Nuclear Astrophysics

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Lecture – 14

Inverse reactions… and Mean life time of a nuclei

Welcome students to today's lecture in which I am going to put a full stop to the topic inverse reactions. Last lecture was fully on inverse reaction only and as I have mentioned 2 more topics in the previous lecture, energy production and mean lifetime. Those topics will be covered in today's lecture. What are the important takeaways from the discussion on inverse reactions.

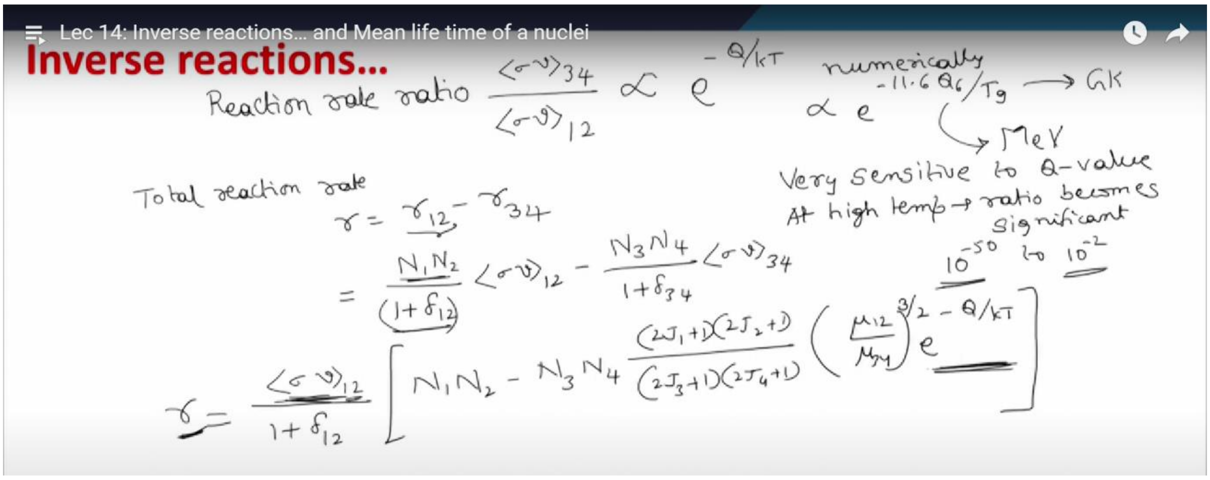
Number one, negative Q value reactions plays important role at high temperatures because at the high temperature particles will gain enough energy to induce reaction in reverse order. So, when reaction proceeds through reverse order whether there will be any change in the cross-section value? Yes, there will be change in the cross section value but the loss governing the nuclear reactions do not change with the direction of the reaction.

So, the principle of time reversal invariance has been discussed in detail in the previous lecture. And you need to come up with a general representation of the cross section of a nuclear reaction. It was the product of 4 terms. Number 1 pi lambda square where lambda is the reduced de Broglie wavelength. Number 2 statistical factor represented by omega. Number 3 term taking into account the Kronecker symbol. Number 4 the matrix elements product.

Because that is the term which considers the interaction mechanisms of the nuclear reaction. Entrance channel to the coulomb compound nucleus exact state can happen through one type of interaction. I mean it could be through weak strong or electromagnetic and compound state to exit channel. Which is giving rise to nuclei of types 3 and 4. It can happen through different mechanisms. They need not be the same but we have seen the ratio of the cross sections does not have any presence of the compound nucleus.

So, that shows clearly that by measuring cross section in one order, one direction, one can measure the cross section in reverse direction by taking the values of reduced mass and spins of the particles. Then I have written the ratio of the reaction rates in terms of Q value and temperature. So, in today's lecture I am going to discuss little bit about inverse reactions. Some main features are left. Then I will discuss mean lifetime of a nuclear reaction. The energy production in stars in terms of reaction rate.

In today's lecture I will compare the inverse reactions by taking into account an example for photo disintegration. In the previous class I have taken 1 + 2, 3 + 4 and 3 + 4 giving rise to 1 + 2. And if gamma is involved how the cross-section ratio and reaction rates ratio will look like. That I will discuss in today's lecture. And then the energy production in stars and the lifetime of the nuclear reaction.



So, the reaction rate ratio when you consider forward and reverse reaction is given as sigma V 34 and sigma V 12. Rate of inverse nuclear reaction divided by rate of forward nuclear reaction is proportional to e to the power of -Q by k T. The terms preceding this exponential term their order is of the unity. Now numerically if I want to express this, it is proportional to e to the power of -11.6 Q 6 divided by T 9.

Where Q 6 is in MeV and T_9 in Giga Kelvins. T_9 is 10 to the power of 9, 1 T_9 is 1 into the power of 9, 2T_9 is 2 into power of 9. So, numerically having this kind of values helps us to come up with some numbers regarding the ratios. Please do calculation by taking different values of Q and by taking different values of T. You will observe that the reaction rate ratio is very sensitive to Q value.

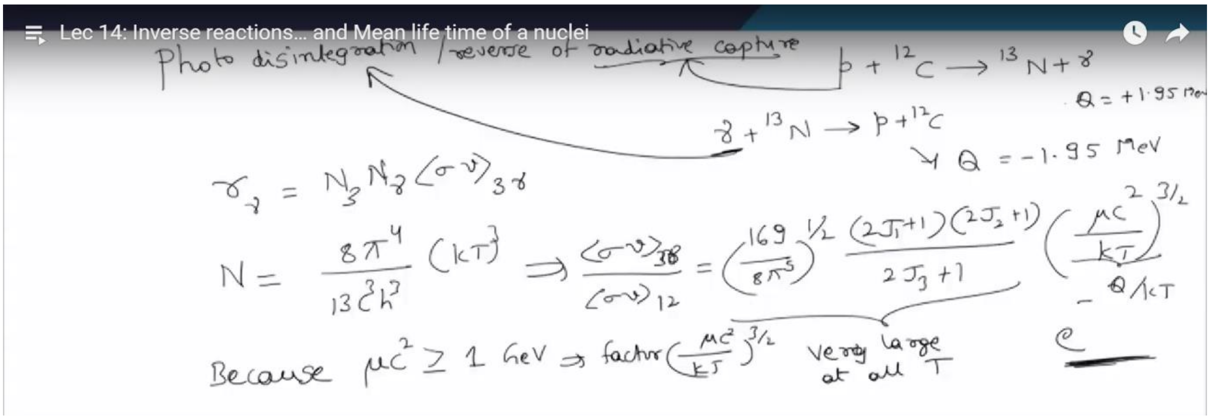
And at high temperatures the inverse reaction that is ratio becomes significant. If I start from 0.1 Giga Kelvin to 10 Mega Kelvin the ratio ranges from 10 to the power of -50 to 10 to the power of -2. So, this signifies the role of inverse reaction at high temperatures. Now we have seen the rate of the reactions in forward and reverse reaction? What about the total reaction rate? I repeat what is the total reaction rate in the star at a particular temperature.

So, the total reaction rate can be expressed as the difference between forward and inverse reaction. Total reaction rate r is equal to r12 - r 34. So, if I expand this, this gives me the number density N_1 N_2 divided by Kronecker symbol delta 12. This is the general expression for the reaction rate. Total reaction rate is number density divided by Kronecker symbol term and reaction rate –N_3, number density of nuclei of type 3 and nuclei of top type 4 1 + delta 34 sigma V 34.

Now using the expression for reaction rate ratio I can write down this total reaction rate as sigma V12 divided by 1 + delta12, N_1N_2 -N_3N_4, $2J_1$ + 1, $2J_2$ + 1 divided by $2J_3$ + 1, $2J_4$ + 1, mu_{12}, reduced mass mu_{34} to the power of 3 by 2, e to the power of –Q by k T. So, again in this expression you can see that the total reaction rate is depending on the e to the power of -Q by k T. Reaction rate of 1 and 2 entrance channel. So, this is how one can express total reaction rate in a star by considering the both forward and inverse reactions.

So, with this let me stop the discussion of inverse reactions. Considering particles as not photons what if one of the species is photons.

I have already discussed what is photo disintegration and what is radiative capture reaction. Photo disintegration is nothing but the reverse of radiative capture. Let me give one example. If you take the proton reacting with carbon-12 it gives rise to 13 nitrogen + gamma whose Q value is + 1.9 MeV. Now if I go for the reaction reversal order. If gamma is available with sufficient energy it can react with nitrogen-13 produced in previous reactions giving rise to proton + 12C and for this the Q value is -1.95 M eV.



So, here a heavier nucleus is disintegrating into splitting into lighter nuclei this is called as photo disintegration. Capture of proton by carbon 12 is giving rise to gamma emission. So, that is called as radiative capture reaction. Now as I have discussed in the previous lecture at high temperatures the photons have enough energy of the order of MeV and they can induce photo disintegration reactions. The rate of nuclear reactions can be given as induced by gammas N_3 N gamma sigma V 3 gamma 3 gamma.

So, if the total reaction rate is like this for the gamma reactions. Then the N gamma is represented according to Planck's radiation law. 8 pi to the power 4, 13 c cube h cube k T cube and this makes the ratio of reaction rates 3 gamma sigma V 12 ratio of reaction rates forward and reverse if gammas are involved. So, this gives us an expression like this 8pi to the power of 5 1 by 2.

So, sometimes you need to handle this kind of mathematical expressions. It is very easy to derive all these things. Please do not leave the derivation part. Here interestingly not only exponential term but you also have rest mass energy and also exponential term. So, this first 2 terms you can say it is of an order unity and the ratio is dominated by exponential term but multiplied by the ratio of rest mass energy to the thermal energy kT.

Now because the product reduced mass multiplied with mu c square is greater than or equal to 1 GeV the factor mu c square divided by k T to the power of 3 by 2 is very large at all temperatures. I am shifting the focus from charged particles which are part of entrance channel and exit channel in general but when gammas are in involved in the exit channel in particular. And when such cases lead to the inverse reaction that is photodisintegration then also we should be in a position to write down the ratio of the reaction rates.

Why this expression is important because majority of the nuclear reactions are happening as radiative capture reactions in the star. Majority of the reactions happening within the star for synthesis of elements it is radiative capture. Though they are slow they play very important role in the synthesis of new elements. So, that is the reason it is important for us to write down expression for the ratio of the reaction rates when gamma rays are involved.

In the expression of reaction rates we have seen that not only exponential term but mu c square by k T rest of energy by thermal energy this is also there. Which is very large at all temperatures. But we have another term that is e to power of -Q by k T that means at elevated temperatures photo disintegration reactions plays a very important role.

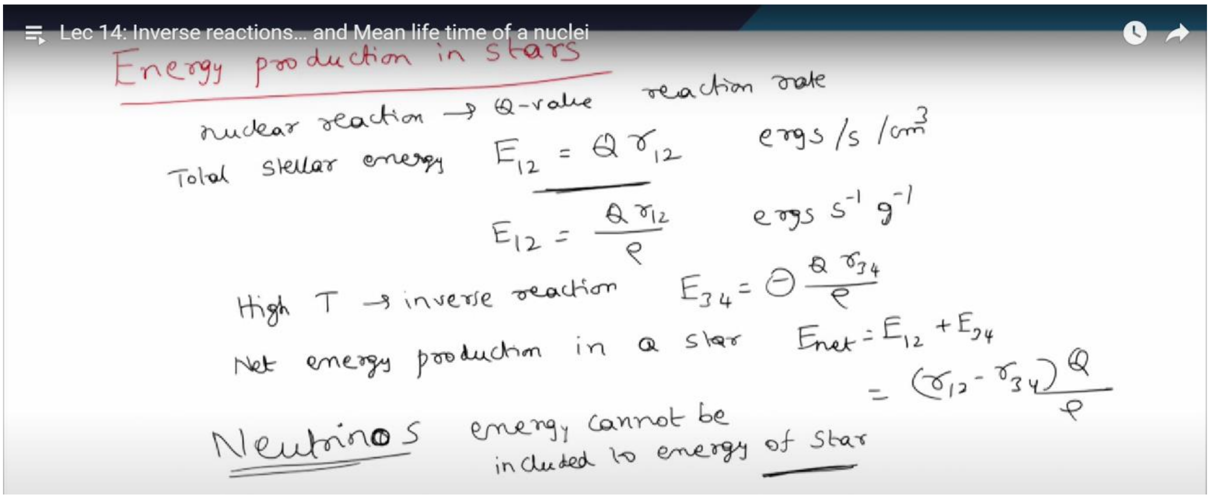
See now we are actually discussing the nuclear physics part of the astronomy or features of the astrophysics. At elevated temperatures it is the photo disintegration reaction but there also sometimes forward reaction can take place. With what rate these two reactions proceed determines the stellar revolution. The evolution of a star have been discussed in HR diagram.

A star on main sequence can go to white dwarf can go to red giant super red gaint can come back to white dwarf like sun. So, this is some qualitative statement but quantitatively to get an idea about which reaction is dominating, it is important to consider the ratio of inverse reaction and forward reaction rates. When gammas are involved, this expression tells us that at elevated temperatures photo disintegration reactions are playing very important role.

So, I hope it is clear to you how to write down the expressions for reaction rates when gamma rays are involved and when gamma rays are not involved. So, after this let me touch the topic energy production in stars. How to write down the expression for energy production stars? If you know the reaction rate that means number of reactions taking place per unit time per unit volume and Q value of the each reaction. May be we can write down simple expression for the energy production in stars. Let us see how it looks like.

Let me write down energy production in stars. One of the important objectives of this course is to understand the energy produced from the stars and energy liberated in nuclear reaction. Energy produced in a nuclear reaction is always characterized by Q value. Now when we mix with the reaction rate, the total stellar energy can be given as E 12 is equal to Q value total reaction rate when 1 and 2 are involved and units are ergs per second per centimeter cube.

Now sometimes this equation commonly is written so that to involve the stellar matter density. So, in that case E_{12} is written as Q r_{12} divided by rho in ergs per second per gram and at high stellar temperatures as I said inverse reactions at high temperatures inverse reactions plays important role because negative Q value reactions also can proceed because particles are having sufficient energy to induce the reaction.



So, in this case E_{34} can be written as -Q r_{34} divided by rho. So, this is how one can have a simplistic expression for energy produced in stars.

Who are responsible for the energy production nuclear reactions what kind of nuclear reactions forward and inverse at high temperatures. And when star evolves from low temperature to the high temperature region different types of nuclear reactions taking place. Now if we consider one particular nuclear reaction in forward and inverse then by knowing the Q value one can have an idea about the net energy production in the star because of one particular nuclear reaction.

I hope you are getting what I am trying to convey in star various types of nuclear reactions taking place and starting from one particular type of nuclear reaction in forward and in reverse direction taking the reaction rate difference r 12 - r 34 and multiplied with the Q and divide with the matter density one can get an idea about the net energy production in the stars. I hope you understood the basic concept of energy production the stars.

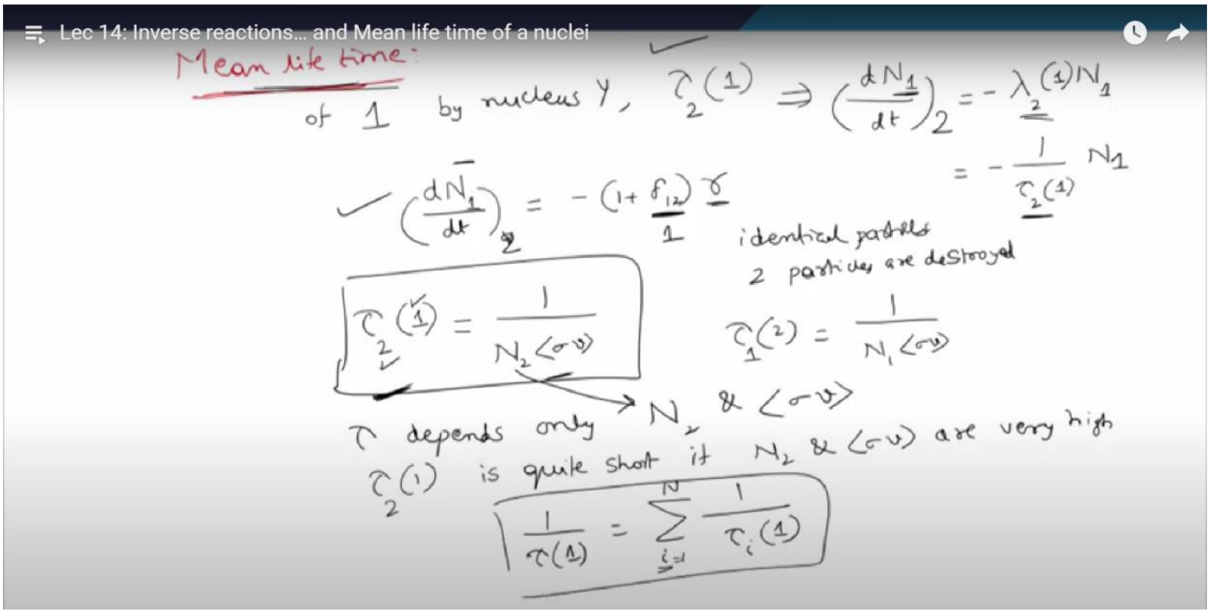
Now one interesting feature of this energy production in nuclear reaction whatever nuclei are produced whatever particles are produced they are absorbed within the star. However there is one exception that is neutrinos. Why it is important when I am discussing this energy production? It will be clear to you soon. Hope you remember when neutrino is emitted in beta minus decay beta plus decay you have seen neutrinos are emitted and many radioactive nuclei within the star they undergo decay.

So, you can have neutrinos but this neutrinos because they travel with velocity of light and they are weakly interacting with any type of nuclei, they can just come out of the star as if nothing is there. That is the reason neutrinos can easily be detected on earth of course you can correlate what I am saying when I presented various facts as evidence of nucleaosynthesis the detection of neutrinos on earth and as I said billions of neutrinos are passing through our body every time.

Where from these neutrinos are coming the nuclear reactions are happening within the stars and neutrinos nobody is there to stop within the star they are coming out. So, the conclusion is that the energy taken out by the neutrinos cannot be included in the energy produced by a star because that energy is not absorbed within the star right. So, neutrinos energy cannot be included to energy of the star because they cannot be absorbed if they cannot be absorbed within the star how can we include in the energy produced by star.

Now you can say in beta plus decay positron is emitted. Well this positron immediately will find electron in the surroundings and it undergoes pair annihilation and two 511 keV gamma rays emits in opposite directions. Again you have the gamma rays and gamma rays can be easily absorbed within the star when compared to neutrinos. So, the conclusion to this topic is following when you are trying to calculate the energy produced from a star do not include the neutrinos energy and we will take this very clearly and without failure when I discuss the energy produced in different nuclear processes within the star.

So, there I will consider total energy produced from the star is this and minus neutrinos energy is the net energy produced by the star. So, after discussing the inverse reactions and this energy production the star let me discuss the last topic in this today's lecture that is mean lifetime of a nucleus. Nucleus once it is produced within the star it will die after certain time by reacting with another nuclei. Can we come up with some mathematical analysis for this mean lifetime. Let us see.



So, let me write down mean life time of a nuclear reaction. So, in the stellar evolution in terms of time an important quantity is this mean lifetime. The mean lifetime of say nucleus one against destruction by nucleus 2. Why I am taking 1 and 2 because I have been considering this 1 + 2 in forward and reverse reaction. So, for the sake of convenience let me represent lifetime of the nucleus 1 nuclei of type 1 when it interacts with the nuclei of type 2 this nuclei one is gone. That is how you develop the mathematical relation for lifetime of the nuclei of type one.

So, lifetime of nuclei one by nuclei y that is tau by interacting with the second nucleus number one nucleus is undergoing destruction. So, this can be represented statistically as dN 1 divided by with respect to time with what rate it is undergoing destruction by interacting with 2 is equal to the decay constant of 1 against 2 and number density of 1. This is a well known relation. You are aware of the decay constant is always inversely proportional to the lifetime of nucleus one against destruction with 2. So, this is N1.

So, we can interpret as the change in the abundance of N1 of nuclei of type one as the result of bombardment of particles with particles of type 2 and decay constant as I said it is inversely proportional to the mean lifetime. Alternatively I can write down this dN1 by dt when it is interacting with 2 second type 2 - 1 + delta 12 reaction rate r. So, the change in abundance of N1 is related to the total reaction rate r by this relationship why we need to take this delta 12 because for identical particles, 2 particles are destroyed.

And this has value one for identical and zero for non-identical nature. Now combining this equation and this equation and the equation that we know for the reaction rate the lifetime is expressed as tau 2. I mean lifetime of one against 2 is 1 by N 2 sigma V. So, the mean lifetime of nuclei one against destruction by nuclei 2 can be represented like this and we, can also analogously can write down tau of 2 by against destruction by nucleus one can be written as 1 by N 1 sigma V.

Remember that identical particles have cancelled out when we write down the equation this is the final one. Now this tau depends only on N2 you see more density of destructing nuclei I mean second part type of nuclei more less lifetime of the first nuclei first type nuclei. So, N 2 and also of course the reaction rate and this tau of one against a destruction by nuclei 2 is quite short if N 2 and sigma V are very high.

And for more than one type of destructive nuclei for example this nuclei of type 1 can undergo destruction not only by interacting with the particle type 2 but also 3, 4, 5 it can undergo destruction by reacting with any type of nucleus right taking into account the overall lifetime of nuclei one can be written like following. So, it is 1 by tau of 1 is equal to sigma of 1 by tau i by x 1. So, I value can be I is equal to 1 to N.

So, to summarize today's lecture what I have discussed in the inverse reactions topic I have expressed the total reaction rate as a difference between forward and inverse reaction rate and then I have considered a special case of photo disintegration reaction all right. Because at elevated temperatures this photo disintegration reactions plays an important role and the reaction rate ratio depends not only on the exponential term but also a very large value at all temperatures that is ratio of rest mass and the; thermal energy rest mass energy and thermal energy.

Then I discussed energy production in stars and finally I have discussed the lifetime of nucleus by taking example of 1 and 2 where lifetime of 1 is represented against destruction by type 2 nucleus. In the next lecture I am going to provide some mathematical expressions for abundance of nuclei abundance of nuclei thank you. So, much for your attention looking forward to meet you in the next lecture, thank you.