

**Electromagnetism**  
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**Lecture – 54**  
**Force on dielectric materials**


Hello, we were discussing about materials mainly dielectric materials under the influence of electric field and we have discussed about the energy in that kind of a situation. Now, let us discuss about what is the Force that a dielectric material experiences being subject to an electric field.

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Forces on dielectrics

Parallel plate capacitor

Fringing field 



$\oint \vec{E} \cdot d\vec{l} = 0$

Let  $W$  be the energy of the system

$dW \rightarrow$  Infinitesimal work to pull out the dielectric material infinitesimally.

$dW = F_{me} dx$   
 $\quad \quad \quad \downarrow$   
 $\quad \quad \quad$  mechanical force

$F_{me} = -F$

Electrical force  $F = -\frac{dW}{dx}$

$W = \frac{1}{2} CV^2 \quad C = \frac{\epsilon_0(\text{width})}{d} (\epsilon_{rl} - \chi_e x)$

Just as a conductor is attracted into an electric field, a dielectric material is also very similar, it also gets attracted into an electric field. Let us consider a simple example for understanding what we are talking about, let us consider a parallel plate capacitor.

Let us try to draw this, this is one plate, this is the other plate and they are parallel to each other. And if we consider finite size of this parallel plate capacitor then in at the middle of this capacitor of course, the electric field is perpendicular to the plates, but if we come to the edge that is not necessarily true, the electric field is not perpendicular to the plates.

We will see some fringing fields there, near the edge. And how do we see that? If we consider a loop at the edge here, then half of the loop is outside this parallel plate capacitor and we want to calculate contour, we want to calculate a closed integral  $\oint \mathbf{E} \cdot d\mathbf{l}$ . So, this integral encloses the surface.

If we try doing that, then we will see that we will be able to prove that there exist fringing field and near the edge, it is the electric field is not exactly perpendicular and uniform to the parallel plates.

So, fringing fields are notoriously difficult to calculate we cannot really find out exactly how those fringing fields are there. And so we need to avoid calculation of the fringing field, we cannot exactly find any expression for this fringing field. If we consider  $W$  to be the energy of the system; if we consider  $W$  to be the energy of the system, Then it depends on the configuration of the entire system.

And if we consider a dielectric material in between these two parallel plates in the shaded region. And now if we try to pull out this dielectric material along this direction, then we will have to apply some force. How do we calculate this force?

So, we need to do some work in order to bring this dielectric material out infinitesimally out and if we consider that the energy that we need to supply the work that we need to perform in order to bring out this dielectric material infinitesimally from the parallel plates. And if we

consider that infinitesimal energy is  $dW$ ;  $dW$  is the infinitesimal energy or infinitesimal work to pull out the dielectric material of course, infinitesimally.

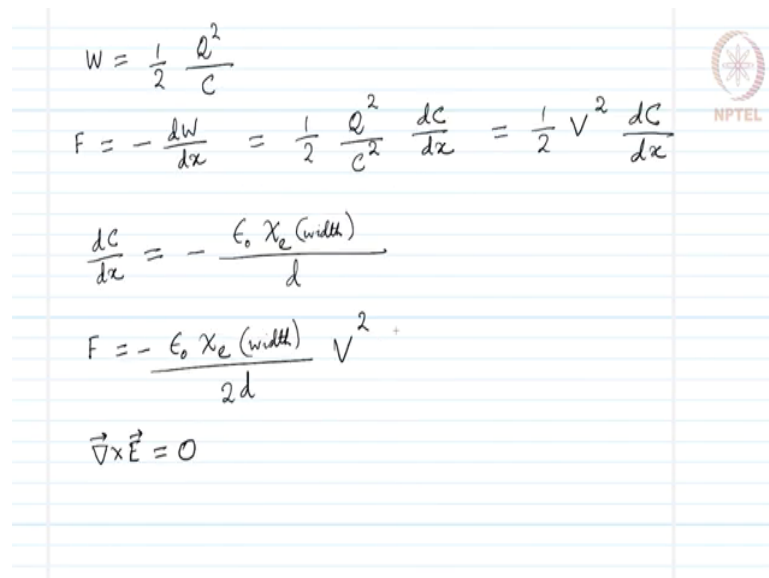
Then we can write the expression for  $dW$  as the force mechanical force times  $dx$ ;  $dx$  is the infinitesimal displacement of this dielectric material. So, this is the mechanical force. Now, according to balancing of the force this mechanical force will be exactly opposite equal and opposite to the electrical force that tries to bring in that dielectric material in between the capacitors.

So, the electrical force on the system on the dielectric material that can be given as  $F$  is equals minus  $dW/dx$  because its equivalent opposite to the mechanical force. Once we get this then we know the energy stored in a parallel plate capacitor that is half  $CV^2$  and once we have this we also know the expression for the capacitance that is given as  $\epsilon_0 \epsilon_r \frac{A}{d}$  or let me write the width completely.

This is the width of the dielectric material, width of the parallel plate capacitor over  $d$ ,  $d$  is this distance between the plates times  $\epsilon_r$  is the relative permittivity,  $l$  is the length of this parallel plate capacitor that is same as the length of the dielectric material minus  $\chi$  times  $x$ , if we take  $x$  distance outside. If we have the dielectric material hanging outside of this parallel plate capacitor by an amount  $x$ , then this is that  $x$ .

With this expression for the capacitance we can write. So, if we pull out this dielectric material what is going to remain conserved? It is not the voltage that is going to remain conserved. The voltage is going to change if we bring in the dielectric material out because the total charge, the charge on each plate will remain constant and if the charge on each plate remains constant and the effective dielectric constant has changed the voltage will change.

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$$W = \frac{1}{2} \frac{Q^2}{C}$$
$$F = -\frac{dW}{dx} = \frac{1}{2} \frac{Q^2}{C^2} \frac{dC}{dx} = \frac{1}{2} V^2 \frac{dC}{dx}$$
$$\frac{dC}{dx} = -\frac{\epsilon_0 \chi_e (\text{width})}{d}$$
$$F = -\frac{\epsilon_0 \chi_e (\text{width})}{2d} V^2$$
$$\vec{\nabla} \times \vec{E} = 0$$

So, we will have to write this expression for energy stored in the dielectric, energy stored in the capacitor in terms of the conserved quantity namely total charge that is half Q squared over the capacitance.

So, now we can find out the force that is given as minus d W dx equals half Q squared over C squared times dC dx. Q squared over C squared is the potential equals half V squared dC, Q squared over C squared is the potential squared actually. So, we have V squared here and if we write now dC dx that is minus epsilon naught chi e times the width over the separation between the plates.

Then we can write the force equal to minus epsilon naught chi e times the width over 2 d V squared, the minus sign indicates that the force is in negative x direction if the dielectric material is pulled out in the positive x direction. If we consider that V remains constant we

will get wrong result as we have already discussed  $V$  does not remain constant if we bring out this dielectric material.

Now, we have calculated this force without really explicitly considering the fringing field the effect of the fringing field, but this is the effect of the fringing field we did not have any expression for the fringing field in our calculation. And this is possible because electric field is a conservative force field; that means, curl of the electric field is always 0.

So, what happens if the curl is 0? We get here integration  $E \cdot dl$  going to 0 and if integration  $E \cdot dl$  goes to 0, we can actually write that the force that we need to. So, then we can balance these two forces the mechanical and the electrical force and that way we can calculate the force on this dielectric material from this situation without explicitly knowing the expression for the fringing field. So, we already got the expression for the force.