Electromagnetism
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Lecture – 69
Calculation of vector potential

Hello, we have already develop the idea of magnetic vector potential, now we will see an example of how to Calculate a vector potential for a magnetic system.

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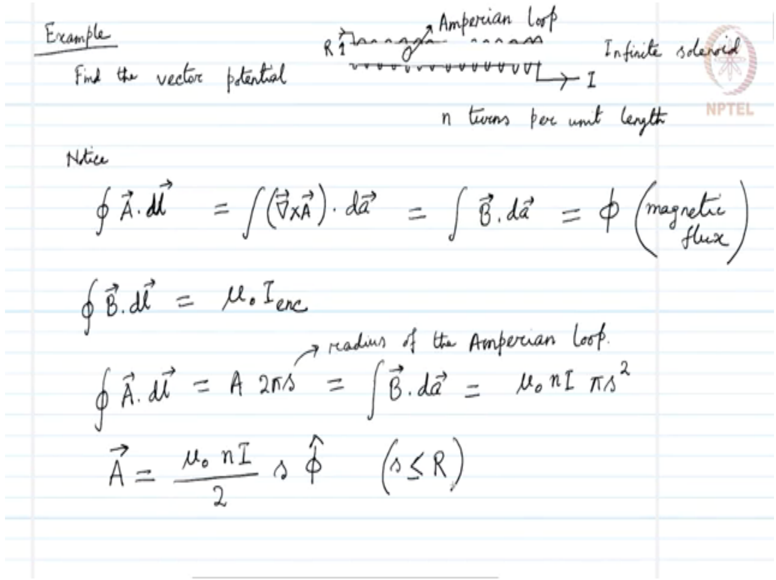

So, we have discussed the idea of an infinite solenoid; let us consider an infinite solenoid here. And we have n turns per unit length on this just like what we considered earlier and the radius of this solenoid is R, this much is capital R, the current flowing through this wire is I

So, we have found out the magnetic field for this kind of a solenoid, now it is our job to find out the vector potential. How do we find out the vector potential? We can see that the current extends to infinity therefore, the way we have calculated vector potential earlier is not valid in this context. But there is something interesting to notice for this problem, that is if we take a closed integral over A dot dl.

We can write that using Stokes law as the curl of A dot da and; that means, curl of A is nothing, but the magnetic field. So, its integration over B dot da the surface integral of the magnetic field this is known as the magnetic flux. Just like in the context of electrostatic field we had an electric flux, this is the definition of magnetic flux in similarity. So, the closed integral over A dot da equals to the magnetic flux.

Now, the closed integral over B dot dl that gives us mu naught times the current enclosed. So, we can see that we have two similar equations closed integral over A dot dl gives us a scalar quantity, closed integral over B dot dl gives us another scalar quantity. And if that is the situation then we can argue that the solution to A should be similar to the solution to B with that similarity argument, if we consider a circular amperian loop inside the solenoid for drawing that I am erasing this part here ok, we have a circular amperian loop inside the solenoid here.

And, once we have that on that loop, we can write down that closed integral over A dot dl equals A times twice pi s, because A by symmetry of these two equations of A dot dl and B dot dl we can argue that A would be uniform inside this region. So, A times 2 pi s and that is equal to the magnetic flux, that is B dot da, which is we know the magnetic field expression for this problem; this problem of infinite solenoid we have already worked that out earlier.

So, we can directly write down that is mu naught n I times the area, that is pi s squared, where s as we have put s here also, so s is the radius of the amperian loop. Once we have obtained this, now finding out the vector potential is not that difficult, we can write down the vector potential equals mu naught n I over 2 multiplied by s, this would be the magnitude of the vector potential. And we have to also find the vector potential, but what we are talking about is inside this solenoid we did not go outside yet.

So, what would be the direction of this vector potential? As we have argued earlier the vector potentials direction would be same as the direction of the current under the gauge that we have considered. So, if we consider the vector potential in its vector form, then phi cap would be its direction because that is the direction of the current. Now, this is valid for s less than equal to capital R that is inside the solenoid not outside the solenoid.

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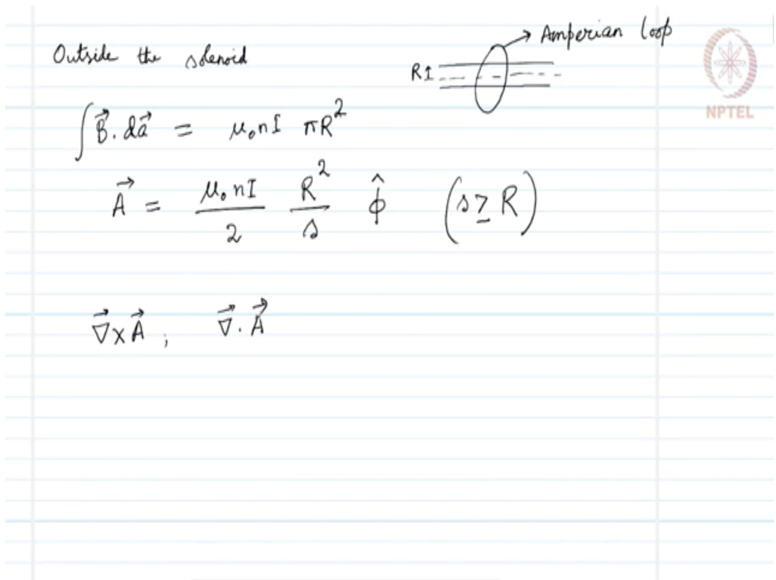

What happens outside the solenoid? Outside the solenoid the same expression is valid, so over any area B dot da that quantity, if we consider something like this is the solenoid and the amperian loop is like this. So, B dot da we will find the flux only in from the part inside the solenoid and outside the solenoid there is no magnetic field. So, this quantity will be mu naught n I this is the field inside the solenoid, so that has to be multiplied with the area of the cross section of the solenoid.

So, R is this much radius and this is the axis of the solenoid. Once we have this we can equate this with the line integral of A and from that we can obtained A equals mu naught n I over 2 R squared over s, and the direction is of course, phi cap, here s is greater than equal to R. So, this is the expression of the vector potential outside the solenoid. Remember the if the field is 0 the potential is not necessarily 0, the potential can have some finite value and this is the value of the potential, but it will give you the field to be 0 outside the solenoid.

Now, one has to find out, the one has to verify by calculating the curl of this vector potential that we have obtained in both cases outside and inside the solenoid, also by calculating the divergence of the vector potential that we have obtained in both cases inside and outside the solenoid, that is a homework for you.