Electromagnetism
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Lecture – 46
Multipole expansion, continuous charge distribution, and assembly of point charges

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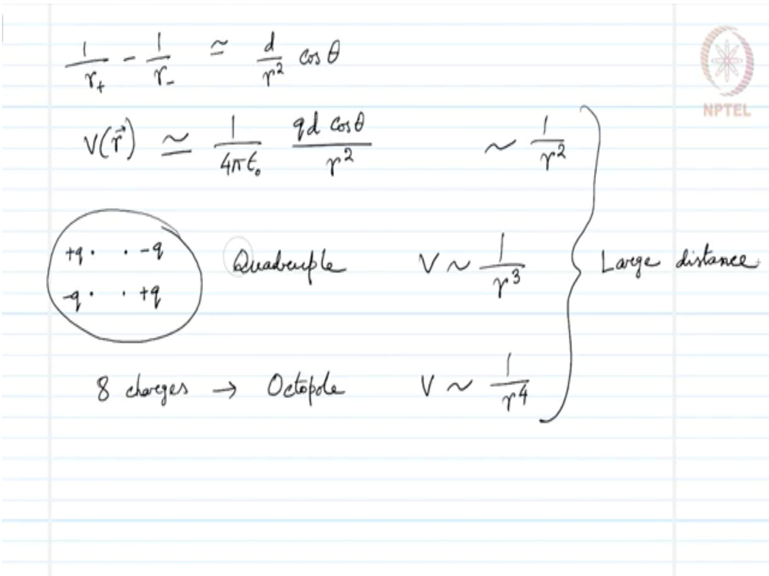

Let us consider something interesting, a Multipole expansion of a potential.

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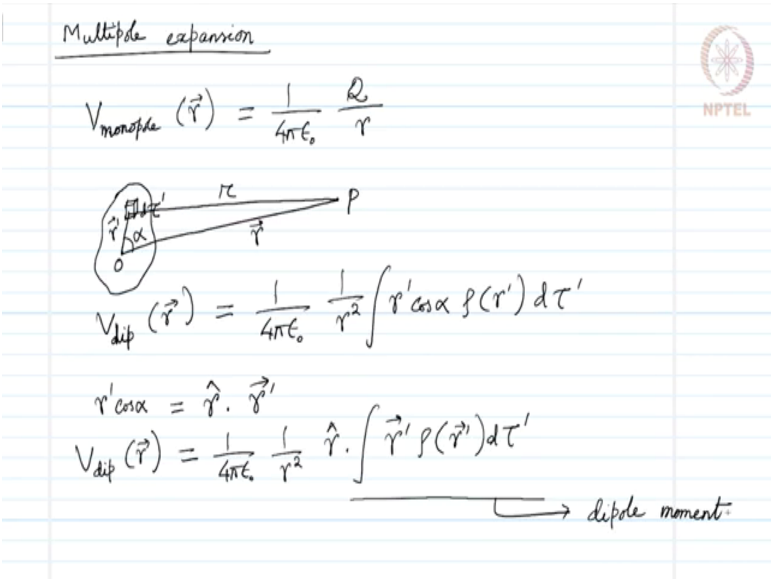

The potential for any arbitrary charge distribution can be expressed by a multipole expansion and this would be valid at large r. So, if we consider the potential due to a monopole, we can write it as 1 over 4 pi epsilon naught q over r that we have seen earlier. If we consider for dipole and if there is a charge distribution like this, on this part. Let us consider this to be the origin. Here, we have a volume element like this. The volume element is marked d tau and this distance is our prime vector.

Our point of observation is somewhere here P and its code this position vector is r the distance from point P to this volume element is curly r and this angle is alpha. If we have this kind of an arrangement, then the potential due to dipole can be expressed as 1 over 4 pi epsilon naught 1 over r squared integration r prime cosine alpha rho r prime d tau primes.

So, this is d tau prime actually; where, r prime cosine of alpha is nothing, but r cap dot r prime vector. Then, V dipole is simplified to the potential due to this dipole is simplified to 1 over 4 pi epsilon naught. 1 over r squared r cap dot r prime rho r prime d tau prime.

So, here we observe something interesting. We did do not have any arrangement of two point charges in this rather we have a continuous charge distribution that is rho a function of r prime and for that if that distribution has a different positive centre and a negative centre, we will still have a dipole moment arising out of this.

So, for a given for any given charge distribution, we can find a dipole moment and we can find the potential due to that dipole term and there is something very interesting although the monopole term can go to 0 at certain situations, the dipole term may be surviving. For example, if we consider a charge distribution that we have done earlier, we have considered two-point charges here plus q and minus q.

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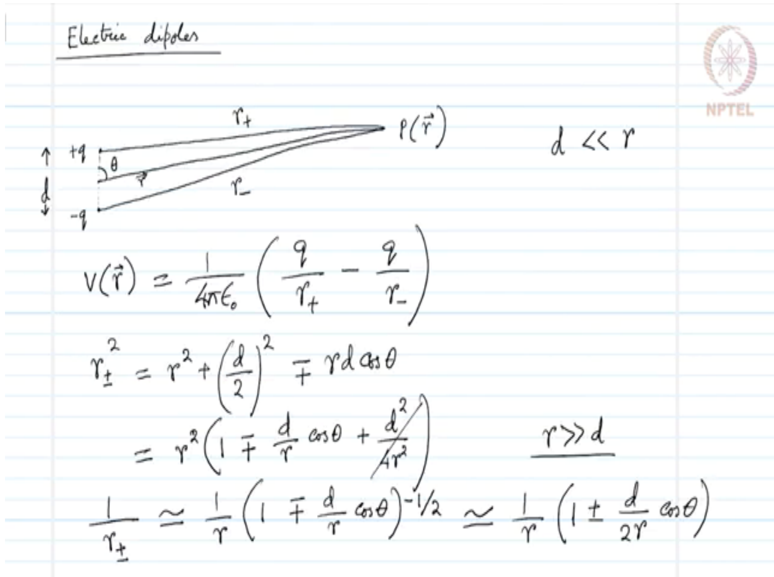

And due to this plus q and minus q charge at a far away distance from these two charges we will have no monopole electrostatic potential. But there would still be an electrostatic potential due to this dipole contribution and the dipole contribution would certainly be the most dominant because quadruple and octopole, they go as 1 over r cubed and 1 over r power 4 respectively. So, dipole contribution will be the most dominant one and still dipole contribution is far less dominant far quickly rapidly decaying as compared to the monopole contribution.

So, if the monopole moment the if the monopole contribution to the electric potential becomes 0, then we have the dipole contribution as the leading one and that often corresponds to no total charge in the system. Still we have an electric potential and an electric field that is quite interesting ok. Now, the integral that we have here, this integral does not really depend on the distance of our point of observation from this dipole and this integral is called the dipole moment.

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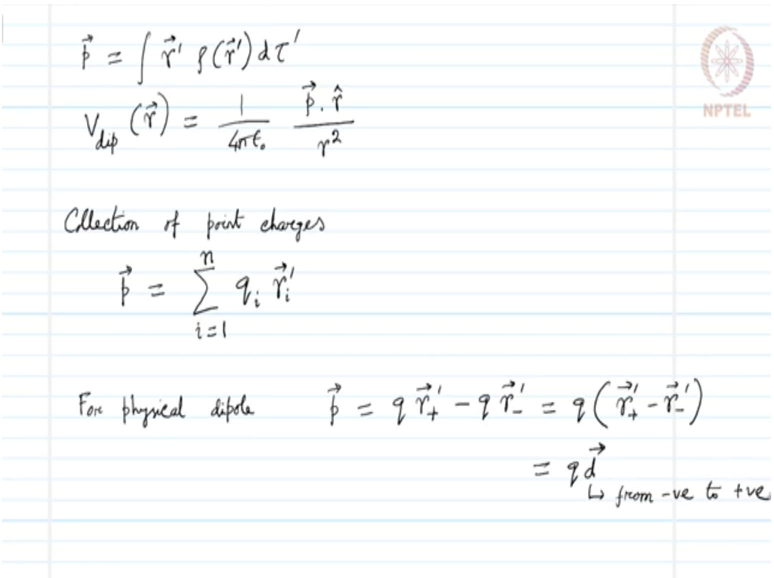

So, how do we express this? It is denoted as small p vector that equals r prime vector times rho as a function of r prime that is the volume charge distribution integrated over the volume. So, in terms of this the dipole potential that turns out to be 1 over 4 pi epsilon naught dipole moment dot the direction of our position vector over r squared.

So, let us try to calculate the dipole moment for a collection of point charges. If we consider a collection of n number of point charges, then the dipole moment can be obtained by making a sum over those n number of point charges like i running from 1 to n q i times the position vector of this charge of this point charge.

Now, for a physical dipole, where we have equal and opposite charges; only two charges we have. Only two point charges we consider in this case; then for those two point charges, we can write the dipole moment p as q times r plus prime minus q times r minus prime and that becomes q times r plus prime minus r minus prime and this vector subtraction gives us this distance d between those two point charges.

That means, this is q times the charge of each point charge times the distance vector between them, where d is the displacement vector from the negative charge to the positive charge. That is how we define the dipole moment in the context of a physical dipole. Now, let us consider the case of a multipole expansion.

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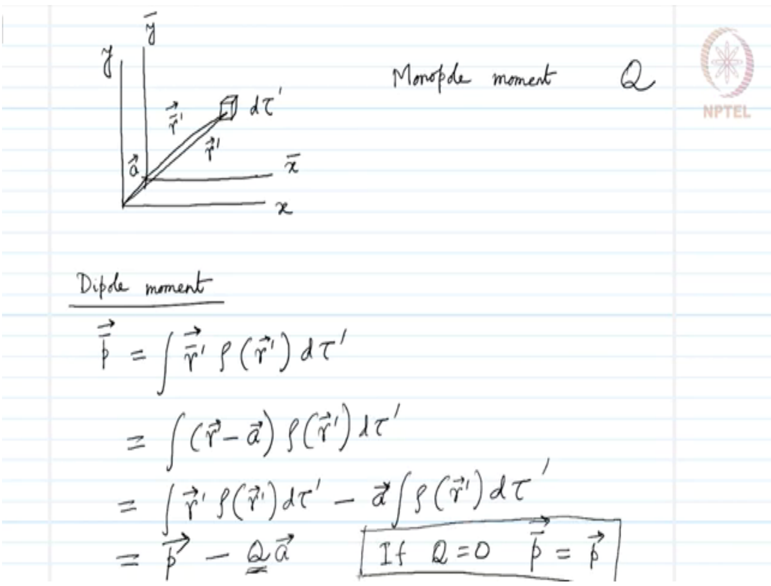

Let us consider a two-dimensional coordinate system. This is x; this is y. Two-dimensional for simplicity and we make a translation of this coordinate system. So, we move to x bar and y bar and we want we have a physical charge distribution. So, this displacement of the coordinate system is made by a vector a. Now, we have a charge distribution somewhere. Let us consider a volume element here ok, in two-dimensional coordinate system, we cannot really consider a volume element.

So, let us assume a three dimensional coordinate system, but we are drawing only two-dimensions here and the position vector of this volume element that is r prime with respect to the original coordinate system and that is r prime bar with respect to the new coordinate system. What is the monopole moment of this arrangement in the different coordinate systems?.

Monopole moment is nothing but the total charge Q. So, the total charge remains unchanged, no matter how you move the coordinate system. How about the dipole moment? We know that the dipole moment is not independent of the position vector. Let us see what happens in this context, when we have moved the coordinate system by an amount a.

So, the dipole moment p bar in the new coordinate system can be written as integration over r prime bar vector rho r prime vector d tau prime because the charge density is whatever physically is there and we can integrate it over any coordinate system does not matter. Only thing is that our position vector of that volume element that changes if we move to a different coordinate system.

So, this is going to give us the dipole moment with reference to the new coordinate system, the one with a bar and this becomes r minus a this is the transformation, we have made on the coordinate system times rho r prime d tau prime which is we can split this into two integrals that is r prime vector rho r prime d tau prime minus a vector which is actually constant of this integration that comes out times rho r prime d tau prime.

So, the left integral is nothing but the dipole moment in the old coordinate system without a bar and the right quantity is total charge that comes from the integral times this a vector. This is the new dipole moment. So, the dipole moment is not independent of the origin. If we shift the origin the dipole moment for an arrangement of charge is going to change, if we have a continuous charge distribution.

But there is something interesting in there, if we have no total charge in that arrangement of charges. If the total charge that is Q here, if that goes to 0, then we have p bar vector that is the new dipole moment equals p vector that is the old dipole moment; only in this context, this situation not otherwise.