Electromagnetism
Dr. Nirmal Ganguli
Department of Physics
Indian Institute of Science Education and Research, Bhopal

Lecture – 75
Magnetization

Now, let us consider the concept of Magnetization.

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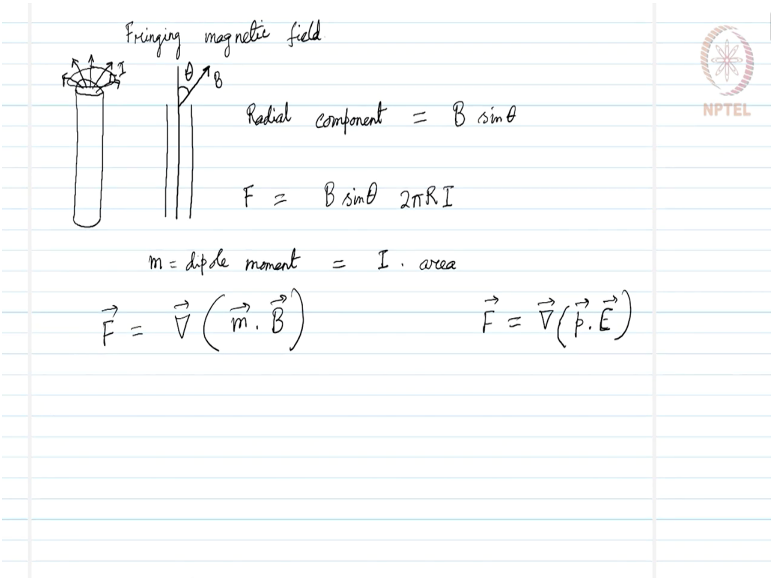

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Magnetization

$\vec{M} \equiv$ magnetic dipole moment per unit volume

Bound currents

$\vec{m} \rightarrow$ dipole moment

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

Dipole moment $\quad \vec{M}\, d\tau'$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}\,') \times \hat{r}}{r^2}\, d\tau'$$

$$\vec{\nabla}'\left(\frac{1}{r}\right) = \frac{\hat{r}}{r^2}$$

Magnetization just like electric polarization is the magnetic dipole moment per unit volume. It is represented as capital M vector. And just like the situation in case of electric dipole moment and electric polarization, we can here also introduce the concept of bound currents. There we had bound charges.

If we try doing that so, let us consider that we have a piece of magnetized material with magnetization capital M and the vector potential for a single magnetic dipole, we know the expression. If small m vector is the vector is the dipole moment, then we can write the vector potential A as a function of position vector r as mu naught over 4 pi m vector cross r cap direction over r squared.

This is the expression for the vector potential and if we have a magnetized object, each volume element d tau prime will carry a dipole moment M d tau prime. So, dipole moment from the magnetized object would be this much for d tau prime volume element.

So, in this case the vector potential can be written as mu naught over 4 pi, we will have to perform an integral M that is a function of r prime over cross r cap over r squared integrated over the volume. And now we have an expression that the gradient of 1 over curly r in the primed coordinate system that will give us r cap over r squared.

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$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \left[\vec{M}(\vec{r}') \times \left(\vec{\nabla}' \frac{1}{\mathscr{r}}\right)\right] d\tau'$$

Integrate by parts

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\left\{\int \frac{1}{\mathscr{r}}\underbrace{\left[\vec{\nabla}'\times\vec{M}(\vec{r}')\right]}_{\vec{J}_b} d\tau' - \underbrace{\int \vec{\nabla}'\times\left[\frac{\vec{M}(\vec{r}')}{\mathscr{r}}\right] d\tau'}\right\}$$

$$= \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}\left[\vec{\nabla}'\times\vec{M}(\vec{r}')\right]d\tau' + \frac{\mu_0}{4\pi}\oint\frac{1}{\mathscr{r}}\left[\vec{M}(\vec{r}')\times d\vec{a}'\right]$$

$$\boxed{\vec{J}_b = \vec{\nabla}\times\vec{M} \qquad \vec{K}_b = \vec{M}\times\hat{n}}$$

If we make use of this on the vector potential, we can express the vector potential as mu naught over 4 pi integration over the magnetization which is a function of r prime cross primed gradient of 1 over r integrated over d tau prime.

And now if we integrate by parts, we arrive at the expression A r equals mu naught over 4 pi integration 1 over r del prime cross that is curl of the magnetization that is a function of r prime; integrated over d tau prime minus integration del prime cross M that is a function of r prime over curly r curl of this vector integrated over d tau prime.

So, we have this expression integrating by parts and if we look at this term here, the second term; this term is a volume integral of a curl. So, we can express this as a surface integral with a cross product with the surface element. So, let us write that in that fashion. We can do this as mu naught over 4 pi integration 1 over r curl of M r prime d tau prime plus mu naught over 4 pi closed surface integral of 1 over curly r M that is a function of r prime cross d a prime.

We can write it this way. And the first term looks like the potential due to a volume current where the volume current density could be expressed as this. And the second term looks like that the potential due to a surface current. So, the surface current density K b that would be expressed as magnetization cross normal to the surface.

Here let me write down J b clearly. Here J b would be curl of the magnetization. These two are the relevant expressions for the bound current; if we want to express the vector potential in the form of bound currents just like what we did in the context of electrostatics.

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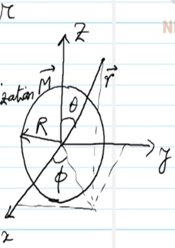
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}_b(\vec{r}\,')}{\mathscr{r}}\, d\tau' + \frac{\mu_0}{4\pi}\oint_S \frac{\vec{K}_b(\vec{r}\,')}{\mathscr{r}}\, da'$$

Example

Find the magnetic field   Uniform magnetization $\vec{M}$



$$\vec{J}_b = \vec{\nabla}\times\vec{M} = 0$$

$$\vec{K}_b = \vec{M}\times\hat{n} = M\sin\theta\,\hat{\phi}$$

$$\vec{K} = \sigma\vec{v} = \sigma\omega R\sin\theta\,\hat{\phi} \qquad \sigma R\vec{\omega} \to \vec{M}$$

Magnetic field $\vec{B} = \frac{2}{3}\mu_0\vec{M}$ inside the sphere

Outside $\vec{m} = \frac{4}{3}\pi R^3\vec{M}$

And then we can write the vector potential finally, as mu naught over 4 pi. We integrate it over J b which is a function of r prime over r integrated over the volume. So, the volume element comes here plus mu naught over 4 pi integral that is a closed surface integral kb. That is a function of r prime over curly r da prime.

So, this comes from the bound surface current and after finding this expression and the expressions for J b and K b the volume and the surface bound currents, let us consider an example where we work out the vector potential where we worked out the volume and surface currents. So, we have a uniformly magnetized sphere with uniform magnetization M and we want to find out the magnetic field due to this uniformly magnetized sphere.

How do we go about this? So, what do we mean by uniform magnetization that does not mean the magnetization can be radial. It must point along one direction and it is better to choose that the z direction. So, we consider in this picture that the magnetization is along the z direction and we want say the magnetic field somewhere here that is r position vector. This is our point of observation and this makes an angle theta with the z axis, the projection is here that makes an angle phi with the x axis and that is the geometry of the problem.

Now, let us try to find out the volume bound current that would be curl of the magnetization. Now our magnetization is uniform; its constant in space and it points along one direction. So, the curl or any such derivative will go to 0. How about the surface bound current density? That would be given as magnetization cross n cap which is M times sin theta times. So, and it is along the phi cap direction because if we consider a surface element here so, that surface element on the curved surface. So, n cap would be along r cap direction along the radius direction radial direction and magnetization is along z direction.

So, if we take a cross product of radial direction and z direction, we will have sin theta phi cap that would be the direction. So, that is what we get here. Now a rotating spherical shell of uniform surface charge density sigma corresponds to a surface current density. So, if we consider that the spherical shell is rotating about the z axis, then and if the spherical shell has the charge density sigma uniform surface charge density, then sigma v would be the surface current density which is nothing, but sigma omega R capital R is the radius of the sphere that we are considering here times sin theta phi cap because phi cap is the direction of the velocity.

So, we find that the field that we have due to this magnetized sphere is identical to the field of a spinning spherical shell. That means, sigma R omega vector this corresponds to the magnetization in this kind of a sphere. These are equivalent and the magnetic field for this kind of a geometry that we can find is B equals 2 over 3 mu naught M and this would be inside the sphere.

There is no M outside, there is no magnetization outside and the field well the magnetic field outside is the same of that due to a perfect dipole. We have a magnetization somewhere away. So, outside we can write the dipole moment for this kind of a current; this kind of a magnetization would be like 4 over 3 pi r cubed times the magnetization vector and the corresponding magnetic field would be due to this M dipole moment whatever we have.