Electromagnetism
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Lecture – 77
Ampere’s law in magnetized materials

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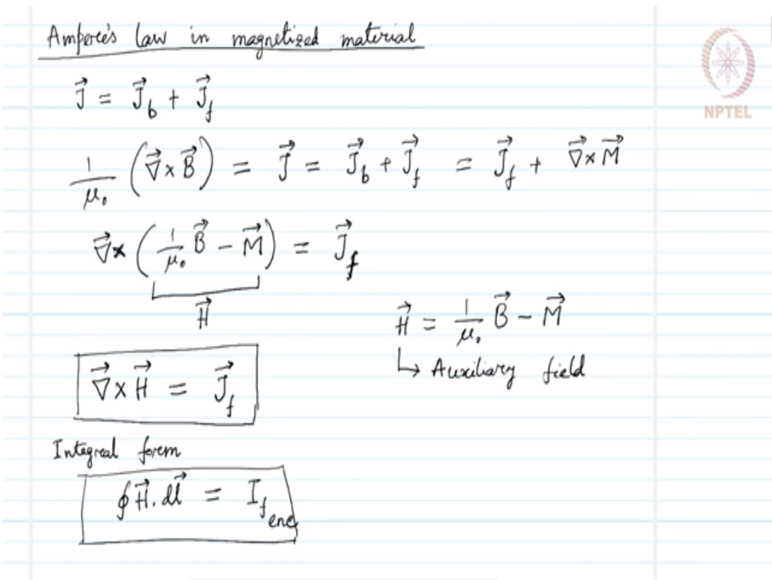

Now, let us consider Ampere’s law in the context of magnetized materials. The total current becomes the bound current plus if there is any free current that is the total current in the system and this would be applicable in amperes law, let us see how. So, we can write 1 over mu naught times curl of B in the differential form that is the total current which is equal to J b plus J f and how do we find J b.

So; that means, its J f plus curl of the magnetization because curl of the magnetization gives us the bound current density. Now, if we have this we can collect the curls in one side and write this as curl of 1 over mu naught times the magnetic field minus the magnetic polarization, that is equal to the free current density due to external source excluding the bound current. Because bound current is not under our control and free current is in under our control this way it would be wiser to write the expression for the magnetic field for the Ampere's law.

And if we do that we can define this vector as a new vector H. So, H is 1 over mu naught times the magnetic field minus the magnetic polarization vector. And then the amperes law takes this form curl of H is given as J f.

So, we had a displacement vector in the context of electric field in polarized material. In the context of magnetic field in material that that gets polarized due to this magnetic field we have this H field. H field can be called as auxiliary field it is more commonly called just H field nothing else. And if we put this into integral form applying stokes law we will find closed integral of H dot dl this will give us the total free current enclosed.

So, this is the amperes law in integral form and this is the amperes law in differential form in the presence of magnetized material. And after finding this we will have to worry about the boundary condition across a surface current.

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Boundary conditions

$$H^{\perp}_{above} - H^{\perp}_{below} = -\left(M^{\perp}_{above} - M^{\perp}_{below}\right)$$

$$\vec{H}^{\parallel}_{above} - \vec{H}^{\parallel}_{below} = \vec{K}_f \times \hat{n}$$

$$\vec{B}_{above} - \vec{B}_{below} = \mu_0 \left(\vec{K} \times \hat{n}\right)$$

Linear medium

$$\vec{M} = \frac{1}{\mu_0} \chi_m \vec{B} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right) = \underbrace{\mu_0 \left(1 + \chi_m\right)}_{\mu} \vec{H} = \mu H$$

If we have, so we know the magneto static boundary conditions. And if we now want to express this for a surface current in terms of H, then we can write that H field the perpendicular component of it above the surface current minus the perpendicular component of H field below the surface current. This is negative of the magnetization perpendicular component of it above the surface current minus the perpendicular component of magnetization below the surface current.

And the parallel component will be given as the parallel component will have two components actually. So, above the surface current the parallel component minus the parallel component of H below the surface current that is the surface free current density cross the direction perpendicular to the surface which is similar to the situation that we have found earlier in the context of magnetic field, we had the boundary condition written as this which is still valid.

Now, let us consider. So, we have considered linear dielectrics in case of magnetic field in material what are the linear medium lets think about that and find out interesting expressions in the context of linear medium. So, linear medium would be where the magnetization is proportional to any externally applied magnetic field; that means, we will have 1 over mu naught chi m that is the magnetic susceptibility times B that will give the magnetic polarization and this means it will be chi m times the H field.

So, if we write this way then we can write the magnetic field B as mu naught times H plus the magnetic polarization which is mu naught times 1 plus chi m times the H field. So, we can write this quantity as mu and then this gives us mu H. So, this expression is very similar to the case of linear dielectric material where, now mu can be called the permeability magnetic permeability of the linear medium that we have under consideration. After introducing this linear medium for magneto statics let us consider an example to see how they actually behave.

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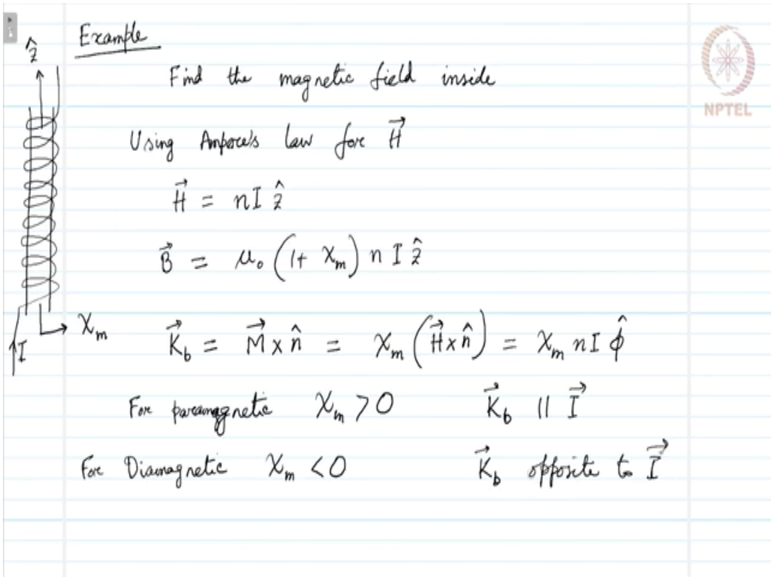

So, the best example that we have in the context of understanding linear medium is that of an infinitely long solenoid because that gives us uniform magnetic field. So, we consider an infinitely long solenoid with n turns per unit length and current I is flowing, but now, this solenoid is filled with a linear dielectric material with a susceptibility chi m.

Here is our solenoid of infinite length we have current flowing on the these winds like this the amount of current is I. And this is our z direction this is filled with a linear magnetic material with chi m susceptibility. And there are n turns per unit length, we are supposed to find the magnetic field inside the solenoid. How do we do that? Since, a part of the magnetic field comes from the bound currents that we do not know yet what kind of bound currents are there in this we cannot directly compute it.

Rather if we apply Ampere's law for H field that would be easier using Ampere's law for the H field we can easily find the expression for H as n that is the number of turns per unit length times the current and this field is along z direction. Once we have this then for a linear dielectric material with susceptibility chi m we can write B easily as mu naught times one plus chi m times m I z cap we have. So, far no reference to the bound current because we have found the H field with external current that is free current I and bound current that information will be somewhere embedded in chi m the magnetic susceptibility.

So, if we have this as the expression for the magnetic field and we will have uniform magnetization in this system because magnetic field is uniform and the material is the it is the linear medium. So, the polarize magnetic polarization is also going to be uniform and for uniform magnetic polarization we have seen that there is no bound volume current density, we will only have bound surface current density.

And that K b can be expressed as the magnetization cross n cap which is chi m times H cross n cap that is given as chi m n I phi cap, this is the bound surface current density. If the medium is paramagnetic the field is enhanced. So, for paramagnetic we have chi m greater than 0 and if it is diamagnetic we have chi m less than 0. And the bound surface current it points along I, if chi m is greater than 0 that is for paramagnet K b is parallel to I and if it is diamagnetic, then K b is opposite to I.