Electromagnetism
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Lecture – 64
Ampere’s law in integral from and its applications

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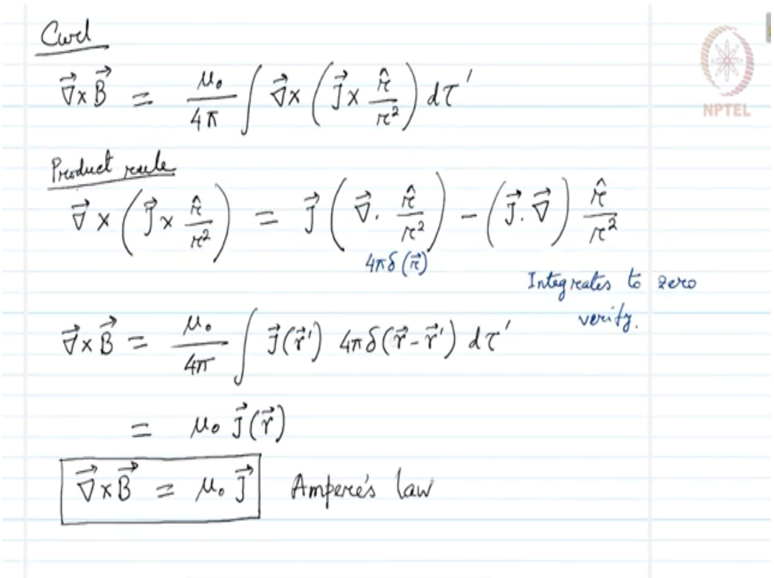

So, since we have a curl involved here we can have a different kind of integral form of this Amperes law, using the Stokes theorem.

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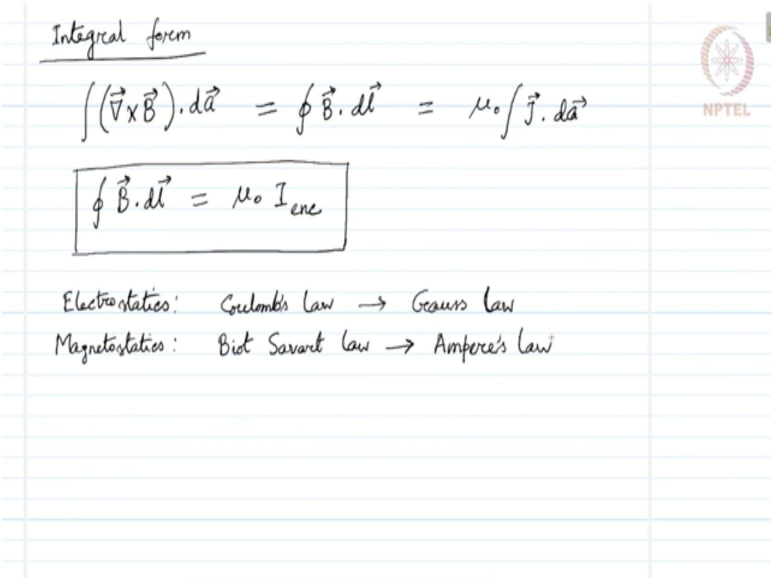

We have actually written this earlier in the context of the example of straight line current let us write this explicitly once again. The integral form will be in integration over the curl of the magnetic field, over a surface element. And this by applying Stokes law we can write as cyclic line integral that encloses an area B dot dl that will be equal to mu naught times integration over J dot da.

So, this would be the integral form of the Amperes law; that means, simplifying we can write cyclic integral closed line integral over B dot dl equals mu naught times the current I that is enclosed by the loop. So, this is Amperes law in Integral form; so if we now compare with electrostatics. In electrostatics we had Coulombs law and from that we could derive Gauss law, Gauss law was always valid, but useful in some symmetric contexts. In case of magnetostatics we have the fundamental law as Biot Savart law and from that we could derive Amperes law.

Amperes law also as we can see here is always valid, but applicable to find out the magnetic field in only certain symmetric contexts. And we will see a few examples of application of Amperes law to give us the magnetic field, if we can apply Amperes law that actually makes things very easy to find out the magnetic field, let us try doing that.

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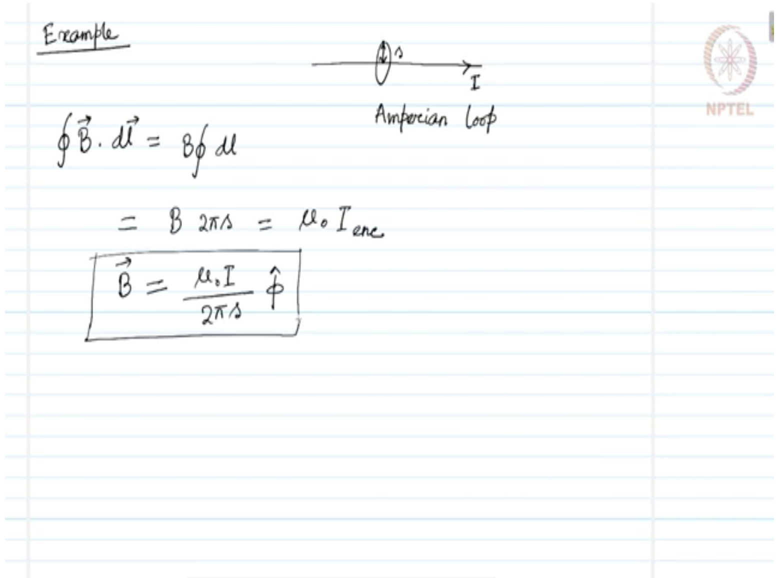

Let us start with a very simple example, so we have a line current I moving in this direction and at a distance s here this distance is s. We want to find the magnetic field due to the steady current I and this where on which the steady current is flowing is very long. So, we will have uniform magnetic field here well we will have the magnetic field that is constant in magnitude on this loop and this loop can be called an Amperian loop.

On this Amperian loop the magnitude of the current would be constant and let us find out its direction it will be I cross r cap. So, I cross r cap will be this kind of a direction; that means, radially outward for this Amperian loop and the line element on this Amperian loop, loop dl prime will be tangential to the loop. So, the magnetic field and the line element no they cannot be tangential please delete this part.

Let us find out the direction of the magnetic moment the direction of magnetic moment is along B sorry I cross the distance. So, I cross the distance that makes it along the direction outside of the volt, so that is tangential to the loop. And, the line element dl that we will consider on the Amperian loop that is also along the same direction tangential to the loop. That is phi cap direction for the loop in cylindrical coordinate system and that is the same, same is the direction of the magnetic moment.

So, we can write line integral over the closed path closed line integral of B dot dl equals because the magnetic field will be constant in magnitude we can take this B outside dl and that in this kind of a geometry. So, integration over dl will just give us the circumference of this circle; that means, B times twice pi s and that equals mu naught times the current enclosed.

And that is I therefore, the magnetic field its magnitude would be mu naught I over twice pi s and if you if we now worry about the direction that is along phi cap. So, this is the same as what we obtained by applying Biot Savart law and integrating over the range from theta 1 to theta 2. And for a very long infinite where we made theta 1 go to minus pi by 2 and theta 2 go to pi by 2 that way we obtained the same expression for magnetic field. And using Biot Savart law it was so easy to find out this expression for the magnetic field. Let us consider another example this time for a surface current density.

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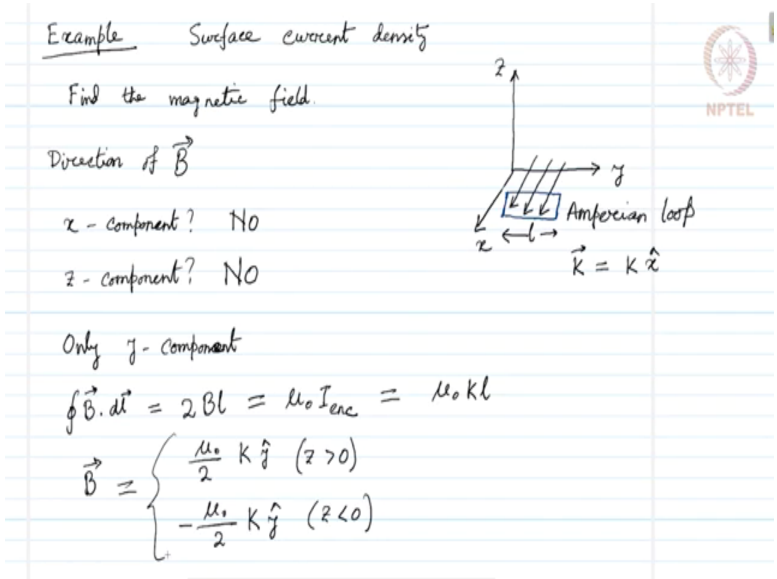

We have a Cartesian coordinate system like this; this is x axis y axis and z axis and we have current flowing in x y plane along x direction, this kind of a surface current is flowing. We want to find the magnetic field of a uniform infinite surface current, so this surface current is flowing over the entire xy plane that we have.

The surface current is not limited it extends to infinity and its constant in magnitude everywhere its uniform. It flows over x y plane and so the surface current density can be given as K equals K x cap K is constant everywhere and it flows along x direction. We want to find out the magnetic field for this kind of a current.

How do we go about doing this we have to find an Amperian loop let us try drawing an Amperian loop like this, let me draw in a different colour that encloses the current. So, we have this line above the plane, this line is below the plane and these two lines intersect the plane x y plane carrying current.

What would be the direction of the magnetic field can it have any x component, x component is not possible because the current is flowing along x direction so the answer is no. Magnetic field will be in the direction of K cross r cap the current direction cross the distance the displacement vector to the point of observation that would be the direction. Therefore, the any component along the direction of the current is not possible it will always be perpendicular to the current direction.

Can it have a Z component? We will have a point of observation that will be away from this plane on this plane we do not want to find the magnetic field we want so. The distance of that point from this current would be along Z direction. So, r cap would be along z direction or it will have certain Z component and if we consider this entire infinite plane then we will clearly see that the Z component will cancel from different parts of this plane.

Therefore, a component along the Z direction is also not possible, so it will only have y component Z component will cancel out from different parts of this infinite plane x component is ruled out because the current is flowing along x direction therefore, only y component is something that is left. And by drawing this kind of an Amperian loop we will have this upper line running along y direction lower line running along minus y direction. And these two arms that intersect the plane they are perpendicular to the y direction that is perpendicular to the magnetic field therefore, B dot dl would not find any contribution from the lines intersecting the x y plane.

Therefore, with this rectangular Amperian loop we can write down integration over a closed line B dot dl on this Amperian loop equals twice B l. Because the direction of B will also flip depending on whether we are above the x y plane or below the x y plane, because the direction of r cap that will change depending on whether we are above the x y plane or below the x y plane above it is positive z cap and below its negative z cap.

Therefore, the direction will change direction of r cap will change therefore, B will flip sign and that will always be in parallel with dl prime therefore, we will have twice B l instead of cancelling the contribution from each other. Now that will be equal to mu naught times the current enclosed. And what is the current enclosed, current enclosed would be if we consider l as the length of this Amperian loop, then the Amperian loop will enclose a current of mu naught times K times l.

So, we will have the expression for magnetic field as mu naught over 2 K y cap if Z is greater than 0 that is above this current carrying plane and it would be minus mu naught over 2 K y cap. When Z is less than 0 that is below this current carrying plane this is what we obtain for this kind of a situation.