Electromagnetism
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Lecture - 06
Curl

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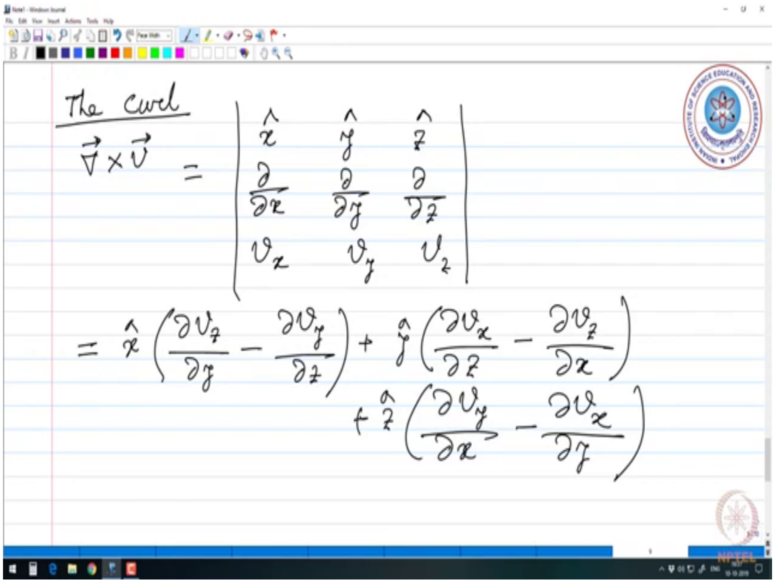

Now, let us consider the cross product that is the Curl, the curl is written as del cross v and is also represented just like the cross product operator in the determinant form x cap y cap z cap del del x del del y del del z v x v y v z. This is the determinant form which is given as if we work out the determinant we will find x cap del v z del y minus del v y del z plus y cap del v x del z minus del v z del x plus z cap del v y del x minus del v x del y.

This is the form of curl and now let us consider the geometric interpretation of curl that is quite interesting. Curl means the whirlpool of a vector. What do we mean by that? Let us consider few examples and try to clarify this.

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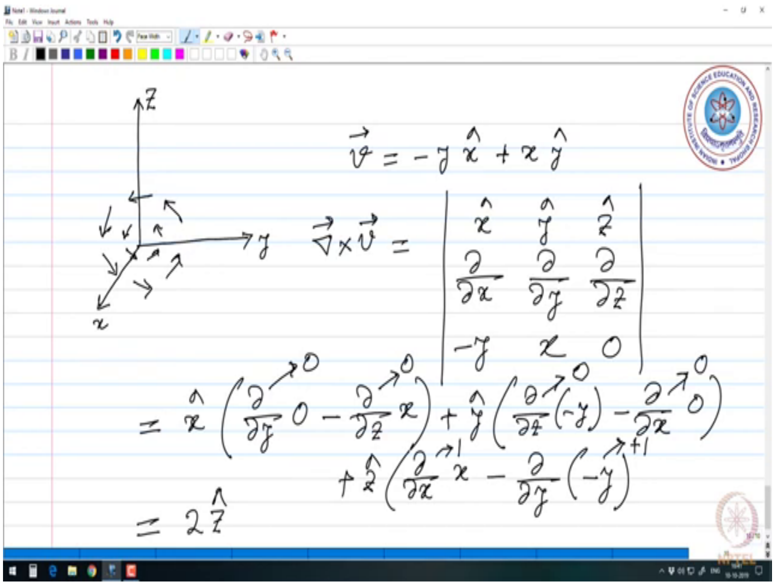

Let us first draw a picture of a Cartesian coordinate system and a vector field in this system. So, this is the x direction, this is y and this is z; as we mentioned curl is a whirlpool of a vector. If we have a vector field that points along the arrows that I am drawing now, we can clearly see that it will have a curl. Let us work this example out. If we have this kind of a vector field, the vector field can be represented as minus y x cap plus x y cap.

Now, if we work out the curl for this vector field, this can be given as the determinant x cap y cap z cap del del x del del y del del z minus y is the x component of it, x is the y component of the vector and the z component is 0; so, this determinant. And, if we work out the determinant it turns out to be x cap del del y of 0 minus del del z of x plus y cap del del z of minus y minus del del x of 0 plus z cap del del x of x minus del del y of minus y.

Now, if we work it out we will see that this quantity goes to 0, here we are taking a partial derivative of x with respect to z. So, this will go to 0 clearly, here we are taking a partial derivative of z sorry y with respect to z this will go to 0. Here we are taking derivative of a constant this goes to 0, this quantity sustains and becomes 1 because it is a partial derivative of x with respect to x.

This becomes minus 1 and then in front of that there is a minus sign so plus 1 and as a result the curl becomes 2 z cap at everywhere in the Cartesian space, that is what we find in this case ok.

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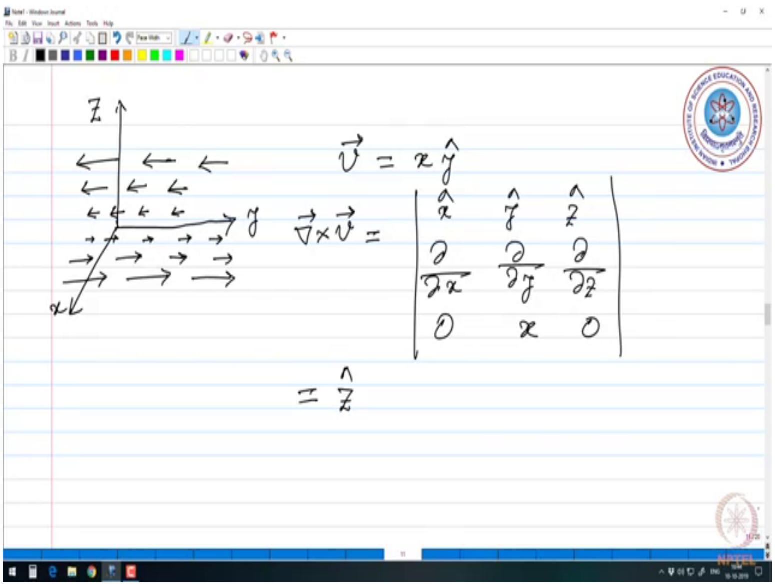

Let us consider another vector field where we will have again a Cartesian coordinate system. This is x direction, this is y direction and this is z direction and this time the vector field looks like this. And, when we reverse x this changes sign, when we go to negative x this thing changes sign something like this. This vector field can be described with the expression vector v is given as x y cap because, y cap is the direction of the vector field.

And, when x value is positive it points along y cap, when the value of x is negative it points opposite to y cap. Now, let us try to evaluate the curl of this vector. So, curl of v in its determinant form can be written, it is very simple. And, although the whirlpool of the vector field is not clear from this picture at all, if you work this determinant out you will get the curl to be z cap.

So, this vector field does have a non-zero curl and; that means, the vector field is has a finite whirlpool that is of z cap amount at every point of it, that is something interesting to see. And, this is how we can also understand the geometric meaning of curve that is very important. Now, let us consider few product rules in case of vector calculus, vector differential calculus.

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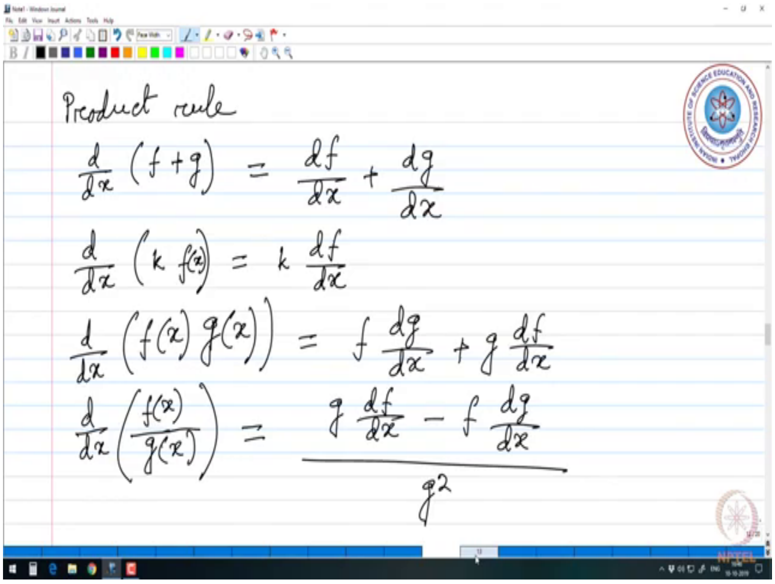

In case of ordinary derivatives if you do not have any vector involved, then the then there are few rules of addition of additional sorry, please remove this later. When we have we considered ordinary derivatives then we have few rules for the derivatives that is if we have d d x of f plus g two functions, that becomes d f d x plus d g d x. We have also product rule d d x of a constant multiplied by the function of x.

This is the constant comes out and then d f d x we are all aware of it and if we take the derivative of the product of two vectors, that is f x times g x we get f d g d x plus g d f d x. And, if we have the quotient then the quotient rule comes that is if we have the vector f divided by sorry the function of x that is f x divided by the function of x that is g x. If we have this kind of a situation we get g d f d x minus f d g d x divided by g squared, that is the numerators sorry the denominator squared.

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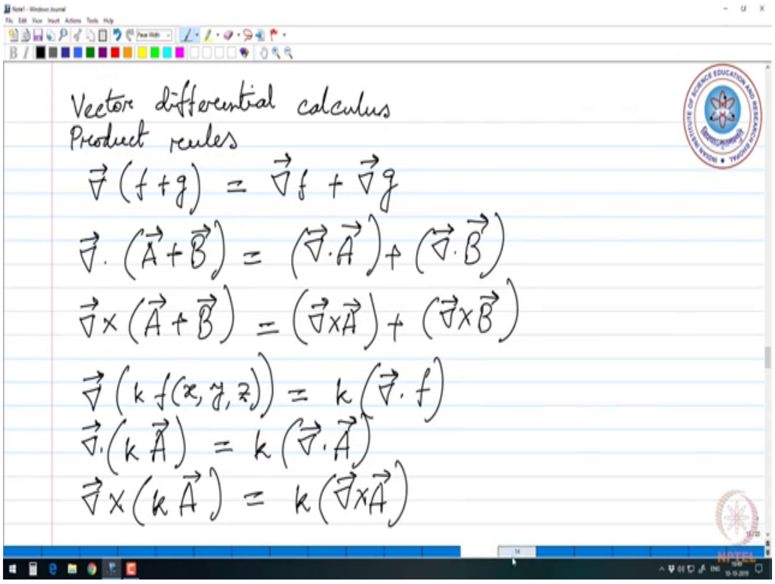

So, there are similar rules for vector differential calculus. The gradient of the sum of two scalar fields, this is given as the sum of the gradients of the scalar fields this. The divergence of the sum of two vectors is given as the sum of the divergence of the two vectors. And, the same thing holds for curl as well the curl of the sum of two vectors can be given as curl of A plus curl of B.

Now, comes the product rules if we have the if we take the gradient of a constant multiplied by a scalar field that is given as the constant comes out multiplied by the gradient of the scalar field. And, the same thing holds for divergence; so, the divergence of a constant multiplied by a vector is given as constant multiplied by the divergence of that vector. Same thing holds for curl as well; obviously, curl of k times A vector is given as k times curl of A vector.

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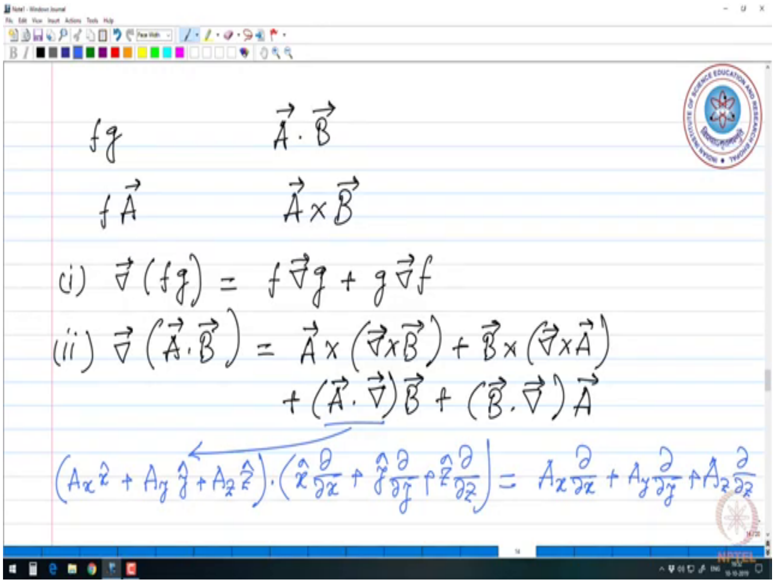

Now, if we have the product that leads to a scalar; that means, product of two scalar fields or the scalar product of two vector fields there are certain rules for this, for vector differentiation. And, there are other kind of products that lead to vectors; that means, there is a scalar field multiplied by a vector field or the cross product between two vector fields.

There are different product rules for that, for the first case we have the gradient of the product of two scalar fields that is given as if gradient of g plus g gradient of f just like what we have seen in case of scalar differential calculus. And, the gradient of the dot product of two vectors, this is given as A cross the curl of B plus B cross the curl of A plus A dot del B plus B dot del A. In this context it is important to understand what is these things like A dot del.

So, if A is written as A x x cap plus A y y cap plus A z z cap and del is given as x cap del del x plus y cap del del y plus z cap del del z. Then this operation gives us an operator itself a scalar operator that is A x del del x plus A y del del y plus A z del del z. This is an operator and we cannot really commute this dot product in the other way around; that means, we cannot really bring this del operator to the left and take the a vector to the right that is not allowed. So, we end up with a scalar differential operator as a result of this dot product ok.

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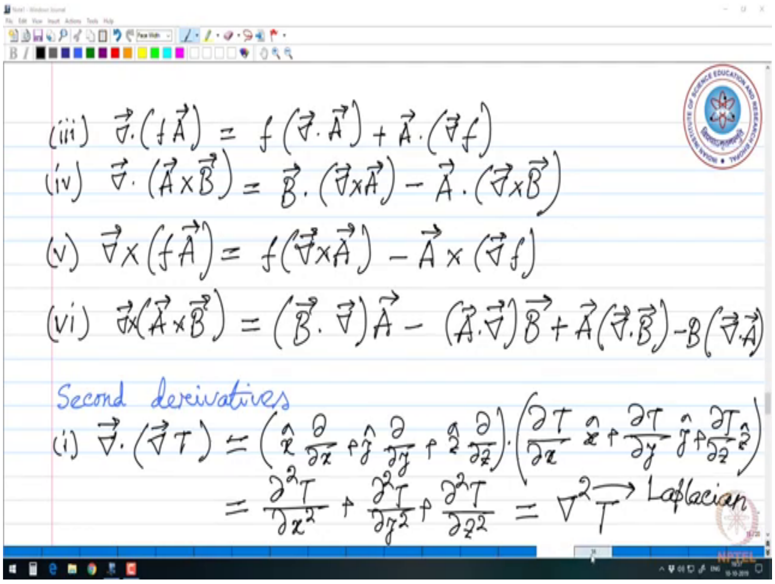

Then there are similar cases for divergence, similar rules for divergence, divergence of a vector field multiplied with a scalar field that is given as; if divergence of A plus A dot the gradient of this scalar field. And, as a result of this we get a scalar quantity out of this divergence. The other rule for divergence is the divergence of A cross B that is given as B dot curl of A minus A dot curl of B.

And, the expressions for the curls can be given as curl of f A is f curl of A minus A cross gradient of f. And, the curl of a cross product is given this way curl of A cross B, this quantity is B dot del A minus A dot del B plus A del dot B minus B del dot A. Now, if we go for the second order derivatives, then we will get the following rules.

The divergence of the gradient of a scalar, we can calculate this one and we will find that this quantity becomes x cap del del x plus y cap del del y plus z cap del del z dotted with del T del x x cap plus del T del y y cap plus del T del z z cap. And, this will be del 2 T del x 2 plus del 2 T del y 2 plus del 2 T del z 2; as expected the divergence gives us a scalar and this is in a short hand notation written as del square of T.

This del square operator is also known as the Laplacian. Remember del square is a scalar operator and hence we did not put any arrow on top of it. Now, we can consider the curl of a gradient.

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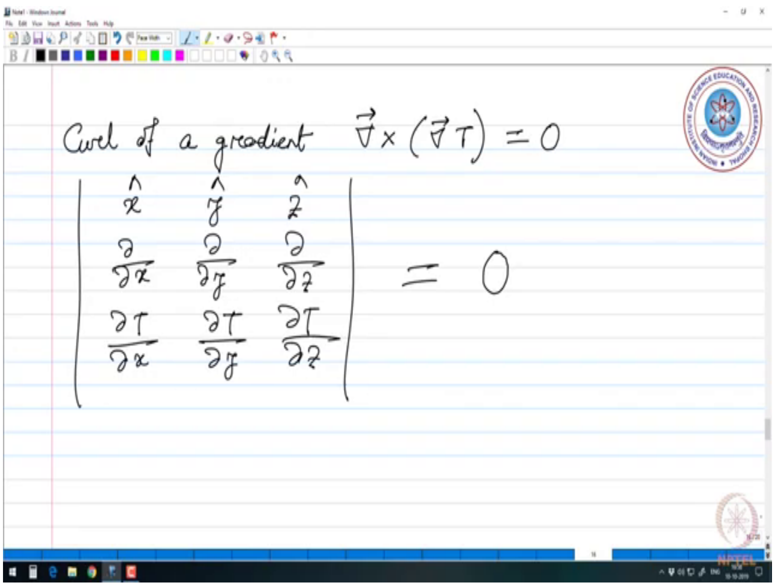

So, this quantity can be expressed in the determinant form as x cap y cap z cap del del x del del y del del z and del T del x del T del y del T del z, the determinant of this which is if you work out carefully you find it must be 0; that implies that the curl of the gradient of a scalar is always 0. And, this result is very important, we will make use of this result in defining scalar potential for conservative force fields ok.

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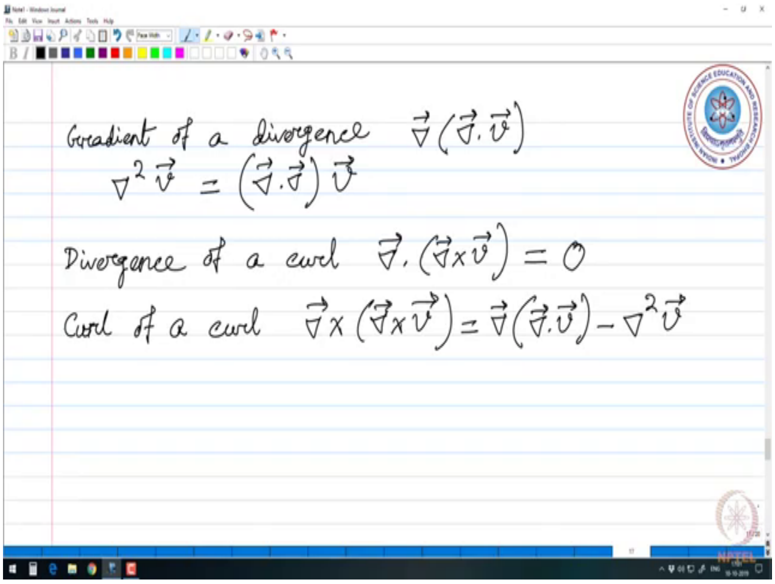

Then considered let us consider the other kind of operation, the gradient of a divergence. If we consider the gradient of a divergence; that means, the gradient of the divergence of vector v, this quantity can be evaluated as. So, this will be del square of the vector which is nothing, but del dot del of the vector v. There can be other kind of operations that is divergence of a curl, this is expressed as the divergence operator of on acting on the curl of v.

And, we can also show that this quantity goes to 0 and this is also very important because we can make use of this result to define vector potential. And, then comes the curl of the curl of a vector, that is written as curl of curl of v. And, we can show that this can be written as the gradient of divergence of v minus the Laplacian operator acting on the vector v. So, these are the expressions that we need for vector differential calculus. And, we will next we will do some examples related to the vector differential calculus.