Nuclear Astrophysics

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Lecture – 12

Gamma Induced Reactions and Inverse Reactions

Welcome students to this lecture today. In today's lecture I am going to cover some of the aspects of photon induced reactions and inverse reactions. So, before that, let me briefly summarize what I have discussed in the previous lecture. You know every time it is important to remember the purpose of this course, what exactly we are expecting, what we are trying to learn in this course.

And as I said one of the important goals is to understand the energy produced from the stars and the synthesis of elements in the universe. It is essential to know the properties of the nuclear reactions which are responsible for the energy production and also nucleosynthesis. And when we say nuclear reactions are happening within the stars which are responsible for the energy production and the nucleosynthesis.

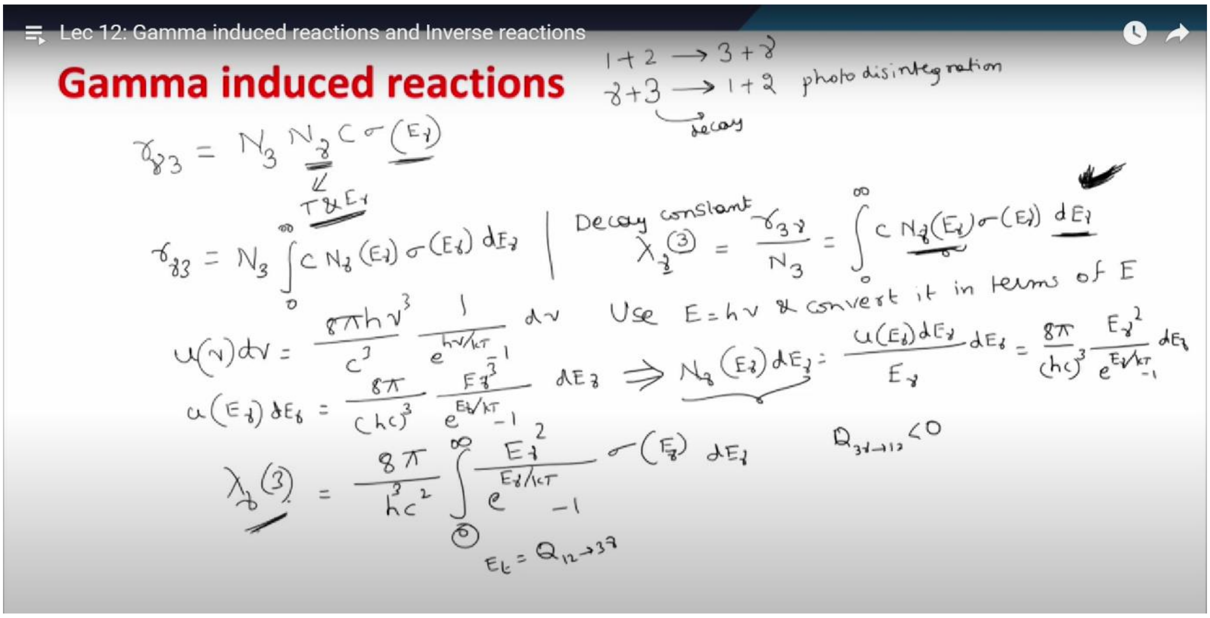
What are the entities in this entrance channel of the nuclear reaction if nucleus 1 and nucleus 2 are considered. So, reaction is happening within certain temperature range and reaction products are formed. In order to understand the synthesis of various elements we should also keep it in mind that how many types of reaction can take place. Majority of the reactions are induced by charged particles, neutrons and gamma rays.

Reactions induced by charged particles, neutrons and gamma rays are responsible for the production of majority of the elements. It is important to know how to calculate the rate of the reactions induced by these particles like protons, neutrons, alpha rays and gamma rays. I have discussed initially the charged particles, whose velocities follow the Maxwell Boltzmann distribution.

After that I discussed neutron induced reaction. The slight dependence of the cross section on the velocity has been included in the mathematical expression. In today's lecture let us see how to write down expression for reaction rate if it is induced by the gamma rays. So, these are the contents in today's lecture I have already discussed the recap and photon induced reactions. So, mainly when I say photons I am referring to gamma-rays.

Mass fractions and inverse reactions are the topics I am going to discuss in today's lecture. If time permits, I will go for the inverse reactions.

Radiative capture reaction is denoted by 1 + 2 giving rise to 3 + gamma. If a gamma interacts with the nucleus 3, 1 + 2 are formed this is called as photo disintegration. This means after absorbing gamma, nucleus 3 is undergoing decay.



The reaction rate of a nuclear reaction is the product of the number densities of the projectile and target and then the reaction rate per particle pair gives you the total rate of reaction.

$$r_{\gamma 3} = N_3 N_\gamma c\sigma(E_\gamma)$$

How do you represent the energy dependence of the gamma-ray. Whether it follows the Maxwell distribution? No. Then what kind of distribution gamma rays will follow. So, keeping that in mind I am going to discuss the reaction rate for gamma induced reactions. So, rate of nuclear reaction induced by gamma and 3.

$$r_{\gamma 3} = N_3 \int_0^\infty c\, N_\gamma(E_\gamma)\sigma(E_\gamma) dE_\gamma$$

The number density of nucleus 3 and the number density of gamma ray. It should be multiplied with velocity of the particle and in this case what is the velocity of gamma ray velocity of light. So, I am replacing v with c and sigma which is a function of energy of the gamma. So, this is the general and fundamental expression for the reaction rate when 2 entities are involved as part of the nuclear reaction.

Now because it is gamma ray one of the species is gamma ray the velocity has to be represented as c. Now the cross section of course depends on the gamma energy. In stellar plasma at equilibrium the number density of gamma is not constant. It depends on the temperature and also on the E gamma at equilibrium in stellar plasma. So, how to rewrite this expression of reaction rate when gamma reacts with nucleus 3 of course N_3 remains same.

Now this N gamma because of its dependence on temperature and energy of the gamma it has to be taken inside the integral. So, velocity C number density of gamma which is a function of E gamma and sigma also as a function of E gamma and dE gamma. Now remember here the nucleus 3 is absorbing gamma it is undergoing excitation right it is getting exited during de-excitation it is giving different products.

When there is a decay process the parameter to quantify is decay constant. So, how do you write the expression for decay constant keeping in mind the reaction rate. So, decay constant when nucleus 3 absorbs gamma ray can be written as lambda absorbing gamma by nucleus 3.

$$\lambda_\gamma(3) = \frac{r_{3\gamma}}{N_3} = \int_0^\infty cN_\gamma(E_\gamma)\sigma(E_\gamma)dE_\gamma$$

Rate of nuclear reaction 3 gamma divided by N3 number density is decay constant.

Remaining term looks same like zero to infinity velocity c number of gamma which is the function of gamma and sigma of E gamma dE gamma. Now the goal is how to write down the expression for N gamma or the term dE gamma. How the distribution looks like. The number density of the gamma ray depends on the energy of the gamma ray but at a particular temperature how the distribution of number of gamma rays looks like.

Planck's law. The number of electromagnetic waves having frequency between nu and nu + d nu is given by energy density d nu.

$$u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT} - 1}d\nu$$

Now use E is equal to h nu and then convert this equation in terms of E.

$$u(E_\gamma)dE_\gamma = \frac{8\pi}{(hc)^3}\frac{E_\gamma^3}{e^{E_\gamma/kT} - 1}dE_\gamma$$

From this expression I want the I want to expand by taking the n gamma e gamma and dg gamma this is nothing but energy density already have written expression energy density divided by energy right multiplied with d gamma when you divide energy density with energy you will get the number density right. So, that is how it looks like eight pi h c cube E gamma square e to the power of E gamma by k T - 1 dE gamma.

$$N_\gamma(E_\gamma)dE_\gamma = \frac{u(E_\gamma)dE_\gamma}{E_\gamma} = \frac{8\pi}{(hc)^3}\frac{E_\gamma^2}{e^{E_\gamma/kT} - 1}dE_\gamma$$

So, now you have this expression substitute in the decay constant formula. So, that you can write as lambda gamma 3 is equal to 8 pi h cube c square you can simplify yourself it is not a big deal 0 to infinity E gamma square divided by E gamma by k T - 1 remaining part is cross section at a particular energy of the gamma and dE gamma.

$$\lambda_\gamma(3) = \frac{8\pi}{h^3c^2}\int_0^\infty \frac{E_\gamma^2}{e^{E_\gamma/kT} - 1}dE_\gamma\ \sigma(E_\gamma)dE_\gamma$$

So, basically in this slide I tried to cover the discussion on reaction rate of gamma induced reactions. The major difference compared to the neutron induced reactions and charged particle induced reactions which follow the Maxwell distribution is that here the number density of the gamma rays is expressed with the help of Planck's radiation law.

So, once you have the idea on the rate of reactions induced by charged particles like protons electrons or alpha particles or some sometimes heavy ions. All these particles comes under charged particles or heavy ions and then neutrons which have mass but no charge. Then gamma which have no mass and no charge. So, you should be in a position to mathematically express the rate of the reactions when they are induced by these 3 types of particles.

Now let me cover 2, basic points connected to the same topic and continuing the cross sections and reaction rate. Initially we have considered general reaction like 1 + 2 is giving rise to 3 + 4. Remember the identical nature and non-identical nature. So, this is the reaction rate and in order to account for the identicalness we have used a Kronecker symbol. It is now in this sigma V this is basically reaction rate per pair and this is the number of pairs of non identical particles.

$$Reaction\ rate = \frac{N_1 N_2 \langle \sigma \vartheta \rangle}{(1 + \delta_{12})}$$

Now as I said the role of experimental and theoretical researchers is to calculate this reaction rate. So, that value can be used as an input to find out the abundance of the element or energy produced within some certain types of stars. However experimentally there are many challenges. Because of various conditions of temperature within the star direct measurements are not possible in all ways.

Reactions are happening at low energies and it is not possible to have it in our laboratory all the time. So, what people do is to make the measurements at high energies and then extrapolate the cross section to the energies of interest. Of course, this extrapolation induces some kind of uncertainty. But this is one of the challenges in nuclear astrophysics. Extrapolation of the cross section and how to circumvent this problem by taking some other quantity that I will discuss in due course.

When you try to find out the reaction rate it is important to have the value of cross section. And cross section cannot be calculated in all cases. In stellar plasma instead of number density researchers go for matter density, grams per centimetre cube. How they are related to each other? Number density is related to the matter density.

$$N_i = \frac{\rho N_A X_i}{M_i} = \rho N_A Y_i$$

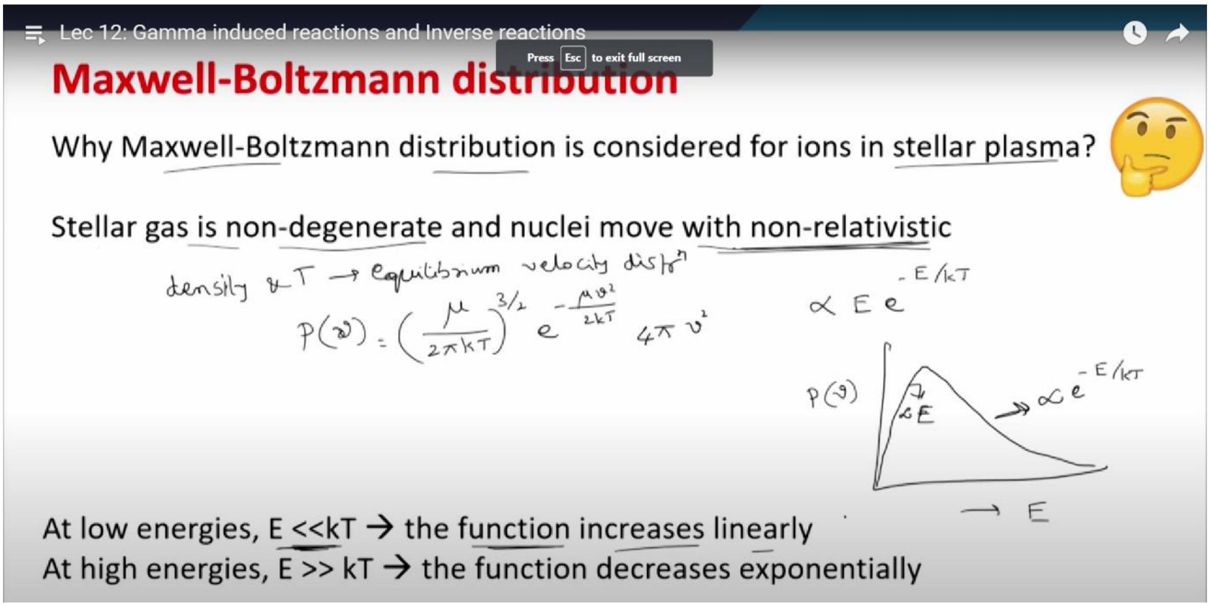
You can cross check the dimensions on both sides. X_i is the mass fraction and this is the mass in amu for example mass of proton if i stands for proton nothing but 1.007825 amu or u and mass of neutron is 1.008665 u. This can also be written as matter density into Avogadro number this mass fraction divided by the mass can be denoted as mole fraction Y_i.

So, in the calculations sometimes you may come across this word mass fraction there you should be in a position to relate this mass fraction to the number density via matter density Avogadro number and the mass of the nucleus.

So, after this let me discuss one more thing, Maxwell Boltzmann distribution. When we do not consider neutrons or gamma rays. Consider proton induced reactions, protons are fermions. And ideally they should follow Fermi Dirac distribution. Why Maxwell Boltzmann distribution is considered?

I repeat the question. When protons that is fermion are considered, the distribution of their velocities and energies should follow the Fermi Dirac distribution but why we are considering Maxwell Boltzmann. Please think about it. This is very interesting. Here I am giving a couple of hints. I am sure you will build up based on them and by reading text books like Rolfs Rodney and Illiadis or some other text books available on the internet.

So, why Maxwell Boltzmann distribution is considered for ions and stellar plasma. So, it is basically non-degeneracy and the motion of nuclei. They are not relativistic, it is non relativistic case. So, let me give you some important features of this Maxwell Boltzmann distribution. See physical conditions in interior of the star that is density and temperature density and temperature they lead to equilibrium velocity distribution which is basically Maxwellian.



P velocity is equal to mu by 2 pi k T to the power of 3 by 2 e to the power of - mu V square divided by 2 k T 4 pi V square. Now this distribution is basically proportional to e into e to the power of - e by k T right at low energies what will happen to this probability. It can be drawn like this, p as a function of velocity when you plot with respect to temperature is well known, it increases up to certain value and then it decreases asymptotically.

So, at low energy that is when energy is less than k T the probability function increases linearly. So, this is proportional to basically E and at high energies when E is much greater than k T the function decreases exponentially. So, this is basically proportional to e to the power of - E by k T decreasing exponentially and asymptotically.

Here I am posing one question to you. When nucleus 1 and nucleus 2 they are undergoing nuclear reaction this nucleus 1 velocity distribution is Maxwellian and nucleus 2 velocities are also Maxwell distribution. And when we take the velocity distribution of both nuclei don't you think the reaction rate should be written as the product of both distributions and we should have double integral.

I repeat the question. When nucleus 1 nucleus 2 in large number they undergo nuclear reaction and the velocity distribution is Maxwellian. Because we are ignoring the neutrons and gamma rays, don't you think we have to consider the double integral to account for the both velocity distributions. Are we doing some mistake, please think about it. So, how this double integration can lead to single integration.

Now after this Maxwell Boltzmann distribution this is the time to discuss one of the interesting aspects of the nuclear astrophysics that is inverse reactions. When 1 + 2 can give rise to 3 + gamma. For example 3 + gamma can also give rise to 1 + 2 there is nothing that stops us. It depends on the Q value. When Q values are positive and energy for gamma is not sufficient then inverse reaction may not take place.

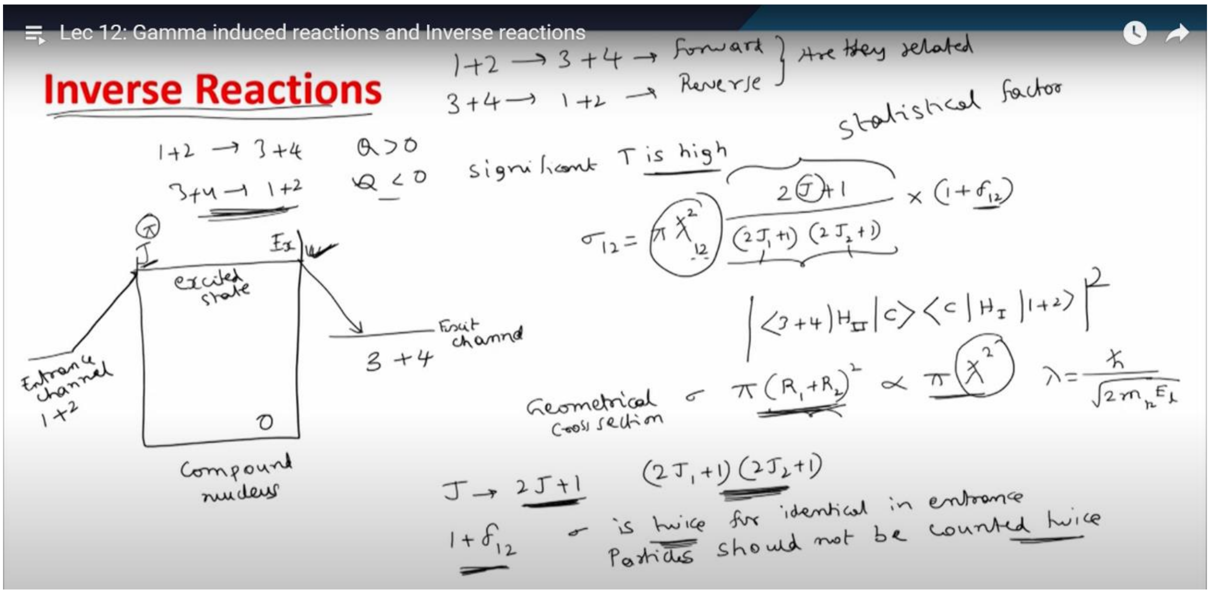
But at elevated temperature when gamma possess high energy there is a probability that gamma also can induce reaction that is what I have discussed right reaction rate representation for gamma induced reactions. Now the question which I am posing now is following if we consider a forward reaction like 1 + 2 giving rise to 3 + 4 or gamma or 3 + 4 giving rush to 1 + 2 if I consider this as a forward reaction and if I consider this as reverse reaction or inverse reaction are they related.

By measuring the cross section for forward reaction can we get the information about the cross section of reverse reaction. If yes how to relate them mathematically. Why it is so important. Many times, it is not possible to measure the forward reaction. But practically if it is not possible and if I will check whether I can perform inverse reaction if it is possible I will go for it. I will measure the cross section of the reverse reaction then using some mathematics I will calculate the cross section of the forward reaction. So, in this topic I am going to cover some of the important features of the inverse reactions. So, already I have written the inverse reaction I have given the inverse reaction background.

And now I am ignoring the gamma thing. I am considering the general case of charged particles which are major in majority within the stars. So, this 1 + 2 giving raise to 3 + 4 when Q is greater than 0 and if Q value is negative 3 + 4 is giving rise to 1 + 2 Q is less than 0 this will become significant when temperature is high. So, in that case also it is important to measure the inverse reaction. How we can draw this in terms of compound nucleus.

Entrance channel this is entrance channel the word entrance channel denotes that nucleus one nucleus 2 undergoing reaction and a compound nucleus is formed a compound nucleus is formed in the excited state when compound nucleus is formed and no compound nucleus is formed generally in ground state if they are formed in excited state and angular momentum spin parity of course from shell model you can designate these values.

And I also will discuss this angular momentum and parity concept in next or next to next lecture and the energy of this level is A x, E x and this is basically excited state. Now this compound nucleus in excited state will not stay for long right it will undergo some kind of decay. So, this is called as exit channel that is 3 + four and this is the zero. Now this is the visualization of a nuclear reaction fine what is the cross section of this nuclear reaction? It is the sum of four terms.



Each and every term I will explain in detail let me write down the complete equation reduced wavelength square 1 2 because 1 and 2 are initiating the nuclear reaction 2 j + 1 where j is capital J is the angular momentum of the excited state of the compound nucleus and spin of the nucleus 1 is j 1 and spin of the nucleus 2 is j 2. I will explain the meaning of this term do not worry and this is multiplied with the identical nature that is I mean if we want to account for the identical nature identical nature we have to use this Kronecker symbol.

And then fourth term is some kind of matrix element 3 + 4 H 2 compound nucleus and compound nucleus H 1 1 + 2 square this double indices 1 and 2 they refer to the particles involved in the entrance channel. So, here I am trying to explain how to correlate the cross section of forward reaction to inverse reaction or cross section of inverse reaction to forward reaction that is the topic of the lecture.

For that I am approaching a way where I am visualizing the nuclear reaction in such a way that 1 + 2 is giving rise to a compound nucleus. This compound nucleus is in excited state and it is decaying to 3 + 4 in this process because it is a quantum mechanical process right on how many terms this cross section depends conceptually. So, this is a very interesting and very important thing you have to understand the meaning of each and every term.

Let me start from the first term in this equation. So, what is this pi lambda square? See when you imagine the cross section you know the geometrical cross section I think already have discussed this but let me quickly do a review geometrical cross section is nothing but pi R square right R is basically radius of the projectile or radius of the target you can use any symbols of your wish this is a geometrical cross section.

But here the reaction whose cross section we are trying to understand is not a classical reaction it is a quantum mechanical case. So, this has to be replaced with a quantum mechanical related term reduced wavelength where lambda is equal to well-known formula reduced Planck's constant square root of 2m that is a reduced mass you can take ok m 12 and then energy of the projectile in the lab.

You are very much aware of geometrical cross section the area projected by the target to the projectile classical case it is simply pi r square where r is the sum of the radius of projectile and target however because it is a quantum mechanical process this radius term has to be replaced with another term. So, that we can take into account the effects of quantum mechanics and that term is reduced wavelength lambda prime and lambda in terms of reduced Planck's constant I have written.

So, this is a number one term if pi lambda square is more cross section is more pi lambda square is more means in lambda you have h by square root of 2 me. So, it depends on the energy and of course reaction mechanism also for that we have to include some more parameters some more parameters.

And number 2 parameter that is a 2 j + 1 ok this is a statistical factor this is a statistical factor where capital j represents the angular momentum of the exited state in the compound nucleus and j1 j2 as I said spins of nucleus one and 2 in the entrance channel. Now we know that in quantum mechanics a state with angular momentum j has 2 j + 1 sub states right 2 j + 1 sub states and the exited state in this compound nucleus represents the compound state for the entrance channel where there are 2 j + 1 sub states right.

Now the entrance channel involves a total of how many sub states 2 j + 1 and 2 j 2 + 1 these many initial sub states are there. Now remember these many sub states are available 2 j + 1 into 2 j + 2. These many sub states are available for the entrance channel what is the probability for the particle to get identified in the one of these excited states it is inversely proportional more number of states less probability for the particle to identify.

So, that is where this statistical factor comes into picture think more about it. So, this characterizes the quantum mechanical nature because nuclei 1 and 2 they have these many sub states and probability for the excited state of the compound nucleus to identify is more if the number of sub states is less if the more states are possible then probability to find it in a particular exited state particular sub state will be very less.

So, that is how this statistical factor comes into picture. So, let us go forward number 3 I hope you are clear for this concept and number 3 is 1 + Kronecker symbol delta 1 2 it reflects the fact that cross section is twice right for identical particles cross section is twice for identical particles in entrance channel isn't it. Now if 1 + delta 1 2 is written in cross section earlier we have written the reaction rate formula do you remember.

Now when you take this cross section having one place Kronecker symbol it will be cancelled when you write in terms of reaction rate. So, there is nothing to worry but to take into account the twice value of the cross section we have to include this term 1 + delta 1 2 ok. So, it will be one and zero depending on the identical nature or non identical nature and last term what is that but here coming back to this 1 + delta 1 2.

Please remember that particles should not counted twice particles should not be counted twice this care you have to take but the cross section doubles because they cannot be distinguished projectile and target. So, to summarize today's lecture I have discussed the reaction rate for gamma induced reactions and then mass fraction concept then something about the Maxwell Boltzmann distribution.

And I started discussing the inverse reactions and next in the next lecture I will continue the concept of inverse reactions how to relate the cross section of the forward reaction to the cross section of the inverse reaction, thank you very much for the attention, thank you.