

Electromagnetism
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Lecture – 43
Force and energy

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Force and energy

$$\vec{F} = -\frac{1}{4\pi\epsilon_0} \frac{q^2}{(2d)^2} \hat{z} \quad \text{From image charge consideration}$$

Energy due to two point charges and no conductor

$$W = -\frac{1}{4\pi\epsilon_0} \frac{q^2}{2d}$$

Actual energy $W = -\frac{1}{4\pi\epsilon_0} \frac{q^2}{4d}$

$$W = \int_{\infty}^d \vec{F} \cdot d\vec{l} = \frac{1}{4\pi\epsilon_0} \int_{\infty}^d \frac{q^2}{4z^2} dz$$

$$= \frac{1}{4\pi\epsilon_0} \left(-\frac{q^2}{4z} \right) \Big|_{\infty}^d = -\frac{1}{4\pi\epsilon_0} \frac{q^2}{4d}$$

Now, let us consider force and energy in this context. The force, on the force between the point charge and the conducting plane that we have, can be written as minus 1 over 4 pi epsilon naught. Minus sign comes because the original point charge and the induced charge distribution surface charge distribution they will attract each other. q squared over 2 d squared z cap. This would be the force if we consider the image charge.

Clearly, we have a minus q image charge at a distance 2 d from the original charge. So, force that we have calculated considering the image charge is actually the correct force. How about

energy? If we consider the energy due to two point charges and no conductor. What would we find? The energy would be minus 1 over 4 pi epsilon naught q squared over twice d.

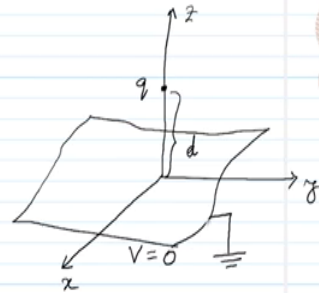
But if we consider a single charge and a conducting plane, the energy is actual, the electric field is actually stored in the upper half of the hemisphere.

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The method of images
Find $V(\vec{r})$ where $z > 0$

Boundary conditions

- $V = 0$ at $z = 0$
- $V \rightarrow 0$ fast from the charge
 $x^2 + y^2 + z^2 \gg d^2$

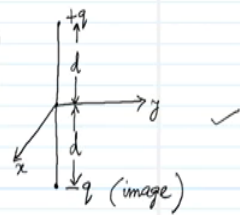


Trick

$$V(x, y, z) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{\sqrt{x^2 + y^2 + (z-d)^2}} - \frac{q}{\sqrt{x^2 + y^2 + (z+d)^2}} \right]$$

Boundary conditions

- $V(z=0) = 0$
- $V \rightarrow 0$ fast for $x^2 + y^2 + z^2 \gg d^2$



So, what we have found using images here in this case? Here we have for this kind of charge distribution that is plus q original charge and minus q image charge, the electric field is in all space. But, in the original problem there is only plus q original charge and there is an induced charge distribution on this plane such that the potential is 0 on the plane and everywhere below the plane the potential continues to be 0 according to Laplace equation.

So, the electric field is found only at the upper hemisphere in this case, in the original case. So, what we find considering two point charges? In case of energy is not correct. Because, this energy is stored in the electric field and electric field below the conducting plane is 0. It is only confined within the upper half of the conducting plane. Therefore, the actual energy would be $\frac{1}{4\pi\epsilon_0} \frac{q^2}{d}$ this would be the actual energy for this kind of a system.

Let us try evaluating this for directly from the original problem. Considering the electric field is strictly localized in the upper half. And in the lower half the electric field is 0. Then the expression for the energy would be $\int_{\infty}^d \mathbf{F} \cdot d\mathbf{l}$. That means, we have the conductor of infinite extent already in there in its place. And we are bringing this point charge from at a distance infinity to the distance d that we have considered here.

This will be given as $\frac{1}{4\pi\epsilon_0} \int_{\infty}^d \frac{q^2}{z^2} dz$. This quantity equals $\frac{1}{4\pi\epsilon_0} \left[-\frac{q^2}{z} \right]_{\infty}^d$. Limits are infinity to d . And putting the limits we find its $\frac{1}{4\pi\epsilon_0} \frac{q^2}{d}$.

So, this finding is in agreement with our consideration about only in the half of the region the electric field is confined. So, whatever energy we have calculated from two consideration of 2 point charges, the actual energy of the system will be half of it. So, this is consistent with our expectation.