

Electromagnetism
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Lecture - 82
Fictitious discussion about Symmetry

Now, let us consider the idea of symmetry in Electromagnetism. We will indulge in a fictitious discussion.

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Fictitious discussion about symmetry

Magnetic charge

Charge and current free region

$\vec{\nabla} \cdot \vec{E} = 0$	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\vec{\nabla} \cdot \vec{B} = 0$	$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Replace $\vec{E} \rightarrow \vec{B}$ and $\vec{B} \rightarrow -\mu_0 \epsilon_0 \vec{E}$

$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho_e$	$\vec{\nabla} \times \vec{E} = -\mu_0 \vec{J}_m - \frac{\partial \vec{B}}{\partial t}$
$\vec{\nabla} \cdot \vec{B} = \mu_0 \rho_m$	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_e + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

↳ magnetic charge
Symmetric

Symmetric
 ↳ Current due to the flow of magnetic charge.

But the idea of symmetry would be clear from this fictitious discussion. Let us consider the idea of magnetic charge. Unlike electric charge, we have never talked about any magnetic charge. Why? And what would happen if we had any magnetic charge? Can we make the

Maxwell's equations symmetric? Let us consider in a charge and current free situation, how the Maxwell's equations read?

If we do not have any charge or current, we can write the divergence of the electric field as 0. The divergence of the magnetic field is anyways 0, the curl of the electric field is minus $\nabla \times \mathbf{B} = -\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ and the curl of the magnetic field that is $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$. When we do not have any charge or any current.

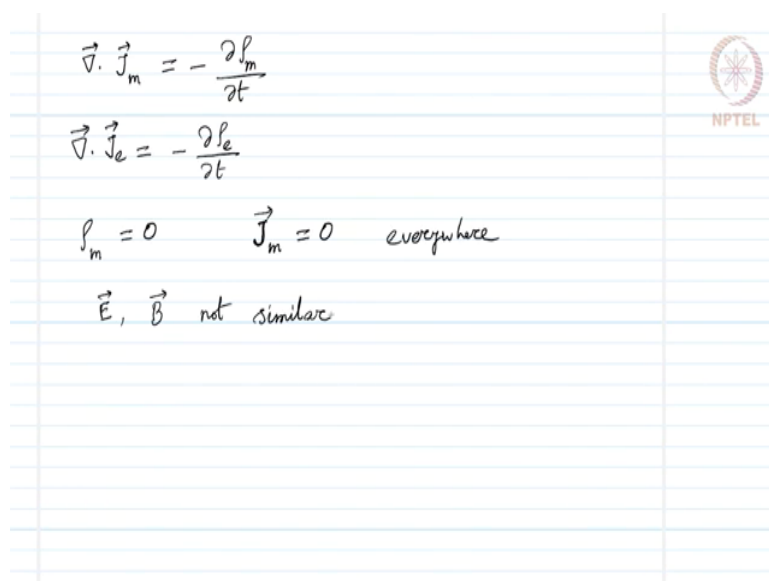
Now, you can see a symmetry in this. The divergence of electric field is 0, the divergence of magnetic field is also 0, the curl of an electric field involves the time derivative of magnetic field and the curl of the magnetic field involves the time derivative of the electric field. If we replace the electric field by magnetic field and the magnetic field by minus $\mu_0 \epsilon_0$ times the electric field then, we can convert one set of equation into the other set of equation. So its symmetric.

But the Maxwell's equations that we are familiar with are not symmetric like this. Can we make them symmetric? If we introduce some magnetic charge and current due to that due to the flow of that magnetic charge, we can make them symmetric. Although, its a fictitious discussion let us go into that because that will tell us many things about symmetry.

So, we can write the divergence of the electric field as $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ times the electric charge density. If we write the divert symmetric to this, the divergence of magnetic field as $\nabla \cdot \mathbf{B} = \mu_0 \rho_m$ times the magnetic charge density. And the curl of the electric field as $\nabla \times \mathbf{E} = -\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}$. What is \mathbf{J}_m ? \mathbf{J}_m is the current density due to the flow of the magnetic charge.

And curl of the magnetic field is as we know is $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_e + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$. \mathbf{J}_e is the current density due to the flow of electric charge plus $\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$. If the Maxwell's equations would have been like this, it would have been perfectly symmetric. But the Maxwell's equations are not like this. So let us write down these quantities and this is the current due to magnetic charge.

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The image shows a slide with handwritten mathematical equations on a blue-lined background. In the top right corner, there is a circular logo with a star-like pattern and the text 'NPTEL' below it. The equations are as follows:

$$\vec{\nabla} \cdot \vec{J}_m = -\frac{\partial \rho_m}{\partial t}$$
$$\vec{\nabla} \cdot \vec{J}_e = -\frac{\partial \rho_e}{\partial t}$$
$$\rho_m = 0 \quad \vec{J}_m = 0 \quad \text{everywhere}$$
$$\vec{E}, \vec{B} \text{ not similar}$$

And in this symmetric equation we must have the charges that is electric charge and magnetic charge both conserved. That means, they will have continuity equation like this, the divergence of the magnetic current is minus del rho m del t and the divergence of electric current is minus del rho e del t, this will give us the continuity equation obeying the condition that the charges are conserved.

However, there have been many efforts to find out the magnetic charge, but no one could find any existence of the magnetic charge. So we can write down that the rho m equals 0 and J m is also equals 0 everywhere. In other words, magnetic charge and magnetic current they just do not exist anywhere. And, this makes the electric field and the magnetic field not similar. There is an intrinsic dissimilarity between these 2 fields and we have to accept this, although it's not symmetric.

