Electromagnetism
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Lecture - 49
Electric displacement vector and Gauss law

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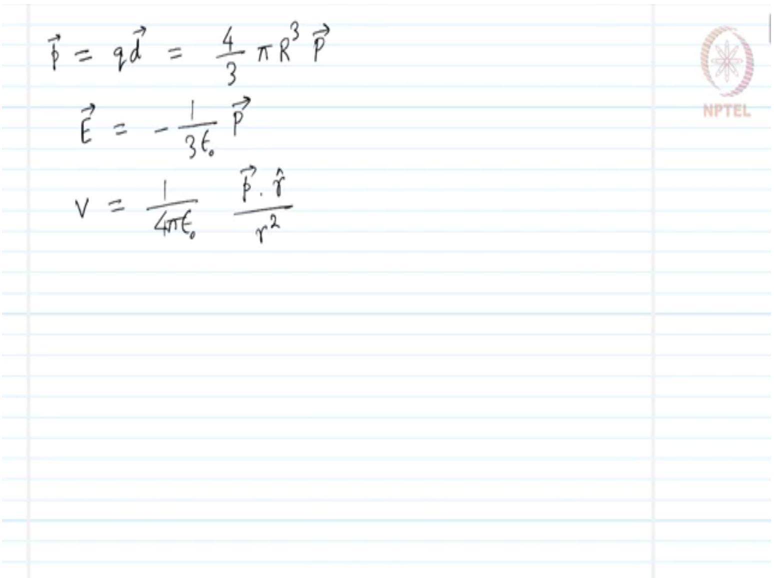

After developing this part lets develop a new concept of Electric displacement. What is electric displacement?

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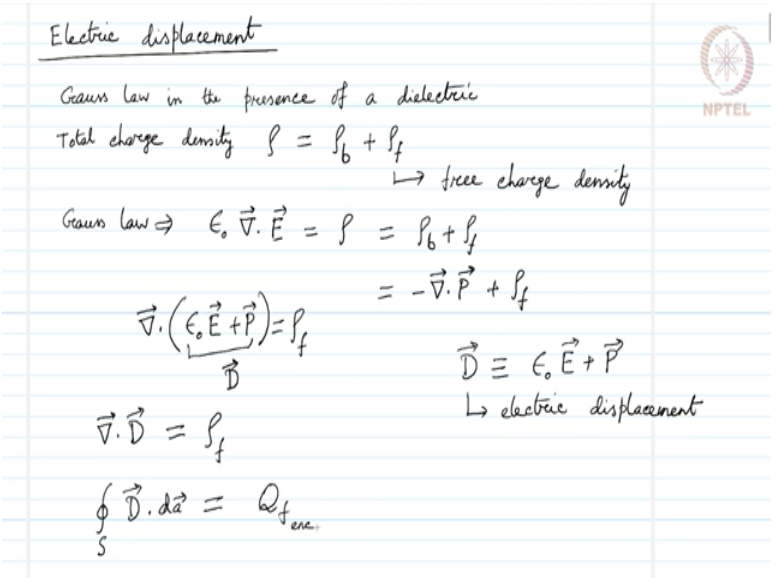

Let us consider using the Gauss law in the presence of a dielectric material. The total charge density that we have now volume charge density we will talk about, because Gauss law in it is differential form is applicable with volume charge density.

So, the total volume charge density rho can be expressed as the bound charge density plus the other charge density that we now call the free charge density with subscript f. Then Gauss law gives us epsilon naught times divergence of the electric field in this case is the total charge density total volume charge density that is rho b plus rho f.

Now rho f is the free charge density that means we have some control over it, rho b is the bound charge density where we have no control over and rho b is also given as the negative divergence of the polarization. Now, if we collect the divergence terms towards the left hand side, what will we get? We will get that the divergence of epsilon naught e plus the polarization vector gives us the free charge density.

So, electric field is not the only field that matters in this case that corresponds to the free charge density, also the polarization corresponds to the free charge density. The divergence of polarization has a correspondence like that and this quantity here let us call that the displacement vector D vector in the presence of a dielectric material. So, D is expressed as epsilon naught E plus the polarization vector and in terms of D the divergence of D becomes the free charge density.

So, this D vector is called electric displacement and the divergence of this expresses the free volume charge density. In the integral form the Gauss law would look like integration over a closed surface S D dot da equals the total free charge enclosed.

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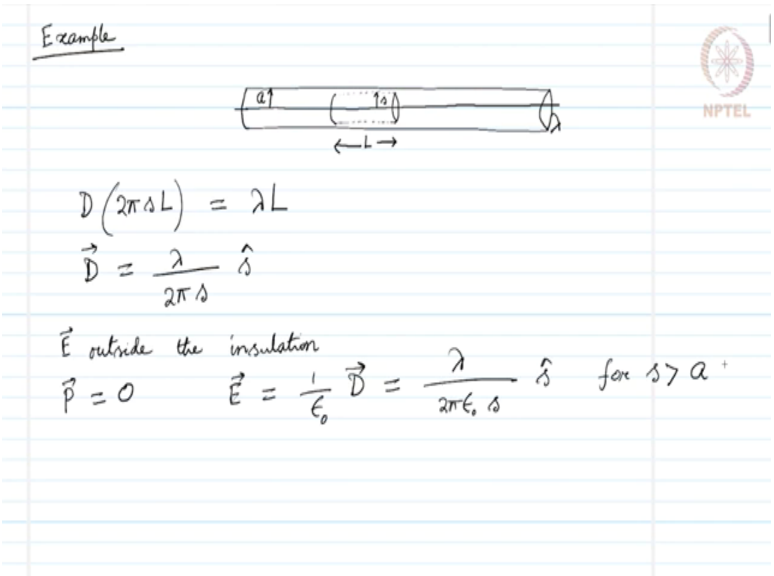

Let us consider an example of this, we have a long straight wire carrying a uniform line charge density lambda and this is our long straight wire goes to infinity. The line charge density on this is lambda and this is surrounded by a rubber insulation like this a cylindrical rubber insulation on this, the radius of this insulation is a.

Now, find the electric displacement corresponding to this arrangement. Let us so we can infer that the electric displacement vector will be radially outward. Therefore we can and it will be symmetric because there is uniform charge density lambda on this line. Therefore, we can draw a Gaussian surface that is cylindrical for cylindrical symmetry like this and let the radius be S for this and let the length be capital L.

On this Gaussian surface we can write that the displacement vector integrated over the surface, that is we will multiply it with 2 pi S times L that is the; that is the area of the cylindrical surface. Because it because the displacement vector is constant on this surface we can just multiply it with the area, that will give us the total charge enclosed by this Gaussian surface according to the Gausss law.

Therefore D can be expressed the magnitude of that D can be expressed as lambda over 2 pi s and if we worry about the direction of it will be along s cap direction as we have argued from the symmetry of the problem. So, notice that the expression holds good within the insulation as well as outside the insulation. If we draw the Gaussian surface outside the insulation we are not going to change for any of the considerations. What we have done in order to find the displacement vector within the insulation.

So, this is this displacement vector works universally, irrespective of whether a dielectric material is present or absent. Displacement vector it is expression is the same, if there is a polarization then the electric field will adjust itself accordingly. But the displacement vector is going to remain unchanged that is something very interesting that we have observed here.

Now, let is try to find out the electric field outside this insulation, where the polarization is 0 polarization vector is 0. So, the electric field after knowing the displacement vector we can write the electric field equals 1 over epsilon naught times the displacement vector, that is nothing but lambda over 2 pi epsilon naught s s cap and this is strictly valid for s greater than a.

For s less than a there will be a nonzero polarization vector and for that this expression would not hold good. But this expression is the same as what we have found earlier for line charge distribution without considering any insulation and we find the expected result in this case.

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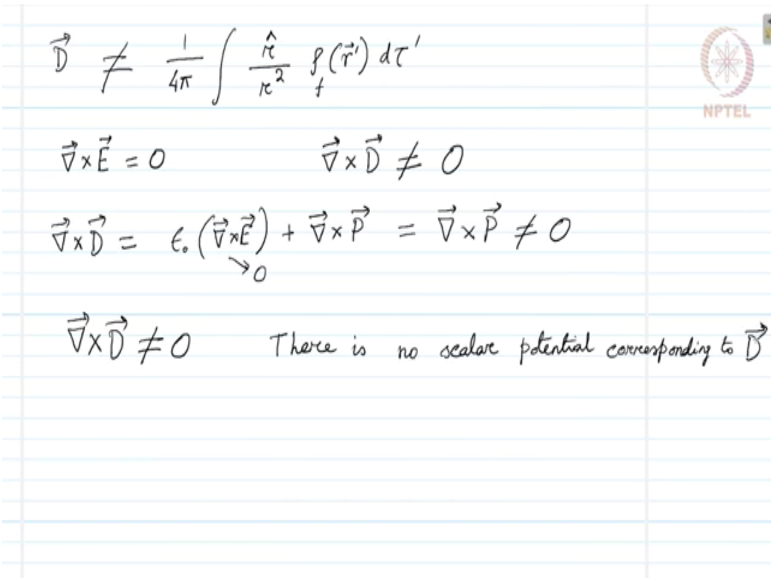

Let us consider something interesting the displacement vector can we equate this to 1 over 4 pi integration of r cap over r squared times the free charge distribution. Can we equate these two? Just like we did for electric field we could integrate over r cap over r square times the charge distribution.

And volume integrate volume integration of this multiplied with the prefactor 1 over 4 pi epsilon naught gave us the electric field can we find the displacement vector, using this no that is not possible. Because curl of the electric field is 0, but curl of the displacement vector that not that is not always 0 let us find out how.

Let us try to calculate the curl of the displacement vector that would be given as epsilon naught times curl of the electric field plus curl of the polarization vector. So, curl of the electric field goes to 0, all fine but there is no guarantee that this quantity curl of the polarization will also go to 0. This may sustain there is no a priori reason that the curl of the polarization is going to be 0. So, this is not in general 0 and if the curl of the polarization vector is not in general 0. So, polarization is a property of the material it will depend on what kind of material we have at hand, it may be 0 it may be nonzero the curl of the polarization may be 0 may be nonzero.

And therefore we cannot express the displacement vector by integrating over the free charge density multiplied by r cap over r squared vector, just like in case of electrostatic field we did it applying Coulombs law that is not possible in this case. Just because the curl of the displacement vector is not 0, there exists no scalar potential for this for the displacement vector. This is something very important to remember.