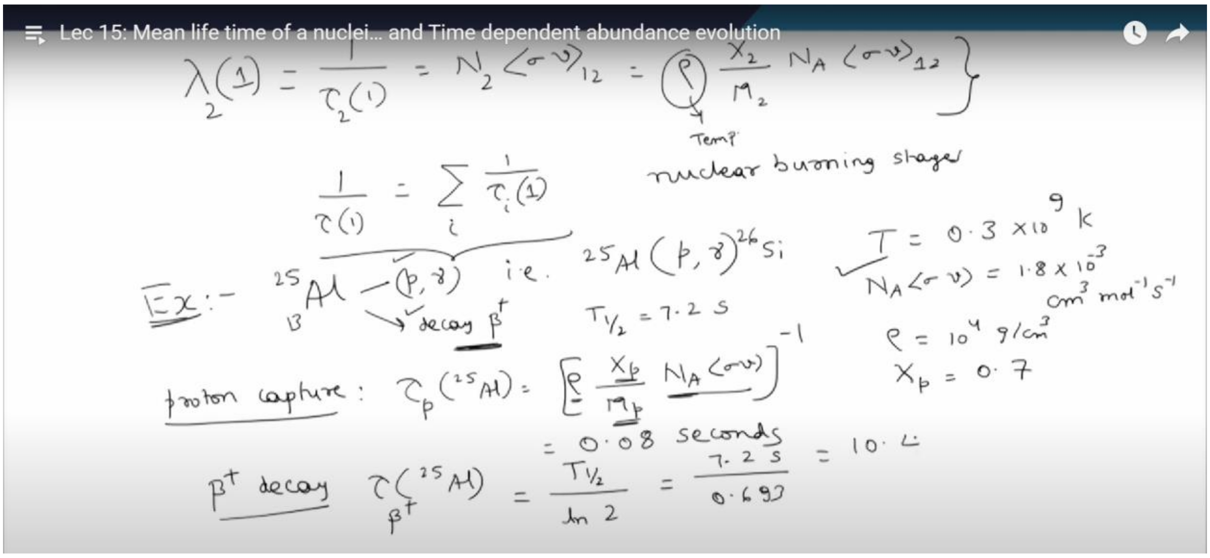
I hope this expression is clear to you. The rate of nuclei of type 1 with which it is undergoing destruction by reacting with type 2 nucleus is nothing but the number density of original number density of the type 1 nucleus and the lifetime of the nucleus 1 where lambda is 1 by tau which is the decay constant. Now I can also write this as dN 1 by dt by interacting with type 2 considering the nature of identicalness delta 12 total reaction rate r 12 is equal to -1 + delta 12 N 1 I am writing the expression for total reaction rate r 12 all right sigma V that is reaction rate per pair which is involving first and second type nuclei 1 + delta 1 2.

I can write down like -N 1 N 2 sigma V 12. So, this Kronecker symbol appears as I said because for identical nuclei each reaction destroys 2 particles. From this I can write down few more expressions like total reaction rate is equal to delta. So, I mean lambda decay constant of type 1 nucleus when it interacts with type 2 nucleus N 1 divided by 1 + delta 12 is equal to 1 by 1 + delta 12 and N 1 divided by tau 2 of 1 this is 1 expression for total rejection rate.

Using this total reaction rate expression I can write down the mean lifetime of nucleus 1 when it interacts with 2 in some useful terms like following is equal to N 1 divided by 1 + delta 1 2. Total reaction rate when first and second type nuclei are involved is equal to 1 by N 2 sigma V 12 because I am writing here by substituting r 12 this expression this equation in this place.

In terms of mass fraction I can write down like this mass fraction of type 2 nucleus and mass of type 2 nucleus in amu and Avogadro number sigma V 12. So, this equation helps us to find out the mean lifetime of a nucleus type 1 when it interacts with type 2 nucleus. This is helpful in terms of some quantities like stellar density, rho, mass of second type nuclei in amu and mass fraction of the type 2 nucleus.

So, this is one interesting relation which helps us to answer the question. Which is the dominant process of destruction when a nucleus can undergo in more than one ways of destruction. This formula I will discuss again in detail by taking one interesting example.


Lec 15: Mean life time of a nuclei... and Time dependent abundance evolution

$$\lambda_2(1) = \frac{1}{\tau_2(1)} = N_2 \langle \sigma v \rangle_{12} = \underset{\text{Temp}}{\rho} \frac{X_2}{M_2} N_A \langle \sigma v \rangle_{12} \Big\}$$

nuclear burning stage

$$\underbrace{\frac{1}{\tau(1)} = \sum_i \frac{1}{\tau_i(1)}}$$

Ex:- ${}^{25}_{13}Al - (p,\gamma)$ i.e. ${}^{25}Al(p,\gamma)^{26}Si$

$\searrow$ decay β^+ $\quad T_{1/2} = 7.2$ s

$T = 0.3 \times 10^9$ K

$N_A \langle \sigma v \rangle = 1.8 \times 10^{-3}$ cm^3 mol^{-1} s^{-1}

$\rho = 10^4$ g/cm^3

$X_p = 0.7$

proton capture : $\tau_p({}^{25}Al) = \left[\rho \frac{X_p}{M_p} N_A \langle \sigma v \rangle \right]^{-1}$

$= 0.08$ seconds

β^+ decay $\tau_{\beta^+}({}^{25}Al) = \frac{T_{1/2}}{\ln 2} = \frac{7.2\ s}{0.693} = 10.4$

So, then let me continue the discussion by writing expression for the decay constant. Decay constant in terms of decay constant of type 1 nucleus when it undergoes reaction with type 2 nucleus is mean lifetime inverse is equal to same relation N2 sigma V12 and rho mass fraction of second type nucleus and mass of second type nucleus NA sigma V of 1 and 2. And the decay constant of the nucleus for destruction via any particle induced reactions like proton, alpha or any other particle induced reaction depends on the stellar density.

Further it depends on the temperature through the reaction rate. If type 1 nuclei can be destroyed by several different reaction the total lifetime of first type nucleus can be represented as 1 by tau i of 1. Now this expression is very useful and frequently they will be applied in several stages of nuclear burning processes. Do you remember in one of the previous lectures I have said there are six burning stages in the star. Hydrogen, helium and then we have carbon burning. Then we have silicon burning then we have s process r process and p process and then finally l process. So, in different burning stages this formula is going to play very important role. So, please bear with me for a few lectures till I use this formula extensively. But before that with the help of 1 numerical let me discuss the importance of this formulae in deciding the dominant way of destruction of a particular nucleus.

For example let me solve one interesting example for you. In stellar plasma consider 25 aluminium. It can be destroyed by proton gamma. That means 25 aluminium reacting with proton it emits 26 silicon. It can also undergo decay. Which kind of decay? Positron decay, whose life is equal to 7.2 seconds here you see aluminium 13 protons are more and neutrons are less number of neutrons is 12. So, 1 proton tries to convert into neutron.

So, as part of that it undergoes positron decay. Now you assume only these 2 processes. Now you have to say which process is dominating given data is t is equal to say 0.3 into 10 to the power of 9 Kelvins that means 0.3 Giga Kelvin. The given reaction rate is 1.8 into 10 to the power of -3 centimetre cube per mole per second and rho is 10 to the power of 4 grams per centimetre cube. And mass fraction of protons or hydrogen say 0.7.

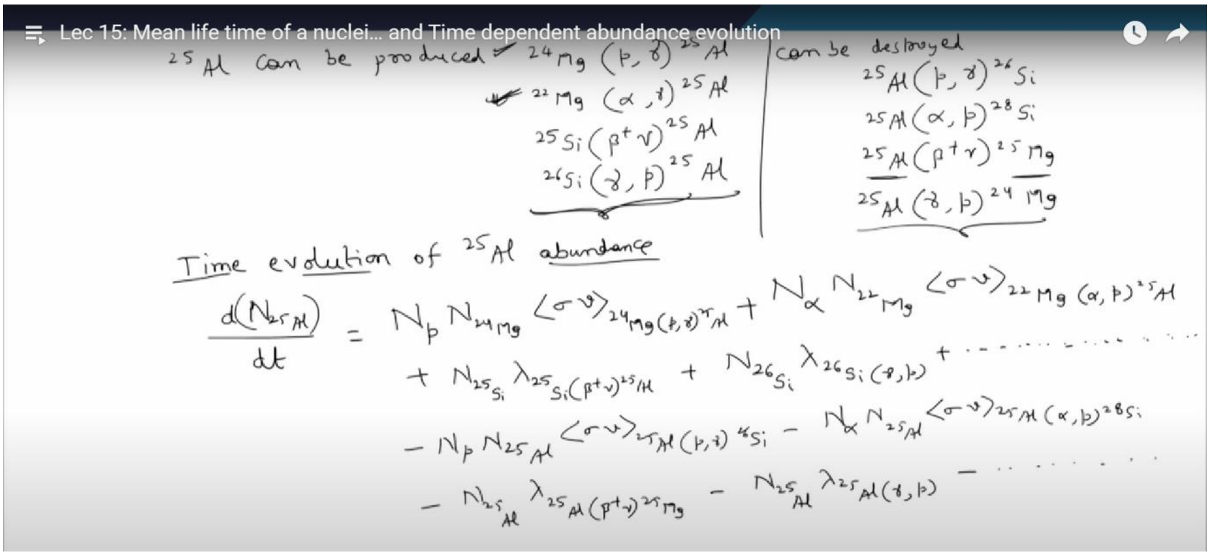
Now let me first consider proton capture reaction. In proton capture reaction what is the lifetime? By reacting with proton type 1 nucleus here it is 25 aluminium and the same formula you can use here. So, rho it is mass fraction of protons Avogadro number sigma V. If you calculate these rho is 10 to the power of 4 and mass fraction is 0.4 this is 1.078 amu and NA into the reaction it is already given here.

If you calculate this it is about 0.08 seconds. Similarly the second way of destruction that is beta decay what is the lifetime of 25 aluminium by undergoing beta plus process is nothing but half life of the beta plus decay divided by logarithm of 2. This is a well known relation to you. So, 7.2 seconds divided by 0.693. This gives me around 10.4 seconds. What does it mean? The lifetime of aluminium 25 via beta plus decay is 10.4 seconds.

Whereas the lifetime of aluminium 25 by reacting with protons. Because protons are in abundant it is 0.08 seconds. So, where is 10 seconds and .08 seconds. So, what is the conclusion? The dominant way of destruction is proton-induced reaction. Remember aluminium 25 is not a stable nucleus it is radioactive. So, it is always free to undergo destruction simply by decaying.

But before undergoing decay because of the mass fraction and number density available the stellar density which further depends on the temperature proton capture proved to be dominant way of destruction mechanism. So, this kind of concept is important to understand the evolution of aluminium 25. Here I have taken one simple example of aluminium 25 I hope it is clear to you how to determine which process is dominant by taking the help of mean lifetime of a nucleus via decay and via particle induced reactions.

So, with this knowledge we are in a position to write down expression for the evolution of aluminium 25 nucleus considering all possible processes. So, for that let me write down the time dependent evolution of aluminium 25.



^{25}Al Can be produced $^{24}Mg(p,\gamma)^{25}Al$

$^{22}Mg(\alpha,\gamma)^{25}Al$

$^{25}Si(\beta^+\nu)^{25}Al$

$^{26}Si(\gamma,p)^{25}Al$

can be destroyed

$^{25}Al(p,\gamma)^{26}Si$

$^{25}Al(\alpha,p)^{28}Si$

$^{25}Al(\beta^+\nu)^{25}Mg$

$^{25}Al(\gamma,p)^{24}Mg$

Time evolution of ^{25}Al abundance

$$\begin{aligned}\frac{d(N_{^{25}Al})}{dt} = {} & N_p N_{^{24}Mg} \langle\sigma v\rangle_{^{24}Mg(p,\gamma)^{25}Al} + N_\alpha N_{^{22}Mg} \langle\sigma v\rangle_{^{22}Mg(\alpha,p)^{25}Al} \cdots\cdots \\ & + N_{^{25}Si} \lambda_{^{25}Si(\beta^+\nu)^{25}Al} + N_{^{26}Si} \lambda_{^{26}Si(\gamma,p)} + \cdots\cdots \\ & - N_p N_{^{25}Al} \langle\sigma v\rangle_{^{25}Al(p,\gamma)^{26}Si} - N_\alpha N_{^{25}Al} \langle\sigma v\rangle_{^{25}Al(\alpha,p)^{28}Si} \\ & - N_{^{25}Al} \lambda_{^{25}Al(\beta^+\nu)^{25}Mg} - N_{^{25}Al} \lambda_{^{25}Al(\gamma,p)} - \cdots\cdots\end{aligned}$$

For that let me tell you same example aluminium 25 can be produced by reaction between Mg and proton which gives rise to 25 aluminium. Not only that, alpha also can undergo with a nucleus that is magnesium which again produces 25 Al. Not only that there is a nucleus 25 silicon which undergoes positron decay giving rise to neutrino that also can produce aluminium 25. Not only that if enough energy is available gamma can react with 26 silicon producing proton and 25 aluminium.

So, these are all ways of production and can be destroyed by reverse processes. For example, 25 aluminium can undergo destruction by reacting with proton giving rise to 26 silicon. 25 aluminium can also react with available alpha particle and producing proton and also 28 silicon. 25 aluminium being radioactive it can undergo positron decay producing 25 Mg. This is not our concern. The main concern is this 25 aluminium how it is undergoing destruction.

25 aluminium it can also undergo reaction photo disintegration process with gamma giving rise to 24 Mg. So, you can see the equivalence between these processes and these processes in inverse and forward direction. So, the time evolution of 25 aluminium abundance is described by the expression number density of aluminium 25 divided by dt d by dt of number density following 25. This is called as time evolution of abundance of Aluminium 25.

Now let me write down number density of protons and a number density of magnesium 24 sigma V. This reaction is between 24 Mg p gamma 25 Al. This is one way of producing the aluminium 25. Second way is this one, alpha + 22 mg. First one we have written and second one number density of alpha and the number density of magnesium and a reaction between both of them that is 22 Mg alpha p producing 25 aluminium.

What is the third way of producing 25 aluminium, silicon 25 can undergo positron decay. So, what is the decay constant? This is the one. beta + nu which is giving rise to 25 aluminium . Not only that, it can also undergo destruction by reacting with gamma. So, what is the disintegration constant 26 silicon gamma p. So, this other than these 4 processes if there are more process I am using the symbol dot dot.

Now we are talking about time dependent abundance evolution of aluminium 25. So, this is not complete expression I have written the possible ways for producing aluminium 25. But the moment they are produced they are also undergoing destruction via these 4 processes which I have written earlier. So, those things also should be included in the expression of abundance evaluation of aluminium 25.

So, how to write down? Like this - N p N 25 aluminium sigma V 25 aluminium by reacting with p gamma it is giving 26 silicon. Second process is aluminium 25 can undergo destruction by reacting with alpha particle. So, sigma V25 aluminium alpha p - at the same time this aluminum 25 nucleus can undergo decay also and the decay constant depends on this particular value. Beta + nu giving rise to 25 mg and here of course 28 silicon.

Now the last process which we are considering at this place is 25 aluminium, lambda 25, aluminium, reacting with gamma proton minus these things. So, I strongly suggest you to write down a general expression for time dependent abundance evolution of a general nucleus considering 3 processes. Number 1 particle induced reactions number 2 decay process number 3 photo disintegration.

And beyond this if any reaction comes to your mind please include that. Like this for every nucleus one has to write down the time dependent abundance evolution. For all the nuclei whose information is available if you write down then that is called as reaction network and depending on the situation one can solve this reaction network analytically or numerically. So, to summarize today's lecture I have discussed mean lifetime of a particular nucleus.

How it can be related to mass fraction and stellar density of another type of nucleus with which it is interacting and undergoing destruction because of which we are discussing the concept of lifetime. And then I have discussed 1 numerical 1 example where by having the data of stellar density, mass fraction, reaction rate one can find out the dominant way of destruction of a particular nucleus.

Finally time dependent abundance evolution equation I have written by taking the example of aluminium 25. I hope these things are clear to you and please do not forget to write down a general expression which is a homework for you a general expression for time dependent abundance evaluation of a specific nucleus considering all general processes like particle induced decay and photodisintegration.

In the next lecture I will start charged particle induced reactions and neutron induced reactions in detail. Thank you so much for your attention.