**Electromagnetism**
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**Lecture – 44**
# Another example of the method of images

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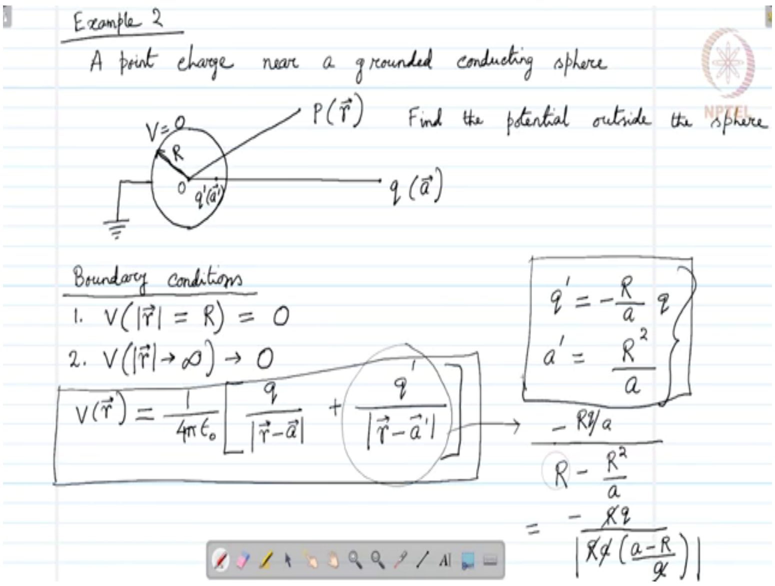


Let us consider Another example with the method of images. So, let us understand that with method of images is a very elegant method in solving the problems of electrostatics, finding the potential in certain regions of interest provided we have a particular symmetry of the problem. Without a symmetry if it is a generic situation then method of images is not very useful, but when there is an intrinsic symmetry in the problem then method of images is very nice and elegant way of understanding the physics behind the problem.

So, here in the second example, we consider a point charge near a grounded conducting sphere. How do we do that? Let us draw it. This is the conducting sphere, not a spherical shell entirely filled sphere and here is the center; center of the sphere and this sphere has been grounded; that means, potential on the surface is 0 everywhere inside is 0. And we have a point charge here, q at a distance, a from the origin of this sphere and we want to find out the potential at a point p with position vector r.

So, the problem asks us to find the potential outside the sphere. Remember with method of images we cannot put an image charge in our region of interest. If our region of interest is outside the sphere, we are allowed to put an image charge inside this sphere, but if we put one outside the sphere then the physics does not guarantee the Laplace equation or whatever we have considered the uniqueness theorem. Nothing guarantees that the solution that we will obtain to this image problem would actually be the solution to the Laplace equation. As long as we put that image charge outside our region of interest, we are guaranteed with that.

So, let us draw this line here this is connecting the origin with the point of observation the radius of the sphere is capital R. So, let us write that down here. This much is capital R and let us consider that this image sits here the amount of image charge is q prime at a location a prime. So, a prime is the position vector of this q prime charge.

And what are the boundary conditions to this problem? We need to find out the boundary conditions in order to solve any image problem. So, the first boundary condition would be given as the potential at if the position vector the distance from the origin is capital R that is on the surface of the sphere that is 0 that is the boundary condition, although we know that everywhere inside the sphere it is the potential is going to be 0, but that will not work as a boundary condition because we are trying to put an image charge inside and with that image charge we will not really satisfy the case that everywhere inside the sphere the potential would be 0, that is not possible.

So, let us put only the boundary condition. Let us only worry about the boundary to our region of interest that is outside the sphere not inside the sphere. So, this is the condition that we must write down. And the other condition that is obvious that if we go to infinity then the potential will also go to 0.

Now, by symmetry of this problem this image charge q prime would be located on the line connecting the center of this sphere O and the real charge real point charge q. We are interested in calculating the potential outside the sphere which from this picture that we have drawn from the arrangement that we have logically arrived at can be written as 1 over 4 pi epsilon naught q over r minus a magnitude plus q prime over r minus a prime magnitude. This is the potential outside.

Now, we must choose q prime and a prime in such a way that the boundary conditions are satisfied. How do we satisfy boundary conditions? Let us put the values of it and let us verify whether it works. Let us put the value of q prime equals minus R over a times q and a prime equals R squared over a. If we have this, then do we have the boundary condition satisfied. The second boundary condition will anyway be satisfied because when r goes to infinity here the denominator will be so large that the potential will tend to 0. Let us put the value of q prime and a prime in this part, second part of this equation for small r equals capital R.

So, this part of the equation will become for q prime we are writing minus R over, R minus R q over a and it is r minus for a prime we are writing sorry small r becomes on the boundary it becomes capital R, R minus R squared over a. This is what we are writing here. That means, this quantity becomes minus R q over R a, this a I am bringing in here and then a minus R over a. So, this a, cancels; this r this r cancels. We are left with minus q over a minus capital R and its absolute value and that is same as the other term with without this minus sign here; that means, they cancel each other and we have the first boundary condition also satisfied.

So, this values for q prime and a prime do satisfy both boundary conditions that we have in this problem. And therefore, this satisfies this is the solution for this image problem, and because of the uniqueness of Laplace equation solution we are sure that this is the only solution the unique solution to this problem. So, outside the outside this sphere that is our region of interest r greater than capital R, we will have the potential expressed by this equation with these values of the image charge and the location of this image charge.