

Electromagnetism
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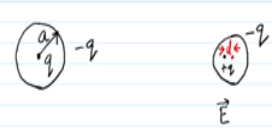
Lecture – 48
Introduction to electric polarization in matter

Now, if we consider a real material and if there is a charge distribution of some kind; that means, if we consider a dielectric material that is insulator and if we bring some charge from outside near it, then it will be subjected to an electric field and that electric field would survive within that material because it is a dielectric material not a conductor.

Conductor can cancel the electric field and develop some induced charge, but if we have a dielectric material, it will still try to oppose the external electric field; that means, it will still try to rearrange its charge centers positive and negative charge centers in such a way that the external electric field would be somewhat mitigated within the material and that way it will develop an electrostatic polarization. That means, in a dielectric material we will have induced dipoles.

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Electric field in matter



Example Calculate the atomic polarizability

We assume the nucleus moves a distance d from the origin

$$E = \frac{1}{4\pi\epsilon_0} \frac{qd}{a^3} \quad \text{due to the dipole}$$

$$p = qd = (4\pi\epsilon_0 a^3) E \quad \text{Atomic polarizability}$$

$$\alpha = 4\pi\epsilon_0 a^3 = 3\epsilon_0 V$$

So, let us discuss the concept of that. If we consider one atom for example, in an atom we can consider that this small dot here is the nucleus with charge plus q and there is an electron cloud like this where the total charge is minus q . Now we are subjecting this one to an electric field; my drawing is not that good. Let me try to redraw this negative charge distribution.

So, the negative charge distribution will be symmetric to the nucleus; that means, the nucleus will be at the center of it. And when we apply an electric field on this arrangement, then the system will try to rearrange itself in such a way that the electric field is mitigated somewhat. So, here we have subjected this to an electric field. This is the positive charge at the center and the negative charge cloud, you can see that the positive charge that is the nucleus is no longer at the center of this negative charge cloud.

This is the way an atom would respond to an external electric field and similarly you can see some polarization is being developed in a dielectric material. If that happens, then let us try to find out the center of this charge this negative charge distribution here. Let me get rid off this plus q written from here. Let us write plus q here and the center of this negative charge distribution will be somewhere here.

And the separation between these two is d . So, you can see that a dipole moment is developed in this atom itself. And let us consider an interesting example with this idea in mind. We consider a primitive model of an atom consisting of a point nucleus with plus q charge surrounded by a uniformly charged spherical cloud of electrons and the radius of that spherical cloud of electrons is a . Now, the problem asks us to calculate the atomic polarizability of such an atom.

How do we do that? If the if at equilibrium, we considered that when we apply the electric field this nucleus moves a distance d from the origin. If that happens, then the electric field due to this dipole the magnitude of that is $\frac{1}{4\pi\epsilon_0} \frac{q d}{a^3}$, where d is the distance between the positive charge and the negative charge center and a is the distance a is the radius of this spherical electron distribution. And so, this is the electric field due to the dipole.

So, we can write in terms of dipole moment. This quantity is nothing but well, let us write down the dipole moment corresponding to this the magnitude of the dipole moment can be written as $p = q d$. And if we now express this dipole moment in terms of this electric field we can write it as $4\pi\epsilon_0 a^3$ times the electric field that we have.

So, the atomic polarizability, we are supposed to calculate. If we express atomic polarizability as α that would be equal to $4\pi\epsilon_0 a^3$ that is 3 times ϵ_0 times the volume of that atom.

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For completely asymmetric molecule

$$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Alignment of a polar molecule

Torque

$$\vec{N} = (\vec{r}_+ \times \vec{F}_+) + (\vec{r}_- \times \vec{F}_-)$$

$$= \left(\frac{d}{2} \times q\vec{E}\right) + \left[\left(-\frac{d}{2}\right) \times (-q\vec{E})\right]$$

$$= q\vec{d} \times \vec{E} \quad \vec{N} = \vec{p} \times \vec{E}$$

Water molecule

And if we consider a completely asymmetric molecule, then we can write the dipole this atomic polarizability has a tensor which obeys this matrix equation p_x p_y p_z , this column vector equals α_{xx} α_{xy} α_{xz} α_{yx} α_{yy} α_{yz} α_{zx} α_{zy} α_{zz} multiplied with the electric field column vector E_x E_y and E_z .

This happens when we have an asymmetric molecule at hand. So, the atomic polarizability is no longer a scalar it is rather a tensor and this matrix equation holds good relating the atomic dipole moment and the electric field that we have. Now, let us consider the alignment of a polar molecule under the influence of external electric field. Let us consider water molecule for simplicity. We all know about water molecule.

How does a water molecule look? There is a big oxygen atom, this is our oxygen atom and there are 2 hydrogen atoms. The hydrogen atoms we are going to colour as blue. Let us

consider this to be one hydrogen atom and this to be another hydrogen atom and if I connect the nucleus from the hydrogen atom to oxygen atom hydrogen atom to oxygen atom, we get an angle here. This angle is 105 degree.

So, let us mark the atoms. This is a hydrogen atom this is a hydrogen atom and if we have this kind of an arrangement we can clearly see that. So, oxygen is 2 minus and hydrogen is these are positively charged. So, this makes an ionic compound, but a polar compound. So, we can clearly see that due to this, the dipole moment will point along this direction that would be the direction of the dipole moment vector.

Now, let us consider this kind of a situation, a bit simplified one. Let us consider a conventional dipole with plus q and minus q charges. Here we have plus q charge, here we have minus q charge and if that makes a molecule an ionic compound, this is the origin of the coordinate system o and the separation is the vector d .

Now, we consider applying an electric field along this direction. If we consider that then what is going to happen? This electric field is going to apply a force on this atom in this direction. Let us call it F plus, same amount of force on this atom in this direction; let us call it F minus and under the influence of these 2 forces, it is going to develop a torque for the geometry given here. If we call that torque as N that would be given as r plus vector cross F plus vector plus r minus vector cross F minus vector.

And that turns out to be $\frac{d}{2}$ cross qE plus $\frac{d}{2}$ cross $-qE$ which is essentially, $q d$ cross the electric field. In other words the torque N is given as the dipole moment cross product with the applied electric field. This is the amount of torque, this molecule due to this arrangement of its subject to compared to the direction of the electric field will experience.

And if possible if this torque is enough to align it along the direction of the electric field, it will get aligned along the direction of the electric field. So, for a simple atom we have considered that the simple atom can develop a dipole moment develop a polarization subject to an electric field by shifting its positive charge center and the negative charge center; that means, the

electron cloud will move somewhat and it will develop a polarization. In case of polar molecules, it is even more interesting. We can see that it can align itself along the direction of the electric field, so it can rotate that molecule.

If that is allowed, if that polar material the polar molecule forms a liquid not a solid in case of a solid tilting itself is very difficult although not impossible. So, in case of a liquid it is very easy to align that molecule in the direction of the electric field and that develops a polarization in that material. So, we have made some effort to microscopically understand how a material can be polarized.

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If \vec{E} is non-uniform $|\vec{F}_+| \neq |\vec{F}_-|$

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q(\vec{E}_+ - \vec{E}_-) = q \Delta \vec{E}$$

$$\Delta E_x = (\vec{\nabla} \cdot \vec{E}_x) \cdot \vec{d}$$

$$\Delta \vec{E} = (\vec{d} \cdot \vec{\nabla}) \vec{E}$$

$$\vec{F} = (\vec{p} \cdot \vec{\nabla}) \vec{E}$$

Polarization \vec{P} is the dipole moment per unit volume in a material.

Now, in this consideration for this molecule the polar molecule that aligns itself, if we consider that the electric field is non-uniform, in that case we will not have F plus and F minus exactly balancing each other.

So, the magnitude of F_{plus} will not be equal to the magnitude of F_{minus} . In that case we will have a total force on this molecule and that will be obviously, the difference between these 2 forces. We are summing the forces because they are opposite in sign and so, the total force is given by this which is nothing but $q(E_{\text{plus}} - E_{\text{minus}})$. So, E_{plus} is the electric field on the positive charge and E_{minus} is the electric field on the negative ion and that is ΔE ; Δ is just $E_{\text{plus}} - E_{\text{minus}}$.

So, let us consider the x component of this ΔE . ΔE_x equals gradient of E_x dot the distance vector d and assuming the electric field in x direction. So, ΔE can be given as $d \cdot \nabla$ applied on the electric field vector; that means, the total force on this molecule that is given as the dipole moment dot ∇ vector times the electric field.

After having discussed this, let us consider polarization. Polarization is represented by capital P vector and small p vector is the dipole moment vector. So, this is the dipole moment per unit volume in a material.