

Electromagnetism
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Lecture – 71
Magnetic dipole

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Compact form

$$\vec{B}_{\text{above}} - \vec{B}_{\text{below}} = \mu_0 (\vec{K} \times \hat{n})$$

Vector potential is continuous.

$$\frac{\partial \vec{A}_{\text{above}}}{\partial n} - \frac{\partial \vec{A}_{\text{below}}}{\partial n} = \mu_0 \vec{K}$$

NPTEL

After analyzing the vector potential in these details let us consider the idea of a Magnetic dipole.

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Magnetic dipole

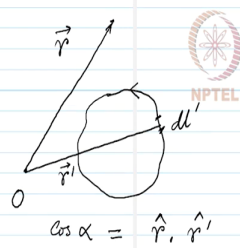
$$\vec{A}_{\text{dip}}(\vec{r}) = \frac{\mu_0 I}{4\pi r^2} \oint r' \cos \alpha \, d\vec{l}'$$

$$= \frac{\mu_0 I}{4\pi r^2} \oint (\hat{r} \cdot \hat{r}') \, d\vec{l}'$$

Now $\oint (\hat{r} \cdot \hat{r}') \, d\vec{l}' = -\hat{r} \times \int d\vec{a}' \rightarrow \text{Homework}$

$$\vec{A}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$\vec{m} \rightarrow \text{magnetic dipole moment}$
 $\vec{m} = I \int d\vec{a}' = I \vec{a}'$



So, what would be the potential due to a magnetic dipole? We can write it in terms of the vector potential. The vector potential due to a magnetic dipole can be given as $\mu_0 I$ over $4\pi r^2$ closed line integral over $r' \cos \alpha \, dl'$. So, what is cosine of α ? What kind of picture are we considering here? If we have a current carrying loop of any arbitrary shape and here we have a line element on this that is dl' . So, and we are trying to find out the vector potential due to this dipole that is this current.

So, a current that encloses an area makes a dipole. So, here if we have some arbitrary coordinate system origin here then this vector would be represented as r and this vector would be r' . Now, this cosine of α is given by $\hat{r} \cdot \hat{r}'$, this is the angle that we are talking about this is the cosine of α . And this is what we are integrating over the

line element on the current carrying loop that is $d\mathbf{l}'$. So, this expression can be written as $\frac{\mu_0 I}{4\pi r^2} \oint \mathbf{r}' \cdot d\mathbf{l}'$.

Now, we can write $\oint \mathbf{r}' \cdot d\mathbf{l}'$ this quantity $d\mathbf{l}'$ we always considered with a vector sign ok. So, this quantity can be expressed as $\oint \mathbf{r}' \times d\mathbf{a}'$.

So, this how this comes? Is a homework for you and once we have this then we can write the magnetic dipole or vector potential as a function of the position vector \mathbf{r} given as $\frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{r}}{r^3}$ where \mathbf{m} is the magnetic dipole moment. And this is given as $I \oint d\mathbf{a}$; that means, current times the area of the loop \mathbf{a} is the vector area enclosed by the current loop.