


Electromagnetism
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Lecture – 45
Electric dipoles

(Refer Slide Time: 00:38)

Electric dipoles



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_+} - \frac{q}{r_-} \right)$$

$$r_{\pm}^2 = r^2 + \left(\frac{d}{2}\right)^2 \mp r d \cos\theta$$

$$= r^2 \left(1 \mp \frac{d}{r} \cos\theta + \frac{d^2}{4r^2} \right) \quad r \gg d$$

$$\frac{1}{r_{\pm}} \approx \frac{1}{r} \left(1 \mp \frac{d}{r} \cos\theta \right)^{-1/2} \approx \frac{1}{r} \left(1 \pm \frac{d}{2r} \cos\theta \right)$$

Hello. So, today we are going to discuss about Electric Dipoles. Dipoles are two point charges separated by a small distance. If we consider a point charge here of the magnitude plus q and a point charge here of the magnitude minus q, of course, the distance between them is exaggerated here to draw the picture.

Let us assume the distance between them to be d, like this. Then from a point of observation p here that is quite far from these two char ges, we can find out the electric field due to this

assembly of two charges that is for due to this dipole. This is the distance from the positive charge and we call it r_+ and this is the distance from the negative charge that we call r_- .

And if we consider a line joining these two charges and consider the point of bisection of on that line, then r is given by this. This is our position vector r . So, we choose the origin of this coordinate system at the point bisecting the line connecting the two charges and this angle marked here is θ .

Now, this is the ideal dipole where the distance d between the two charges is much less than the distance of these charges from the point of observation that is r . And in case of physical dipole this may be a little different, not exactly like this because there is no point charge in real world and their separation that depends on exactly how the dipole is created.

Let us consider this situation and try to find the potential for this arrangement of charges at a distance r from the midpoint of the line connecting these two charges that is the distance from the dipole at our point of observation p . So, if we write down the expression for potential then that would be $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_+} - \frac{q}{r_-} \right)$, minus sign comes because this charge is minus q and for here its plus sign because this charge is plus q . The two charges are equal and opposite in magnitude.

Now, from the law of cosine we can write down $r_+^2 = r^2 + \frac{d^2}{4} - rd \cos\theta$ and $r_-^2 = r^2 + \frac{d^2}{4} + rd \cos\theta$. We can write this down which turns out to be $r_+^2 = r^2 \left(1 - \frac{d}{r} \cos\theta + \frac{d^2}{4r^2} \right)$ and $r_-^2 = r^2 \left(1 + \frac{d}{r} \cos\theta + \frac{d^2}{4r^2} \right)$. Since, we are interested in the limit r is much much greater than d , in this limit we see that this third term is negligible.

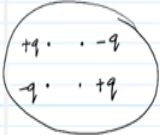
So, we leave out the third term that is this term here. And if we neglect the third term then we can write that $\frac{1}{r_+} - \frac{1}{r_-}$ is nearly equal to $\frac{1}{r} \left(1 + \frac{d}{r} \cos\theta \right) - \frac{1}{r} \left(1 - \frac{d}{r} \cos\theta \right)$. And square root of this quantity, $\frac{1}{r}$'s, actually $\frac{1}{\sqrt{r^2 \left(1 - \frac{d^2}{4r^2} \right)}}$, actually $\frac{1}{\sqrt{r^2 - \frac{d^2}{4}}}$.

so power minus half. And that is nearly equal to 1 over r, 1 plus minus d over twice r cosine of theta. This is using the binomial expansion.

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$$\frac{1}{r_+} - \frac{1}{r_-} \approx \frac{d}{r^2} \cos \theta$$

$$V(\vec{r}) \approx \frac{1}{4\pi\epsilon_0} \frac{qd \cos \theta}{r^2} \approx \frac{1}{r^2}$$


 Quadrupole $V \sim \frac{1}{r^3}$

8 charges \rightarrow Octopole $V \sim \frac{1}{r^4}$

Large distance

If we have this, then we can write down that 1 over r plus minus 1 over r minus that we have in the expression for potential can be expressed approximately as d over r squared cosine of theta. And if we can do that then the expression for potential at point p with position vector r can be expressed as 1 over 4 pi epsilon naught q d cosine of theta over r squared.

Now, this quantity that we are writing is at rest. So, our point of observation is far away from the dipole, then this is valid. And what we see here is quite interesting. The potential due to an electric dipole, goes as 1 over r squared, while the potential due to a monopole that is one point charge goes as 1 over r. So, here we find that the potential due to dipole goes as 1 over square; this is very interesting observation in the context of dipoles.

Now, if we put together a pair of equal and opposite dipoles. So, what we have considered earlier was two point charges like this, this was plus q and this was minus q . If we put together two equal and opposite dipoles; that means, here we will have minus q and here we will have plus q this arrangement is called an octopole, sorry this arrangement is called a quadruple. And in case of quadruple the potential goes as 1 over r cubed.

If we have 8 such charges alternatively arranged that will lead to an octopole and in case of an octupole the potential will go as 1 over r power 4 and so on. And all these things are valid, this the way the potential goes at large distance. So, only when the point of observation is far from the dipoles or quadruples or octopole in general multiples, then this expression is valid not otherwise.