Nuclear Astrophysics

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Lecture – 11

Reaction Rate… and Neutron induced reactions

Welcome students to today's lecture. Before going to discuss some new topics in today's lecture let me quickly summarize what I have discussed in the previous lecture. I hope you are following the sequence which I am trying to follow in the lecture. Initially I have started with the observational astronomy, some important features. And then selected general properties of the universe. As part of that I have given you the salient features of elemental abundance curve Hubble's law, remnants of 2.7 Kelvin CMB, HR Diagram, which gives you the idea about the relation between luminosity, temperature or colour.

I started discussing thermonuclear reactions because nuclear reactions in the stellar plasma within the stars all charged entities get their energy because of the thermal motion.

When these ions are undergoing gravitational contraction, after certain stage, the gravitational contraction leads to the development of thermal pressure. Thereby in that region temperature will increase. The ions will get kinetic energy and they will undergo nuclear reactions. We termed these reactions as a thermonuclear reaction. I started with the basics of the nuclear reactions like atomic Q value, nuclear Q value. When the binding energy correction term should be considered or not.

Then I started discussing the concept of cross sections of the nuclear reactions. It means the probability for nuclear reaction to happen. Under what conditions cross section will be more or less, that depends on various parameters like energy and the nature of interaction. I have also given you some numbers considering some examples regarding the cross section considering weak force, strong force and electromagnetic force.

So, you should understand how the cross section is going to change because of the interaction mechanism involved in the reaction. It depends upon the nature of the particles in the reaction entrance channel. One of the important goals of this course is to know the abundance of the elements. Why one element is having more abundance and why some elements are less abundant?

For that we need to know the number of reactions that take place. More the number of reactions, more the production of the elements and more the abundance. Less abundance or more abundance depends on the number of reactions taking place. The number of reactions taking place depends on the number of particles undergoing the interaction. It also depends on the cross section of the nuclear reaction, the velocity of the particles, product of number densities and the reaction rate.

In today's lecture what I am going to cover is reaction rate for neutron induced reactions and reaction rate for photon induced reactions. Here I am referring to gamma rays. Then depending on the time availability, I will discuss the concept of mass fraction. The goal of today's lecture is to understand how to mathematically calculate the reaction rate if the reactions are induced with neutrons and with gamma rays.

Before that I will spend some time to explain the salient features of the equations.

We have used Maxwell Boltzmann distribution for the velocity distribution of the particles. Then by using the number densities we have written expression for the reaction rate. The Maxwell Boltzmann distribution

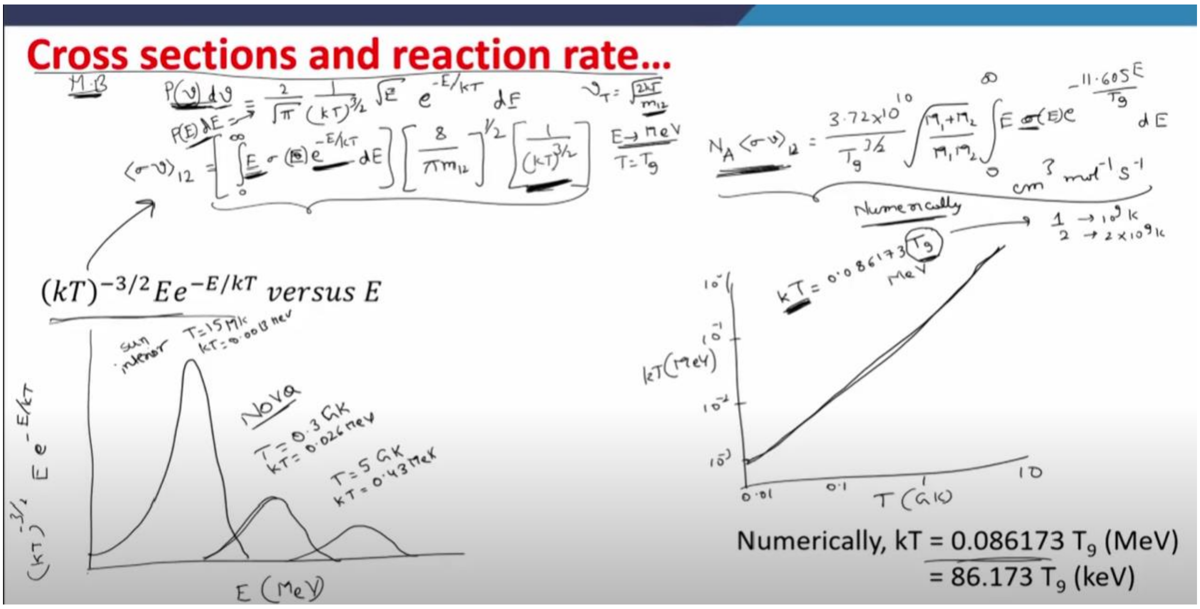
$$P(E)dE = \frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\sqrt{E}e^{-E/kT}dE$$

Reaction rate per particle pair when nucleus 1 and nucleus 2 undergoes nuclear reaction is given by

$$\langle\sigma\vartheta\rangle_{12} = \sqrt{\frac{8}{\pi m_{12}}}\frac{1}{(kT)^{3/2}}\int_0^\infty E\sigma(E)e^{-E/kT}dE$$

$$N_A\langle\sigma\vartheta\rangle_{12} = \sqrt{\frac{m_1+m_2}{m_1 m_2}}\frac{3.72\times10^{10}}{{T_9}^{3/2}}\int_0^\infty E\sigma(E)e^{-11.605E/T_9}dE\ cm^3mol^{-1}s^{-1}$$

At low energies the factor increases linearly and it reaches a maximum at a value of kT. At T equals to 15 Mega Kelvin you can see that kT is equal to 0.0013 MeV or 1.3 keV. If I go for more temperatures then it looks like this. This is for nova T is equal to 0.3 GK.



For low energies it increases linearly then the kT the maximum value corresponds to 0.026 MeV. So, it has increased from 1.3 keV to up to 26 keV. Now if I go to supernova then the variation looks like this. The temperature is about 5 Giga Kelvins and the value of kT is given as say 430 keV that means 0.43 MeV. So, this is how the factor in the expression looks like when it is plotted with respect to E.

Now, at different temperature scenarios like suns interior nova and supernova. So, at different temperature scenarios you are having different kT values. So, how the plot of kT versus temperature will look like and for the sake of convenience I will consider the temperature in terms of Giga Kelvins that means T_9 that means temperature is 10 to the power of 9 Kelvins. So, numerically you can get this kind of value and graphically if I want to express.

So it looks like this. So, when you plot kT in terms of MeV and temperature Giga Kelvins. So, if you start from 0.01, 0.1 and one logarithm scale 10 and starting from 10 to the power of -3, 10 to the power of -2, 10 to the power of the -1 then 10 to the power of 0 you will get an expression a linear relation kind like this. And this relation if you fit this linear relation if you fit it looks like kT is given as 0.086173 T_9.

Remember, if T_9 is equal to 1 it means 10 to the power of 9 Kelvins. T_9 is equal to 2 means 2 into 10 to the power of 9 Kelvins and kT is in MeV. So, this is how one can understand the salient features of reaction rate equation when 2 nuclei are non identical and at a different temperature scenarios how the factor of k T to the power of -3 by 2 whatever the term is there how it varies with respect to the energy.

So, it is clear that the maximum occurs at kT value. This distribution is going to play a very important role to understand what range of energies should be considered if we want to perform an experiment in Earth's laboratory. In stars nuclear reactions are happening at various energies.

But majority of the reactions are happening within what energy range, we have to identify. Otherwise in all energy ranges available within the star if you want to perform the experiment it is quite difficult moreover it makes no sense because not all energy ranges gives equal probability for the production of the elements in the universe. That is what we understood when we say cross section depends on the energy.

At the same time velocity distribution is also there and for each velocity its cross section is also changing. So, considering these 2 parameters for time being it is sufficient to understand that the maximum is occurring at the value of kT. Whether at this energy kT more reactions are taking place that we have to see. I repeat the question because I am going to answer this after a few lectures my question, listen carefully.

Whether the energy corresponding to k into T, k is Boltzmann constant and T is the temperature in Kelvins. Whether at this particular energy most of the nuclear reactions are taking place or not, to understand this, this kind of mathematical background is very important. I have shown you how kT changes with T if T is in Giga Kelvins. And after this let me discuss something else.

Like I have said in the; initial part of the lecture how to write the rate of the reactions if they are induced by neutrons. What is the point in considering the neutron induced reactions. Different types of particles are available within the stars. Why only neutrons because in the elemental abundance curve. If you remember elements beyond iron majority of them are because of the neutron induced reactions.

Within the star neutrons and another nuclei when reaction takes place there is nothing like one is projectile another is target. But that kind of scenario cannot be created in Earth's laboratory. So, you have to consider one entity as projectile and another entity as target and the reaction between projectile and target. The data analysis has to be done by converting into centre of mass system can give us the final idea about the energy produced within the stars.

So, now elements beyond iron in the elemental abundance curve have been synthesized because of the neutrons. Where from these neutrons are coming, from previous reactions. So, neutrons produced in the previous reactions they are acting as the seeds. They are inducing the reactions, neutrons which have no mass and which have no charge but having mass they will they are going to play a major role in the elemental abundance.

So, that is the reason we need to understand mathematical expression for neutron induced reaction rate. Already we have written some general expression for the reaction rate equation. We have written something general by considering 2 entities as non-identical, which has cross section and other terms.

The cross section in terms of neutrons is not like charged particles for which mainly we have written the previous expressions. So, for neutrons one has to consider the average cross section. Why average? All those things I will discuss in due course.

Neutron reacting with any nucleus giving alpha as ejectile or neutron giving some kind of gamma ray or neutron in interacting with some nucleus giving rise to proton, for all these kind of reactions how to calculate the rate. As I said Maxwellian average cross section because mostly at low energies and within a small narrow range only neutrons are inducing the reactions.

So, that is why one can go for the Maxwell Maxwellian average the cross section. So, same representation like Avogadro number and here it is not depending on the energy. So, that is a major difference between previous discussion here and mostly at some kind of thermal energy range. Now there is no velocity because I am taking the average in terms of energy. But at the same time I can write in terms of V by inserting V on the in the numerator and V T in the denominator.

$$N_A\langle\sigma\rangle_T = \frac{N_A\langle\sigma\vartheta\rangle}{\vartheta_T} = \frac{1}{\vartheta_T}N_A\int_0^\infty \vartheta\, P(\vartheta)\sigma_n(\vartheta)d\vartheta$$

$$= \frac{4}{\sqrt{\pi}}\frac{N_A}{\vartheta_T}\int_0^\infty \vartheta\sigma_n(\vartheta)\left(\frac{\vartheta}{\vartheta_T}\right)^2 e^{-\left(\frac{\vartheta}{\vartheta_T}\right)^2} d\vartheta, \qquad \vartheta_T = \sqrt{\frac{2kT}{m_{12}}}$$

So, 0 to infinity ϑ sigma n cross section of neutron which depends on velocity even slightly ϑ by ϑ_T square e to the power of - ϑ by ϑ_T square d ϑ. So, this is the expression for rate of the nuclear reactions induced by neutrons where ϑ_T is in general corresponds to some kind of a thermal neutrons which has this narrow range of energies. After this neutron induced reactions we need to discuss photon induced reaction.

So, the next topic is photon induced reactions. These are the 2 topics I told in the initial part of the lecture that I am going to discuss. The mathematical representation of reaction rate when the reactions are induced by neutrons and photons. Now why this photons are important because in majority of the nuclear reactions happening within the stars the outcome is gamma rays. Now reactions having positive q values they dominate at low temperatures. If the reaction is represented like 1 plus 2 giving rise to 3 plus 4. Now 4 is replaced with gamma ray this kind of reaction is called as radiative capture reaction.

$$1 + 2 \rightarrow 3 + \gamma$$

It can be induced by proton or it can be induced by a neutron. But if one of the outcome is gamma ray in addition to the reaction product then we call this as radiative capture reaction.

Now as the temperature increases these gamma rays emitted in one reaction can induce a reaction with other nucleus may be same nucleus in reverse order. So, if gamma reacts with some nucleus say 3 and gives rise to 1 plus 2 this is called as photo disintegration reaction photo disintegration reaction because it is induced by photon.

$$3 + \gamma \rightarrow 1 + 2$$

So, I am using the word photo and gamma plus 3 basically you are getting a nucleus in exited state. Gamma does not alter the proton number and neutron number. Once the nucleus 3 absorbs gamma-ray, it undergoes excitation. There is no change in the atomic number, mass number, but during de-excitation this nucleus undergoes disintegration depending on the energy available.

Where from this energy availability is coming to picture. The gamma whose energy is coming from one of the previous reactions and at high temperatures there is a more probability for gamma to have more energy. So, slowly I am trying to explain this in terms of positive q value and a negative q value. If the reaction has positive q value energy is released and if reaction has a negative q value, energy is to be supplied for the reaction to happen. Most of the reactions that is photo disintegration they are the negative q value reactions.

When gamma reacts with the nucleus 3. After absorbing gamma this nucleus 3 gets excited and during the excitation it may emit the same gamma ray or depending upon the energy levels available, it can emit multiple gamma rays. The formation of new nuclei is via one plus 2 this is called as photo disintegration reaction. So, if I can measure the cross section of a radiative capture reaction I can find out the cross section of the photodisintegration.

Vice versa somehow if I have the facility to measure the cross section for photo disintegration reaction. That means incident projectile is gamma ray. Very difficult to obtain experimentally though not impossible. Reacting with the nucleus 3 it is giving same 1 plus 2. Now the question is whether the cross sections of this forward and inverse reaction forward and inverse reaction are they related to each other or not.

Those things we will I will discuss. Just wait for some time. In the next lecture I will discuss. Now in this photon induced reactions the question is very simple what is the expression for reaction rate and what kind of changes you would like to make in the general reaction rate equation. So, to summarize in today's lecture I have discussed some of the salient features of the reaction rate equation when particles of non-identical nature are involved and at a different temperature scenario.

And I have shown you the values of kT starting from 1.3 keV, 40 keV, 140 keV starting from sun interior to supernova. So, far was basically some explosion stages of the star at its last stage. Then I discussed the neutron induced reactions how to express reaction rates and then photon induced reactions I just started I will discuss more in the next lecture. Thank you very much for your attention.