Electromagnetism
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Lecture – 65
Magnetic field in a long solenoid

Let us consider another example, the example of a solenoid which is very important in the context of magnetostatics.

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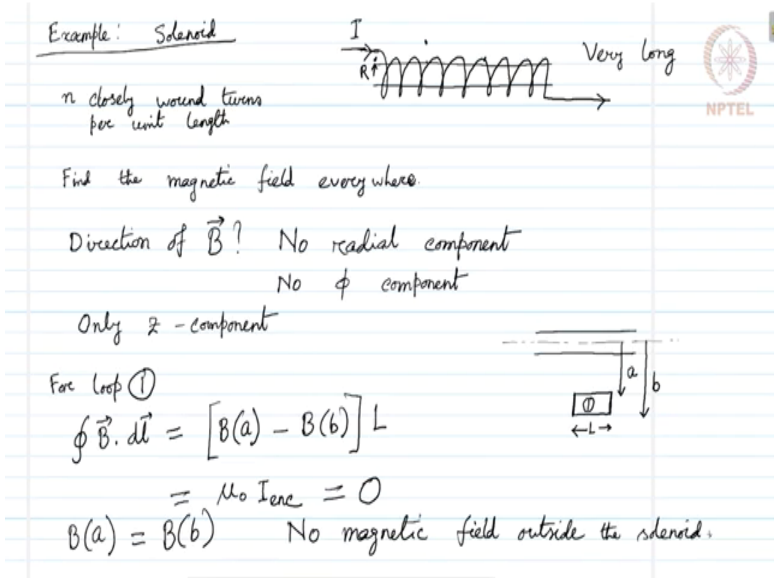

We consider having a long solenoid. What is solenoid? Its a cylinder and on this cylinder, we have a current carrying wire winded like this. Many of you have seen something like this. A current I comes from this direction and leaves from this direction. So if the winding is very close, we can assume that a surface current is flowing on the cylindrical surface of the solenoid. And for this kind of a system, if we consider that a steady current I is flowing on this wire, and there is there are n closely wound turns per unit length of the solenoid.

If this is the situation and if we have the radius of this solenoid as capital R, then we want to find the magnetic field everywhere due to this solenoid. So our solenoid is very long. That means, we will assume that it extends to infinity for every practical purpose.

What would be the direction of the magnetic field? Would it have any radial component? Our current is flowing along phi direction in the cylindrical coordinate system. So can it have a radial component? Let us see. If we have our point of observation here, let us say then, at this point of observation, the part from the current, if we try to find out the magnetic field, the current is along phi cap and this is along R cap.

So, it does not show any radial component there. If we considered the current right here. But, if we considered the current here, that is along phi cap and this way, there would be some radial component. But, that radial component will be exactly cancelled out by some contribution from here exactly opposite direction. That way, it will not survive any radial component for an infinitely long solenoid. Had it been finite, there would have been some radial component. For infinitely long solenoid there would be no radial component.

How about phi component? Because we have a surface current density along the phi direction, there is no question of a phi component. So the only component left with the left is the z component. And what would be the z component at different places? Let us consider lets for simplicity, draw a cylinder like this of infinite extent of course, and lets consider an Amperian loop somewhere here, a rectangular Amperian loop. And from the axis of this cylinder here the distance of the closest arm is let us say a, the distance of the furthest arm is let us say b.

And this is our first Amperian loop, loop 1. For this loop, we can write down the closed line integral over B dot dl that is, so only z component of magnetic field is there. Therefore, we do not have any contribution from the short arms of this Amperian loop. And if we consider that l is the length of this Amperian loop rectangle, then we can write this as the magnetic field magnitude at the point at distance a minus the magnitude of magnetic field at distance b times the length of this Amperian loop. And that equals mu naught times the current enclosed.

This loop here does not enclose any current whatsoever. So this quantity goes to 0. What does that mean? That means, B a equals B at distance b. Which essentially means no there is no magnetic field outside. Entire magnetic field is inside the solenoid. Remember, it extends to infinity and everything whatever we can find is inside.

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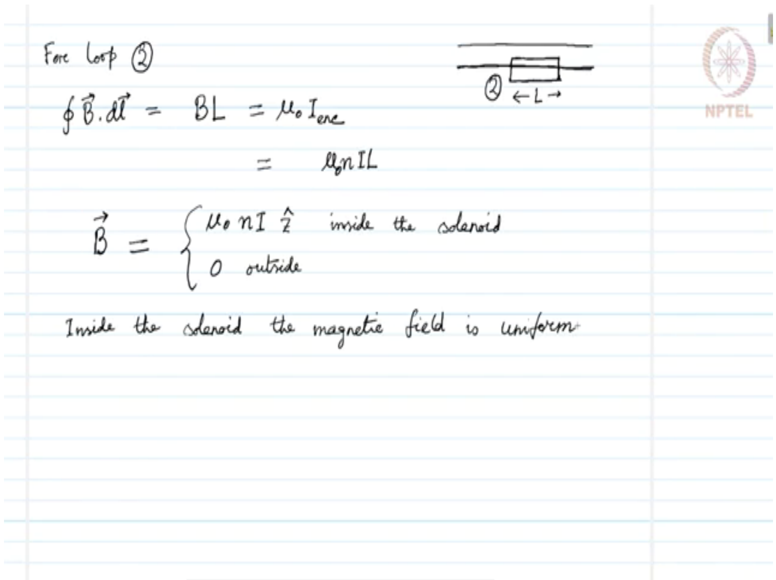

Now, let us consider the situation of inside the solenoid. Let us draw another one here, this is our solenoid. It extends to infinity and this time we have an Amperian loop like this. Here, this is the second Amperian loop and for this loop what we can write is the same thing closed integral over B dot dl equals BL.

We have found out that outside there is no magnetic field, because if we go at infinite distance from this solenoid, infinite radial distance, then the magnetic field has to be 0 because any non-zero value there would be unphysical. Therefore, at 2 different distances outside the solenoid, if we find the same value of the magnetic field then it must be 0 everywhere outside the solenoid.

So on this line here, the long arm lying outside the solenoid, the magnetic field is 0. So, that does not contribute to the integral, only this arm that lies inside the solenoid that contributes. So this closed line integral B dot dl gives us B times L. Where L is the length of this rectangle. And that equals mu naught times the current enclosed how much current is enclosed? Its a surface current, but there are closely wound wire, there is a closely wound wire and that wire carries a current amount of I.

So per unit length it will carry n I amount of surface current density. So, the total current over length L can be given as nIL that times mu naught. So that is BL. Therefore, the magnetic field would be B can be expressed as mu naught nI. L from both sides will cancel. If we worry about the direction then this would be along the z direction and it would be 0 outside.

So, this would be the expression for magnetic field everywhere in space. We notice here that inside the solenoid, the magnetic field is not a function of distance from the surface or from the axis. That means, inside the solenoid, the magnetic field is uniform. Earlier in case of electrostatic field, we have found some way of generating uniform electrostatic field. That is by taking 2 infinite parallel plates and charging them with equal and opposite amount of charge, and that must be distributed uniformly over them.

If we have this kind of a situation then in between those plates we had uniform electrostatic field. And here we have found that if we have infinitely long solenoid, there is uniform winding of wire and if a steady current flows through that wire, then inside the solenoid we will have a uniform magnetic field along the z direction, it would not depend on how far our point of observation is from the axis of the solenoid, axis of the cylinder.

But outside that solenoid, the magnetic field is 0. Outside those 2 parallel plates in the context of electrostatic, electric field was also 0.