Electromagnetism

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Lecture - 51

Boundary conditions on the displacement vector and linear dielectric materials

We were discussing about electric field in a material and in the by the word material we mean; a dielectric material in this context because, in the dielectric material we will have a polarization developed in the material. If we have a conductor then, there will be no electric field and we have already understood how a conductor behaves under the influence of electric field.

So, we have already developed a concept of polarization, the bound charges and how electric field is modified given the existence of this dielectric material, polarized dielectric material and we have also developed the concept of a displacement vector D. And we have modified the Gauss law for accommodating this displacement vector D and after doing all these let us see; what are the boundary conditions across a surface if we have a dielectric material that is polarized.

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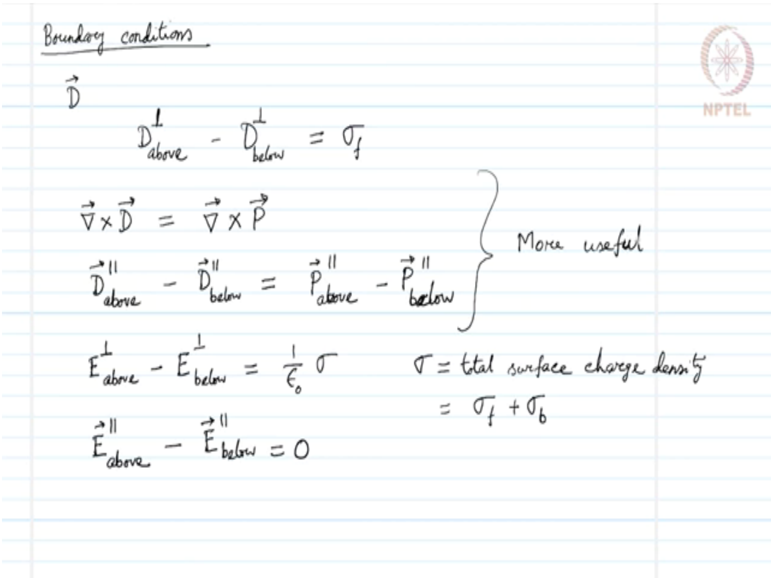

So, if we have the vector displacement vector D, then we can write down the boundary condition in terms of the displacement vector. So, the charge that is relevant to in the context of displacement vector and we have found out it is the free charge not the bound charge that comes due to polarization. So, in terms of the displacement vector; we can write that across a surface free surface charge density sigma f. We can have the displacement vector perpendicular component above the surface minus the displacement vectors perpendicular component below the surface, that is equal to the surface free charge density.

We can easily derive this from whatever we have derived earlier in the context of electric field and the surface charge density. So, then what we do is; we write down the curl of the displacement vector and that as we have found out earlier is equal to the curl of the polarization vector. And that means that; the parallel component of the displacement vector above the surface charge density minus the parallel component of the displacement vector below the surface charge density earlier for electrostatic field, this quantity was 0 because curl law of electrostatic field was 0.

But now, that the curl of displacement vector is non zero; this is going to be the difference between the polarizations above and below. The parallel component of the polarization above and below. The difference between that is the difference between the parallel component of the D field, the displacement vector above and below the surface charge density. And in the presence of dielectrics these are sometimes more useful than the corresponding boundary conditions on electric field, that we have written down earlier and that holds good even in the presence of a dielectric material.

So, let us write that down in case of electric field we wrote that the perpendicular component of the electric field above minus the perpendicular component of the electric field below, that equals 1 over epsilon naught times sigma. Now this sigma is the total charge density, total surface charge density. That means, in the presence of a dielectric material; this would be sigma free plus sigma bound.

And for the parallel component because the curl of the electric field is 0 therefore, E parallel above minus E parallel below will still be 0. Now, we are saying that these equations are valid for electric field they these equations are also valid, but this one is more useful in the context of a dielectric materials. The other one has limited use. After this, let us introduce the concept of linear dielectric materials.

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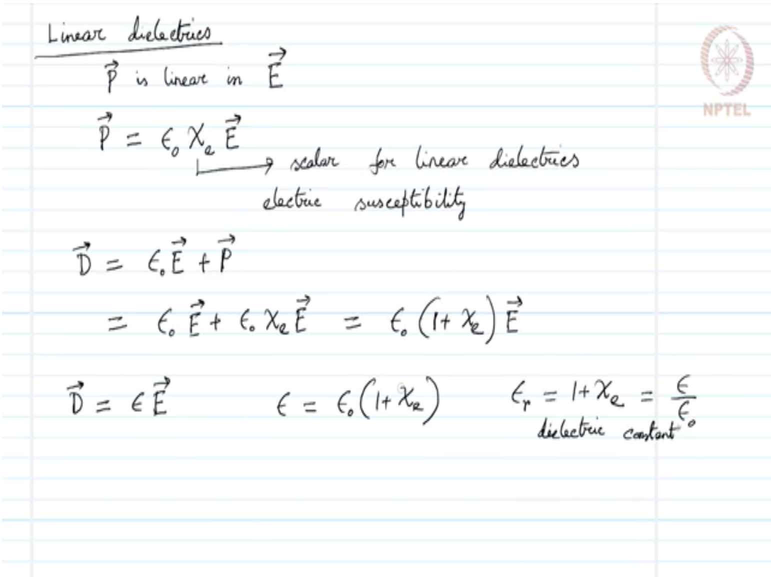

So, what is linear dielectric material? That means; the polarization would be linear in the electric field and if that happens; we can write down polarization as, so polarization is the response of the material when subjected to an electric field and it can be written as epsilon naught chi e times the electric field. Where chi e is a scalar for a linear dielectric material and it is a constant for that material, this is called the electro electric susceptibility. And we have defined the displacement vector as epsilon naught times the electric field plus the polarization vector.

Now, this polarization vector can be we can put this expression for linear dielectrics for the polarization vector and we will get the displacement vector equals epsilon naught E plus epsilon naught chi e times the electric field and that would mean; this is equal to epsilon naught 1 plus chi e times the electric field. In general for a non-linear material chi e would make a tensor. It is not necessary that it is the response of the material is linear in the electric field it is also not necessary that the response is in the direction of the electric field. It may be along different directions.

And in case of linear dielectric; we define linear dielectric in such a way that the response of the material that is polarization is along the direction of the electric field and chi e is a scalar is a constant for that material. That is how we define a linear dielectric material for which the displacement vector is also along the direction of electric field and it is just a constant times the electric field for only for a linear dielectric material not otherwise.

So, let us write the displacement vector as epsilon times the electric field, where epsilon is given as epsilon naught 1 plus elect the electric susceptibility. And we also define another quantity dielectric constant. Dielectric constant is 1 plus the relative susceptible there is a electric susceptibility and this equals epsilon over epsilon naught, we call it the dielectric constant. Let us consider an example to understand the concept of a linear dielectric material.

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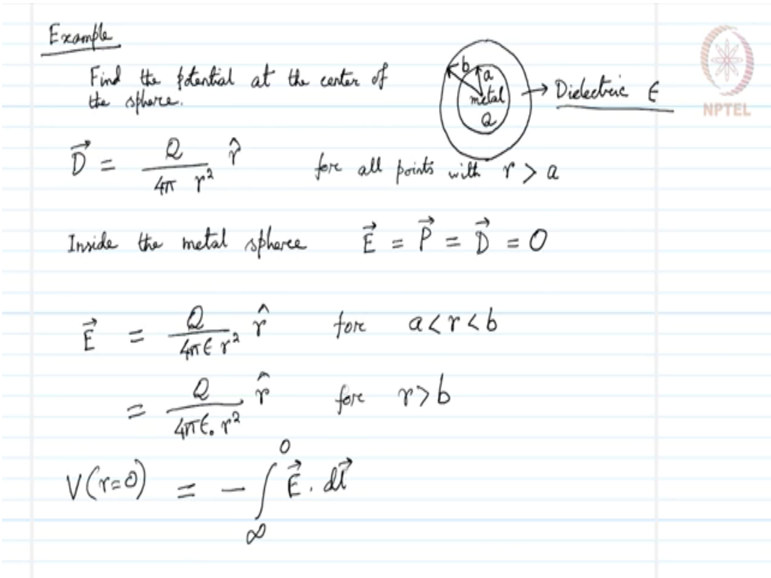

We have a metal sphere like this and outside that sphere; there is a coating of a dielectric material like this. This part is a dielectric material. Now if this is the origin, if the radius of this metal sphere is a and the radius of this dielectric coating is V. And so, this dielectric material has a permittivity of epsilon and we put an amount of charge capital Q on this metal sphere. If we consider this kind of a situation, then what would be the potential at the center of this metal sphere, that is both spheres. So, we are supposed to find the electric potential at the center of the sphere.

How do we approach? We want to find the potential and we do not know the electric field, we do not know the polarization as well and we know given the metal, we know the charge distribution. But what do we do with that charge distribution. So, here is the spherical symmetry. There is a metal at the core and that meant if we put amount of charge Q that metal sphere will have the charge Q distributed on the surface, but outside that metal sphere the electric field or displacement vector would be similar to the situation where there was a point charge at the center of this metal sphere.

So, can we write assuming the origin of a spherical coordinate system at the center of this metal sphere, the displacement vector can we find that out. The displacement vector in this context outside the metal sphere can be written as Q over 4 pi epsilon naught r squared along r cap direction.

And this is valid for all points with small r; that is the distance of our point of observation from the center of this sphere that is greater than the radius of the metal sphere. That means, outside the metal sphere it will be valid, it will be valid outside the dielectric sphere as well and also inside the dielectric spherical region whatever it is. It would not be valid in the metal sphere itself ok.

And this we can obtain using spherical symmetry and Gauss law on this spherical structure. And if we have this displacement vector, then we can write down that inside the metal sphere. Inside the metal sphere we know that the electric field would be zero there will be no polarization and therefore, the displacement vector will also go to 0 inside the metal sphere.

So, we know the displacement vector everywhere in space. And if we know the displacement vector everywhere in space then we can find out the electric field also everywhere in space. Electric field would be Q over 4 pi epsilon naught r squared r cap sorry, this is outside in between a and b; we will have epsilon naught epsilon naught. Where epsilon is the permittivity of this dielectric material.

So, this is valid for a less than r, less than b and we have found out its value Q over 4 pi epsilon naught r squared r cap, that would be the value for r greater than b ok. With this we can write down the potent we can calculate the potential at the center as minus integral from infinity to 0 r equals infinity to 0 E dot d l. And how do we evaluate this?

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$$= -\int_{\infty}^{b} \frac{Q}{4\pi\epsilon_0 r^2}\,dr - \int_{b}^{a} \frac{Q}{4\pi\epsilon r^2}\,dr - 0$$

$$= \frac{Q}{4\pi}\left(\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon a} - \frac{1}{\epsilon b}\right)$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} = \frac{\epsilon_0 \chi_e Q}{4\pi\epsilon r^2}\,\hat{r}$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = 0$$

$$\sigma_b = \vec{P}\cdot\hat{n} = \begin{cases} \dfrac{\epsilon_0 \chi_e Q}{4\pi\epsilon b^2} & \text{Outer surface} \\ -\dfrac{\epsilon_0 \chi_e Q}{4\pi\epsilon a^2} & \text{Inner surface} \end{cases}$$

We can evaluate this by doing minus integral from infinity to b; there we have the electric field Q over 4 pi epsilon naught r squared d r minus integral b to a, q over 4 pi epsilon naught sorry, epsilon r squared d r minus, there is no electric field, so, it is there is no electric field inside the metal sphere. So, this is going to be the last term is going to be 0 and that gives us the potential as 1 over 4 pi, 1 over epsilon naught b plus 1 over epsilon a minus 1 over epsilon b.

Once we have found the potential we also have found the electric field we can write down the polarization vector as epsilon naught chi e times the electric field. And that turns out to be epsilon naught chi e times the total charge Q over 4 pi epsilon r squared r cap.

Now, if we want to find the bound volume charge density for this problem; we can do that by taking the negative divergence of the polarization vector and that will give us we can see its the divergence of r cap over r squared and we are not really considering any place where r equals 0. We excluding the center of the sphere. So, there in the center of the sphere that is a metal we cannot reach there. So, there is any way no polarization.

So, the bound charge in the polarized part of the material goes to 0. How about the bounds surface charge density? The bounds surface charge density is P dot n cap and that is different for the outer surface of the dielectric material and the inner surface of the dial dielectric material. And the outer surface its epsilon naught chi e q over 4 pi epsilon b squared and it is minus epsilon naught chi e Q over 4 pi epsilon a squared for the inner surface. This is how it looks.