Electromagnetism
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Lecture - 58
Surface and volume current

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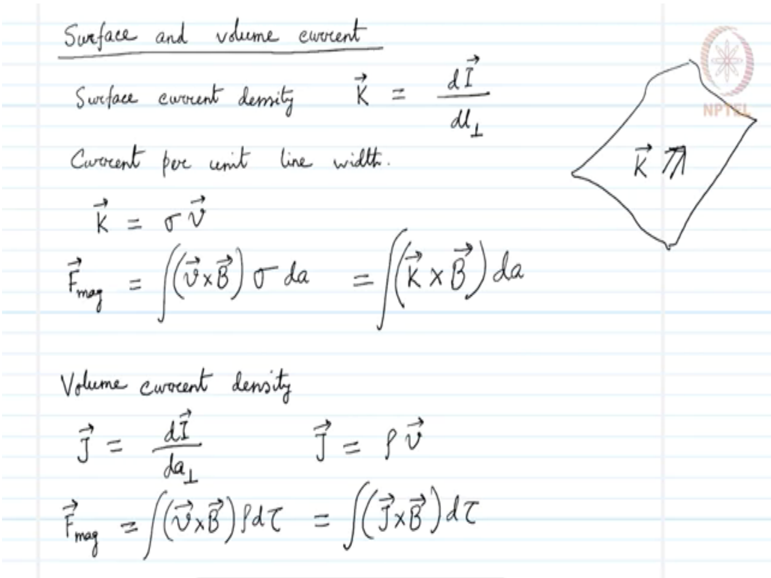

Let us now consider the concept of surface and volume current. So surface current density is represented as K vector which is nothing but if we have a surface like this, then it so the charges are restricted on this surface to move and they move along this direction that gives us this current density K which is in terms of the current the total current I dI dl perpendicular to the dire; dl is along the perpendicular direction to this current.

In other words, surface current K is the current per unit line width. So we can get the surface current density, the expression for the surface current density from the surface charge density as K equals sigma times v. Therefore, the magnetic force for a surface current density, can be given as v cross B; v is the velocity of the charges times sigma and over an infinite decimal surface area integrated over that entire area this would be the surface current density, which is, sorry this would be the magnetic force which is equal to so, v times sigma is K as we have seen the here, cross B da integrated over this.

And, the volume current density this is represented as J which is the differential of the total current to the perpendicular area. Now, the cross section through which the charges are flowing the perpendicular flowing perpendicularly; that means, the volume current density J can be represented as the volume charge density times v. Similarly, to earlier magnetic force can be expressed as v cross B rho d tau integrated over which is nothing, but J cross B d tau integrated over.

Now, in the context of line current for magnetic force we have written down that the here. We have written down also an alternative expression for the magnetic force that is I times dl vector cross the magnetic field vector. For surface current and volume current, that is not possible. That kind of expression is not possible we are forced to consider surface current density and volume current density as vectors. There is no scalar interpretation for surface and volume current density. Let us consider an example to understand this better.

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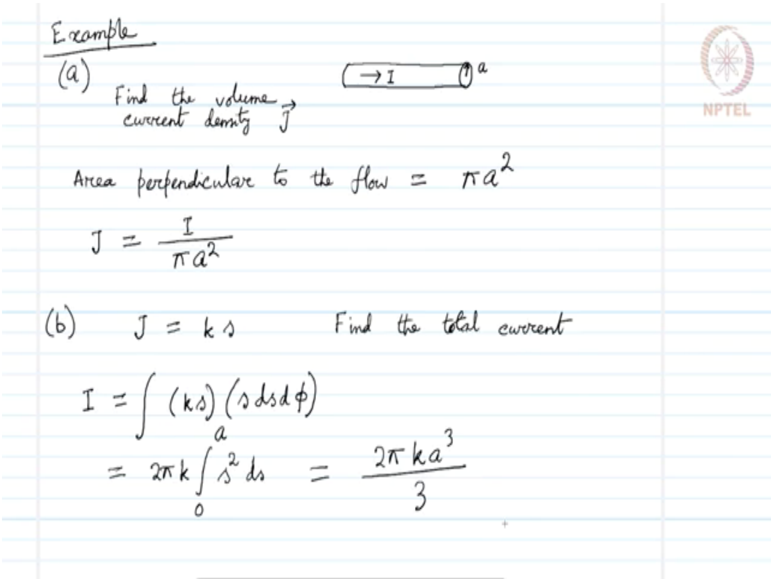

Let us consider a current I is uniformly distributed over a wire of circular cross-section. So, we have a wire of circular cross-section like this. Cylindrical wire and the current I is flowing and this current is uniformly distributed over this cross-section. Now, the radius of this cross-section is a. We are supposed to find the volume current density J. How do we find this? Because its uniformly distributed and the area perpendicular to the flow is pi times a squared. So, the volume current density can be given as I over pi a squared.

And let us consider similarly, another example. Suppose the current density in the wire is proportional to the distance from the access. For the same wire, we have a current density, but this time the current density is not uniformly distributed over the cross-section rather, we have this expression J equals a constant K this is not the surface current density just a constant, multiplied by s, s is the distance from the access. The cylindrical s coordinate system, cylindrical s coordinate usual one.

Now, we are supposed to find the total current. How do we go about doing this? Current as we have defined earlier, would be integration over the volume current density times da and on this cross-sectional area. So, the cross sectional area in this cylindrical coordinate system can be given as s d s d phi.

So we will have to perform this integral and that gives us. Because there is azimuthal symmetry, we have the volume current density a function of only s not phi. We can write we can already perform the phi integral and get 2 pi outside, K is a constant, goes outside integration is from 0 to the dimension of the wire that is a s square d s. And this integrates out to twice pi k a cubed over 3.

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$$I = \int_S J \, da_\perp = \int_S \vec{J} \cdot d\vec{a}$$

$$\oint_S \vec{J} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{J}) \, d\tau$$

Charge is conserved

$$\int (\vec{\nabla} \cdot \vec{J}) \, d\tau = -\frac{\partial}{\partial t} \int \rho \, d\tau = -\int \frac{\partial \rho}{\partial t} \, d\tau$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}} \quad \text{Continuity equ}^{n}$$

So, this is the total current density. Current I that is in general, given as surface integral over J da perpendicular which is also surface integral over J dot da. Ok, now if we consider a closed surface integral, a surface enclosing a volume over J dot d a. What are we suppose to get? We are supposed to get, applying the divergence theorem, a volume integral of the divergence of this volume current density J times d tau. And, we know that the charge is conserved.

So, the amount of charges flowing that comes from a source and the total charge always remains unchanged. With the charge being conserved, whatever follows out must come in and it at the it is at the expensive of what remains inside. So, the volume integral over the divergence of J that can be written as minus del del t of integral over rho d tau.

And that is equal to so, we have a time derivative and integral over space, that is volume. So these 2 quantities are commutative. we can take the time derivative inside, that is equal to minus integral of del rho del t d d tau. That means, this will this expression will hold good provided the argument, that is the kernel of the integral divergence of j and del rho del t are equal minus del rho del t

So, we will have divergence of the volume current density is negative of del rho del t in a given volume. And this equation is very important. This is nothing but the conservation of charge, this in known as the continuity equation.