

Electromagnetism
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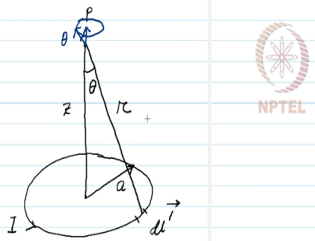
Lecture - 60
Biot Savart law with surface and volume currents

Hello, we have already learnt the Biot-Savart law and the situation with the force in between two parallel currents, we will consider an example of that kind of a situation now.

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Example

$d\vec{B}$

$$B(z) = \frac{\mu_0 I}{4\pi} \int \frac{dl'}{r^2} \cos \theta$$
$$= \frac{\mu_0 I}{4\pi} \frac{\cos \theta}{r^2} 2\pi a$$
$$= \frac{\mu_0 I}{4\pi} \frac{a^2}{(a^2 + z^2)^{3/2}}$$


So, we have a circular loop somewhere like this and this loop carries a current, let us say along this direction. And here is the centre of this loop here we are at distance z from the centre of the loop. And we are supposed to find the magnetic field at distance z above the

circular loop, the circular loop has radius a and it carries a steady current I . This is the situation we are trying to find out.

Now, if we consider a line element here dl prime, let us put vector sign on this. So the magnetic field that we have at this point of observation P due to this line element here on the loop that can be given as dB , let us write that the different the element of magnetic field and we can see that it will depend on this distance. If we call this distance curly r then we can see in which direction the magnetic field survives.

The horizontal component of the magnetic field would cancel because it will be summed over from different dl primes from the entire loop. So, the horizontal component will cancel and we will be left with only a vertical component.

And the vertical component of the magnetic field as a function of z if we integrate over all such dB 's, can be written as B_z equals μ_0 times the current over 4π integration over dl prime over r squared. But, we have to consider the vertical component. So, let us mark this angle as θ . So, this is the total amount of the magnetic field, but we are interested in the vertical components so, if we multiply this with cosine of θ that will give us the vertical component of this magnetic field.

That means, we want the magnetic field along this direction, let me use another colour for this, blue. The magnetic fields survives along this direction and if we consider this kind of a loop, then this angle that is generated is also θ . So hence, we get $\cos \theta$ here and because we have dl prime and this curly r vector, these are perpendicular to each other and we will have cross-product of it. Then we will get these as the vertical component. And if we perform this integral, we will find this to be $\mu_0 I$ over 4π , its integration over dl prime. So, cosine of θ remains as it is over r squared.

That also is not changed times the circumference of this circle. The circumference is twice πa , that makes the magnetic field equals, so, we have to represent this in terms of z and r square is not fine, we have to convert it into z . In order to do that, we can write this as $\mu_0 I$ over $4\pi a^2$ over $a^2 + z^2$ power $3/2$. We have use the

relation of cos theta in terms of z and a and this curly r and then this is the expression that we got for the magnetic field as a function of z, ok.

After working out this example, let us consider Biot-Savart law in the context of surface and volume current.

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Biot Savart law for surface and volume current

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}') \times \hat{r}}{r^2} da' \quad \text{Surface current}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \hat{r}}{r^2} d\tau' \quad \text{Volume current}$$

For a moving point charge

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2} \quad ? \quad \times$$

If we have a surface current then the magnetic field can be expressed as mu naught over 4 pi integration over the surface current density K, that is the function of r prime that is the source coordinate system cross r cap over r squared d a prime. This is for surface current, if we have a volume current then the magnetic field can be expressed as a function of r as mu naught over 4 pi integration over J r prime cross r cap over r squared d tau prime for volume current.

There is something important to note, from surface current and volume current if we want to find the total current I , we integrate surface current over line and we integrate volume current density over some area, the cross-sectional area. And we any way in order to find the magnetic field integrate $I \times r \text{ cap}$ over $dl \text{ prime}$. So, the here this $da \text{ prime}$ that is area element for in the context of surface current and volume element in the context of volume current that keeps everything dimensionally fine, dimensionally same.

Now if we consider a moving point charge, what would be the amount of magnetic field? Can we write the magnetic field B as a function of r as $\mu \text{ naught over } 4 \pi q v \times r \text{ cap over } r \text{ squared}$, is that possible? If you think carefully, if there is a point charge of magnitude q and that moves with the velocity v , it does not give rise to a steady current. And since it does not give rise to a steady current, we cannot apply Biot-Savart law for finding out the magnetic field for that kind of a situation because, Biot-Savart law is strictly applicable for steady currents. So, this does not work. This is wrong.