Electromagnetism
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Lecture - 07
Tutorial on differential vector calculus

Now, we are going to have some tutorials to see how the problems related to electromagnetisms; Electromagnetism can be solved. So, without being able to solve problems electromagnetism will be or hardly of any use. So, tutorials are very important and after covering a significant portion of the topic we will always have a tutorial.

The first Tutorial is about differential calculus with vectors, because most of you were already familiar with vector algebra, we do not have any tutorial on vector algebra it starts with vector calculus. And in differential calculus we have learnt gradient divergence and curl mainly these three aspects of vector differentiation. So, this tutorial is about the gradient divergence and curl of a vector ok.

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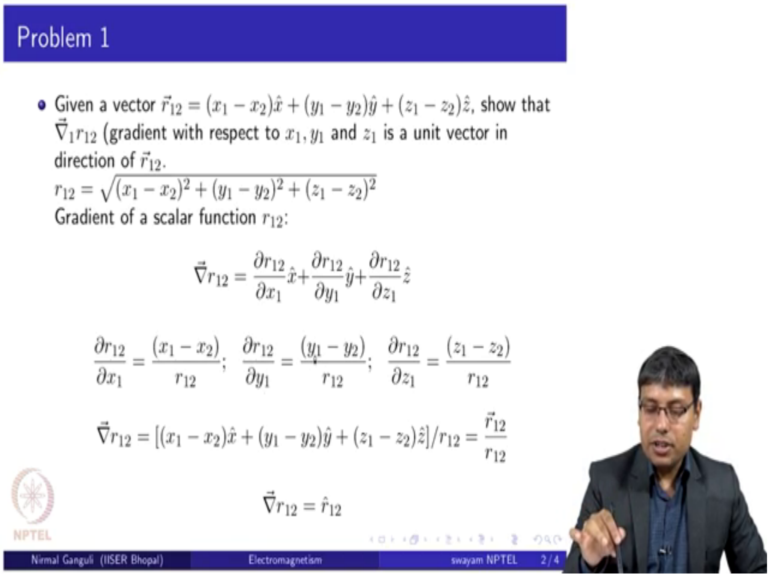

So, during the tutorials what I will do is after explaining the problem I will pause for a while you can pause the video during this time and you can try solving the problem. After you have attempted to solve the problem if you have solved it completely fine you can move to the next problem, if you could not solve it I will explain how the problem can be solved you can go through that and there will also be parts. So, the problem will be divided into different parts and you can pause the video after some parts and then try to solve the problem remaining part of the problem and then look at the next part of the video you can proceed this way.

So, we have this problem that we take the gradient with respect to x 1 y 1 and z 1 that is gradient 1 of the magnitude of this displacement vector r 1 2; how do we. So, you can now pause the video and try solving it yourself ok. After you have tried solving the video let us try it solving the problem, let us see how it can be worked out.

So, the magnitude of r 1 2 vector can be given as x 1 minus x 2 whole square plus y 1 minus y 2 square plus z 1 minus z 2 square, square root of this quantity and we can write the gradient of a scalar function r 1 2 as r del r 1 2 del x 1 along x cap direction plus del r 1 2 del y 1 along y cap direction plus del r 1 to del z 1 along z cap direction. So, if we calculate 1 component of it the first component, let us say there del r 1 2 del x 1 we will obtain x 1 minus x 2 over r 1 2. Similarly, for del r 1 2 del y 1 we will obtain a y 1 minus y 2 over r 1 2 and for del r 1 2 del z 1 we will obtain z 1 minus z 2 over r 1 2.

Now, if we add all these components with appropriate unit vector multiplied to the components we will get x 1 minus x 2 along x cap direction plus y 1 minus y 2 along y cap direction plus z 1 minus z 2 along that cap direction over r 1 2 magnitude the magnitude of this r 1 2 vector.

So, this is this 1 at the numerator is nothing, but r 1 2 vector and that is divided by r 1 2 magnitude, that means, the gradient of r 1 2 magnitude is a unit vector along r 1 2, that is what we have obtained by solving this problem ok.

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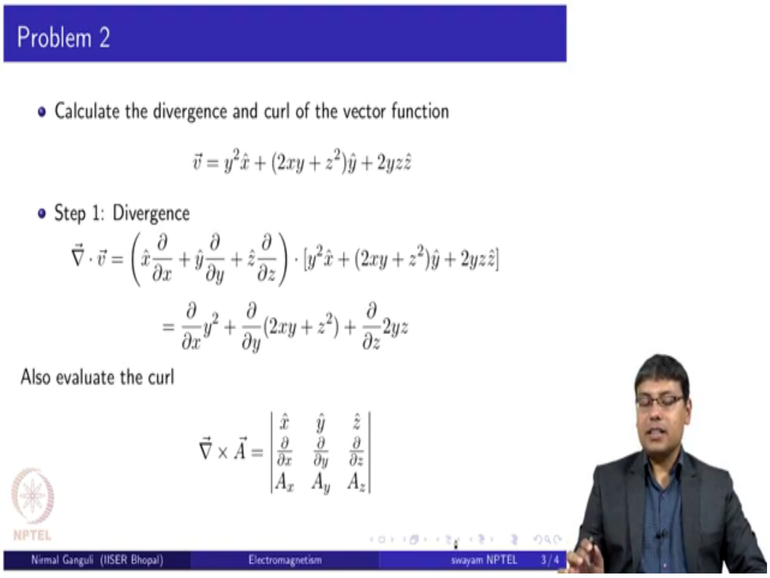

After that let us move on to the second problem. In the second problem you are asked to calculate the divergence and curl of a given vector field, v is the given vector field that is expressed as y squared x cap plus 2 xy plus z squared y cap plus 2 yz z cap. This is the vector field and you are asked to find out the divergence and curl of this vector field.

Now, you can pause the video try it yourself and then replay the video ok. If we; if you need some help in solving this problem this is the way we can calculate the divergence in the first, step that is the divergence operator is x cap del del x plus y cap del del y plus z cap del del z dot the vector field y square x cap plus 2 x y plus z square y cap plus 2 y z that cap, that means, del del x will operate only on the x component here.

Similarly, del del y will operate on the y component that is this one and del del z will operate on the z component this one ok. So, this is the way we can calculate the divergence. And if we want to calculate the curl, then curl of A vector a is given as this determinant x cap y cap z cap del del x del del y del del z A x A y A z, where A x A y and A z are the x y and z components of the vector respectively.

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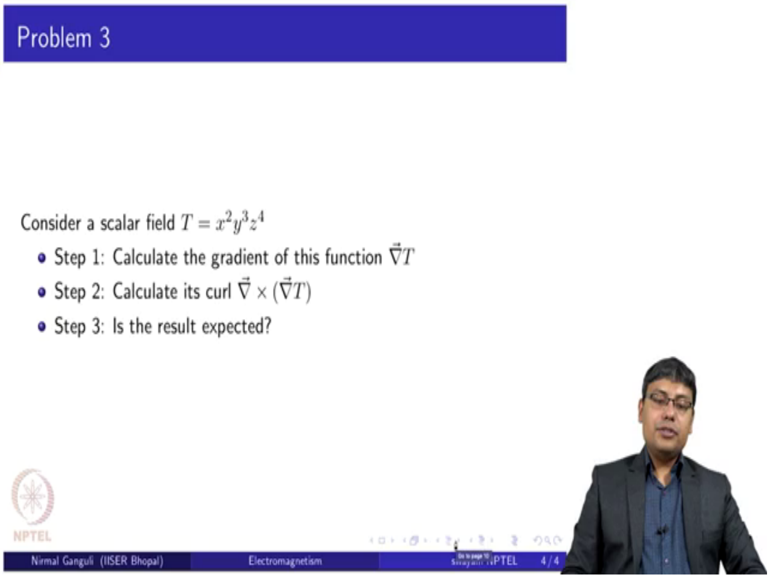

So, for this given vector at our hand we can calculate also the curl of the vector using this expression ok. Now, let us move on to the third problem, here we consider a scalar field given by T equals x square y cube z power 4. And in the first step you are asked to calculate the gradient of this function that is grad of T the scalar field and the second step you are asked to calculate the curl of the gradient of this function.

So, we have a scalar function if we calculate its gradient we are going to have a vector at hand. And in the second step you are supposed to calculate the curl of that vector quantity. Then the third step is to see whether the result of this curl that we got was expected did you expect this kind of a result here you can pause the video and try the problem out. And for this problem I am not going to offer a solution this is for you to work out and see whether you get the expected solution.

I will only give you one hint that we have found out that the curl of the gradient of a scalar field is always 0. So, in step 2 you are supposed to get 0 as a result and if you have got that that is the expected result if you have got something else some nonzero value in the second step, then you better recheck your calculation.