Electromagnetism
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Lecture – 59
Biot Savart law

After discussing continuity equation, let us move on to finding the magnetic field for a given current and the law. So, this is like the coulombs law in the context of magnetostatics and this is known as the Biot Savart law.

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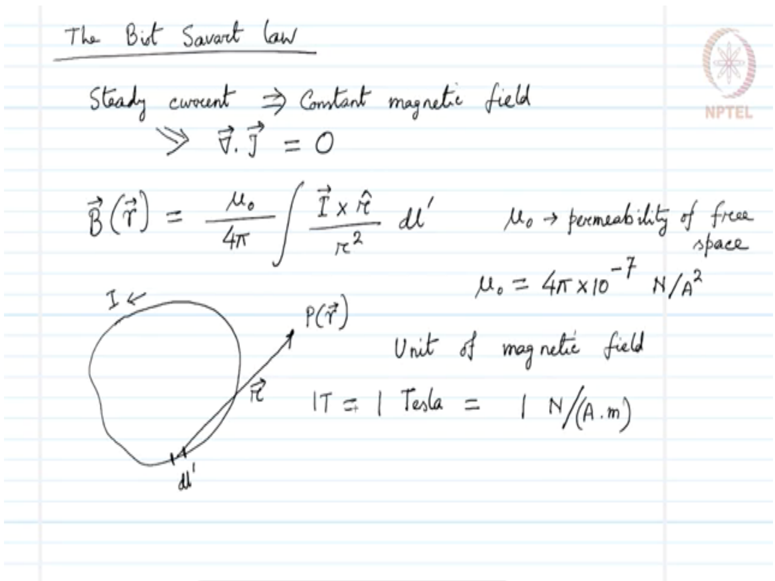

If we have a station if we have stationary charges we get constant electric field. And similarly if we have steady current that will lead to a constant magnetic field and forced what do we mean by steady current steady current means that divergence of the volume current density thus that goes to 0.

And in this kind of a situation the magnetic field in it is vector form as a function of the position vector can be expressed in terms of the current as mu naught over 4 pi integration current cross r cap over r squared dl prime.

Now, I will have to explain what it means let us consider a current carrying loop like this; let us consider the current I is flowing in this direction and we have a point of observation here, with position vector r and here we consider a line element dl prime and this vector is r vector in this context we can write this equation where we have mu naught is known as the permeability of free space; magnetic permeability of free space. So, the value of mu naught is 4 pi times 10 power minus 7 N per A squared. What is the unit of magnetic field? Unit is Tesla, which is 1 Tesla equals 1 Newton per ampere times meter; Tesla is also written as T.

Now, let us see an example of how to calculate the magnetic field using Biot Savart law.

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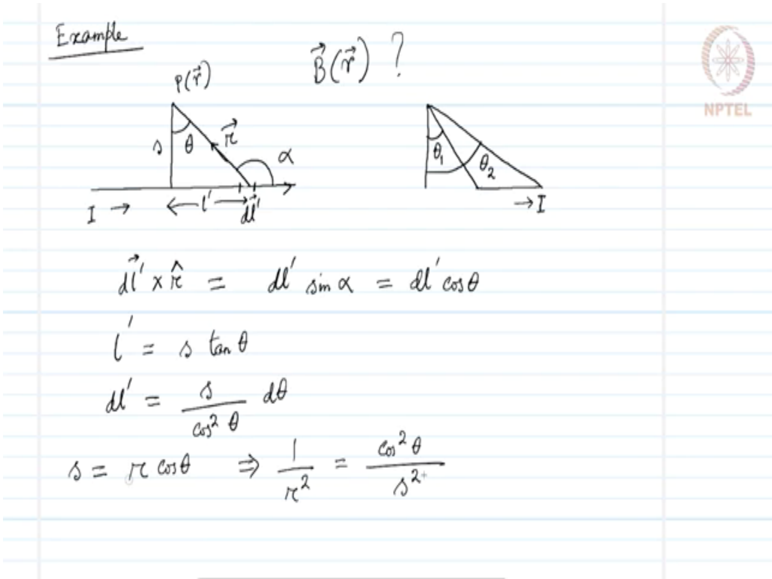

Let us consider a current on a straight line like this, this is a wire and it is carrying steady current this wire is very long. So, we will assume it well the wire is not very long please delete that part. So, this wire is carrying current amount I.

And we are interested in finding the magnetic field at a point here our point of observation marked as P at position vector r that is at a distance s from this current carrying wire. How do we find that? We want to find B at the position vector r; that means, at the point of observation P. So, let us consider an element of line that is element of current here it is dl prime.

And the distance of our point of observation from that line element. Let me try drawing a better line is r this is the vector and let us say this angle is theta, and this angle is alpha. If we have this arrangement then we can try finding the magnetic field due to any segment of the line.

If we consider this segment and if we consider our point of observation here; this is the perpendicular distance say this angle is theta 1 and this bigger angle is theta 2. So, we can find for any theta 1 and theta 2 the magnetic field due to this current I ok. How do we do that? In this diagram, we can see that I cross r cap. So, r cap is along this direction and the current is along r cap is along this direction here and the current is along this direction here.

So, I cross r cap that points out of the screen I because I cross r cap points out of the screen that will be the direction of this magnetic field. And the magnitude of the magnetic field would be r cap cross d dl prime cross r cap. It will it is magnitude is dl prime sine alpha that equals dl prime cosine of theta the direction points outside of the screen.

Now, if we write l prime as s times tangent of theta; that means, this length is l prime. Then we can write dl prime as s over cosine square theta d theta. With this expression we can write with the geometry we can write s equals r times cosine of theta that implies 1 over r squared equals cosine squared theta over s squared.

So, we have everything that goes into the expression for the magnetic field. Now it is our job to calculate the magnetic field.

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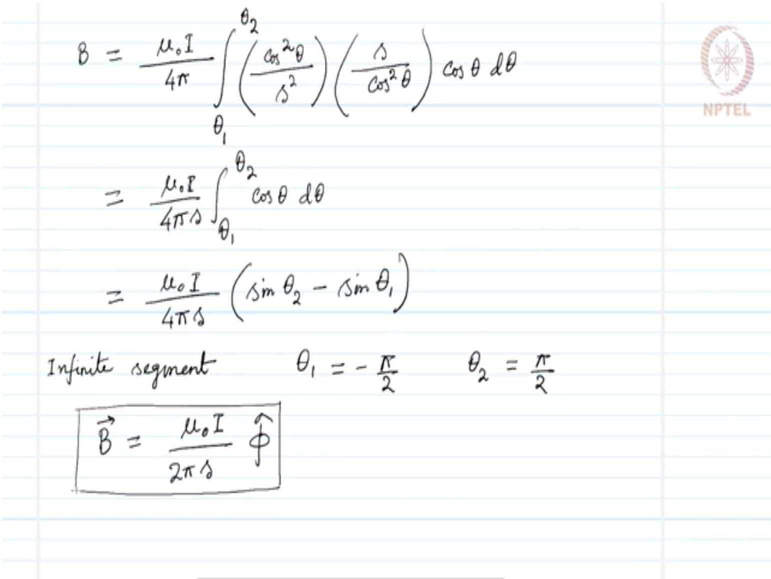

Let us do that we have to integrate the magnitude of b will be given as mu naught I over 4 pi integration, if we consider the line segment with angle that makes angles theta 1 to theta 2.

Now, this these theta 1, theta 2 could be any arbitrary angle and you can cover any length of the wire by adjusting these two angles. This will give us cosine squared theta over s squared times s over cosine squared theta times cosine of theta d theta and that is nothing, but mu naught I over 4 pi integration theta 1 to theta 2 cos theta d theta.

And this integrates to be mu naught I 4 and yeah. So, we missed one s earlier here we had 1 over s square here in the numerator we had s. So, in the denominator we will have s and s comes out of the integral because integration is over theta we have mu naught I over 4 pi s sine theta 2 minus sine theta 1 this equation gives the magnetic field for any straight wire segment in terms of theta 1 and theta 2.

Now, if we consider an infinite segment of an wire; of a wire. For an infinite segment we will have theta 1 equals minus pi over 2 and theta 2 would be pi over 2. Now if we put this then the magnetic field becomes mu naught I over twice pi s and what is the direction? The direction in this case in our cylindrical coordinate system will be along phi cap.

Because if we consider this situation here it will the magnetic field will point out of the screen and out of the screen is phi cap direction. So, this is the expression of the magnetic field for this kind of a system that we have calculated using Biot Savart law and integrating over the line element through which the current is passing.

Now, if we consider 2 parallel wires that are carrying current I 1 through 1 wire and I 2 through another wire.

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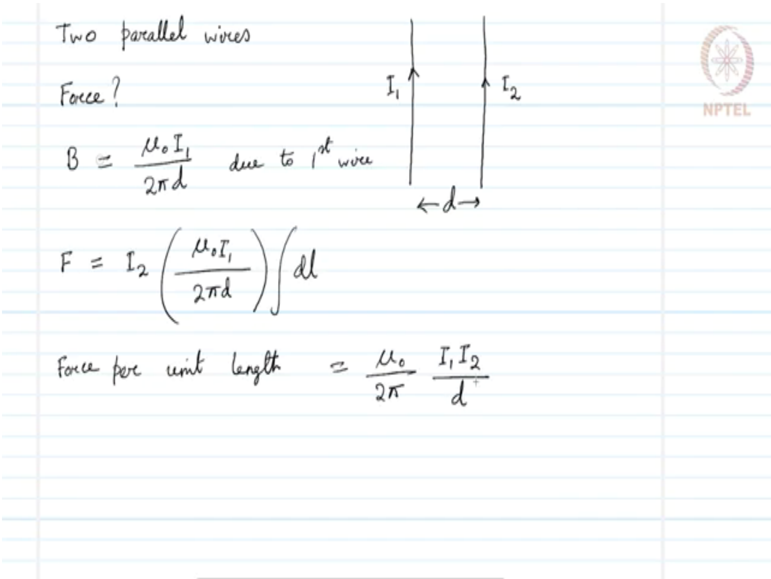

We have 2 parallel wires carrying current this is I 1 and this is I 2. We are supposed to find the force between these 2 currents. The magnetic field on the second one, due to the first wire; on the second wire due to the first wire can be given as mu naught I over 2 pi times.

The distance between these two wires let us call it d this is due to the first wire. So, we have I 1 here. So, the force on the second wire is I 2 times mu naught, I 1 over twice pi d integration over dl. Now, the total forces; obviously, infinite if we have infinite extent of these wires the force per unit length that can be given as mu naught over twice pi I 1 I 2 over the distance between these 2 currents.