Electromagnetism
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Lecture - 67
Magnetic vector potential

Now, here in case of magnetic field we have the curl of the magnetic field nonzero. So, scalar potential is out of the question. We cannot define a scalar potential for the magnetic field that would not work. And we can see that the magnetic field is not a conservative force field therefore, scalar potential that concept if is out of the question. What we see is that the divergence of the magnetic field is 0 and we know that if we have a vector given by curl of another vector, then the divergence of that will go to 0.

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Comparison between $\vec{E}$ and $\vec{B}$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ Gauss law

$\vec{\nabla} \times \vec{E} = 0$ No name $\rightarrow$ define a scalar potential

$\vec{\nabla} \cdot \vec{B} = 0$ No name $\vec{\nabla} \times \vec{A} = \vec{B} \rightarrow \vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ Ampere's law

$\vec{F}_{em} = Q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

We know that, if we have a vector A and the curl of this vector gives us B, then divergence of B will always be 0 for any generic vectors A and B, B is not necessarily with the magnetic field. Then can we define a scalar potential A in the context of sorry, can we define a vector potential A in the concept of context of magnetic field where the curl of this vector potential will give us the magnetic field.

But what is the advantage of doing that? In the context of defining a scalar potential we mentioned that instead of dealing with three components of a vector with scalar potential we can just deal with one component. Here with vector potential we still have to deal with three components. What would be the advantage of doing something like that could there still be some advantage?

Let us see, how it works out? If we try doing this trying try defining a vector potential corresponding to the magnetic field and the curl of that vector potential will give us the magnetic field itself then how do we define this vector potential and how does it help us at all.

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Magnetic vector potential

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$\hookrightarrow$ vector potential

$\vec{\nabla} \cdot \vec{A} = 0$ Can we ensure?

$\vec{A} = \vec{A}_0 + \vec{\nabla}\lambda$ $\quad \lambda \rightarrow$ scalar

$\vec{\nabla} \cdot \vec{A} = 0 \Rightarrow \vec{\nabla} \cdot \vec{A}_0 = -\nabla^2 \lambda$

$\vec{\nabla} \times \vec{A} = \vec{\nabla} \times \vec{A}_0 \qquad \vec{\nabla} \times (\vec{\nabla}\lambda) = 0$

$\nabla^2 \lambda = -\vec{\nabla} \cdot \vec{A}_0$ similar to the Poisson equation $\nabla^2 V = -\rho/\epsilon_0$

So, we are going to define a magnetic vector potential and magnetic vector potential would be defined as we have discussed in this way; the magnetic field will be expressed as the curl of this vector potential A. And if we have that can we also make so, can we ensure that the divergence of A goes to 0.

Let us see, how we can do that and what are the advantages of doing this. This way we can have some unique determination of the vector potential. This is one of the possible gauge transformations in the context of vector potential, vector potential is defined with some arbitrariness in it just like the scalar potential we have arbitrarily defined a reference here in vector potential also we are happy as long as the curl of the vector potential gives us the magnetic field and that is all physical about this vector potential.

If we have some irrotational vector added to this vector potential curl of which goes to 0, we do not mind doing anything like that and that is a gauge transformation. So, if we want to make sure that the divergence of the vector potential goes to 0, what way we can do it? Let us consider that we have a vector we construct and a vector potential A with the vector potential A naught that gave us the correct magnetic field, but it is divergence did not go to 0 plus the gradient of a scalar.

So, the gradient of a scalar lambda is always going to be irrotational; that means, the curl of this gradient is going to be 0 therefore, by adding a gradient of a scalar, we did not change anything physically. We will retrieve the same magnetic field by taking the curl of this new A vector.

And if that happens we can always come we can always choose some lambda in such a way that the divergence of the vector potential goes to 0. So, this is one of the possible gauge transformations that will actually require the divergence of A naught to be minus del square lambda. We choose lambda this way so that, this condition is satisfied and when we take the curl of the vector potential that is curl of the old vector potential before any gauge transformation because, curl of the gradient of lambda is always 0.

So, we can write down that del square of this scalar gauge lambda is minus divergence of A naught, the vector potential before making this gauge transformation. And this is similar to the Poisson equation. I the context of electrostatics, we had the Poisson equation as del square V equals minus rho over epsilon naught rho being the volume charge density. So, this the gauge condition is similar to the Poisson equation.

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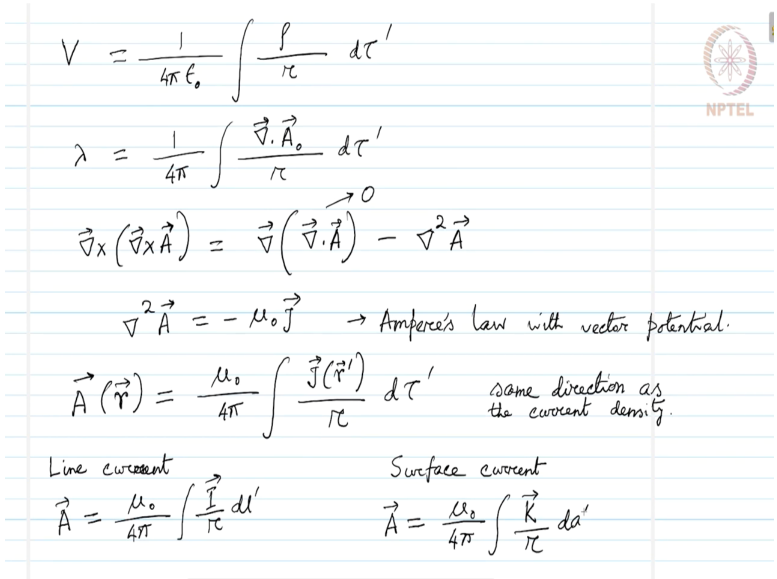

Now, because this condition is similar to the Poisson equation let us write down the solution to the Poisson equation. If we want to find out V, the potential the electrostatic scalar potential for a charge distribution rho, we could do that by writing it as the potential as 1 over 4 pi epsilon naught, integration over this volume charge density divided by r d tau prime.

Similarly, because this vector potential equation is also very similar to the scalar potential equation, Poisson equation; can we determine lambda as 1 over 4 pi integration over the divergence of A naught over divided by r d tau prime. We should be able to do this and we notice that curl of the curl of A, that is using a vector differentiation product rule del the gradient of the divergence of A minus del square A.

And if we make if we construct it in such a way that the divergence of A goes to 0, then under such kind of a construction we will have from amperes law del square A is minus mu naught

J. So, this is the form of Ampere's law using vector potential with the appropriate gauge transformation.

So, having this we can then find out this equation is also similar to Poisson equation, we can then find out a similar solution to what we obtained for Poisson equation that is the vector potential can be given as mu naught over 4 pi integration the volume current density, that is a function of r prime over curly r, the distance d tau prime.

Where we see that the vector potential is along the same direction as the current density. So, this is for volume current, if we consider a line current we can write the vector potential as mu naught over 4 pi integration I over r d l prime and if we consider a surface current similarly, we can write the vector potential A as mu naught over 4 pi integration K over curly r d a prime.

So, everywhere we have the vector potential along the direction of the current and this would be valid if we assume the current goes to 0 at infinite distance from our point of observation or any point of interest.