Electromagnetism
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Lecture – 36
General idea of energy in electrostatics

Hello, we were discussing the energy associated with electrostatic systems. We have already found out the energy for assembly of a number of point charges and now we will work out the energy for a continuous charge distribution.

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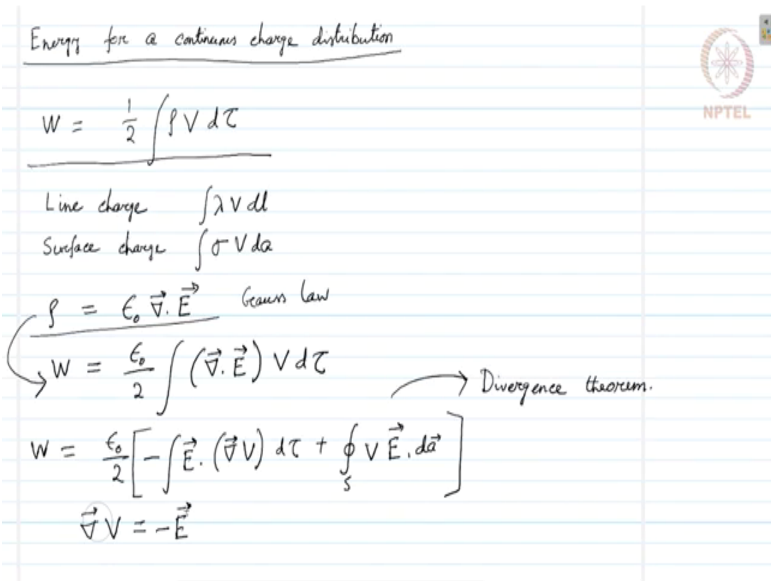

And, how should this be different let us see. So, if we have a volume charge density rho and if this volume charge density is uniform and if the corresponding potential that we have is V, then the work done in assembling this volume charge density would be half integral over rho V d tau. If we consider a line charge density; so, we will prove this statement that we have meant, if we consider line charge density then it can be given by lambda V dl, if we consider a surface charge density then this would be given by sigma V da.

So, if we consider a volume charge density rho then this from Gauss law is expressed as epsilon naught times the divergence of the electric field. Therefore, the work done to assemble this charge density would be given as epsilon naught over 2 divergence of the electric field times the potential times the volume element.

So, what we have seen earlier is that the if we come from our consideration of point charges that we have done we have seen that it is half times sum over all point charges and the potential that gave us the work done to assemble this charge that is how we have obtained this expression here the work done for assembling a volume charge density rho over a given volume would be rho times the potential integrated over the volume and half factor in front of that.

And, if we now consider Gauss law given here this is just the differential form of the Gauss law, we can write using this the work done as a function of the divergence of electric field. And, if we have this then we can integrate by parts this expression we have the divergence of electric field and potential both are function of space. So, we can integrated by parts and write down W equals epsilon naught over 2 minus integral E dot gradient of V d tau plus closed surface integral of V E dot da.

Of course, in order to find this term, the second term here, we have used the divergence theorem ok. Now, gradient of the potential we have here. So, the gradient of the potential is nothing, but minus negative of the electric field.

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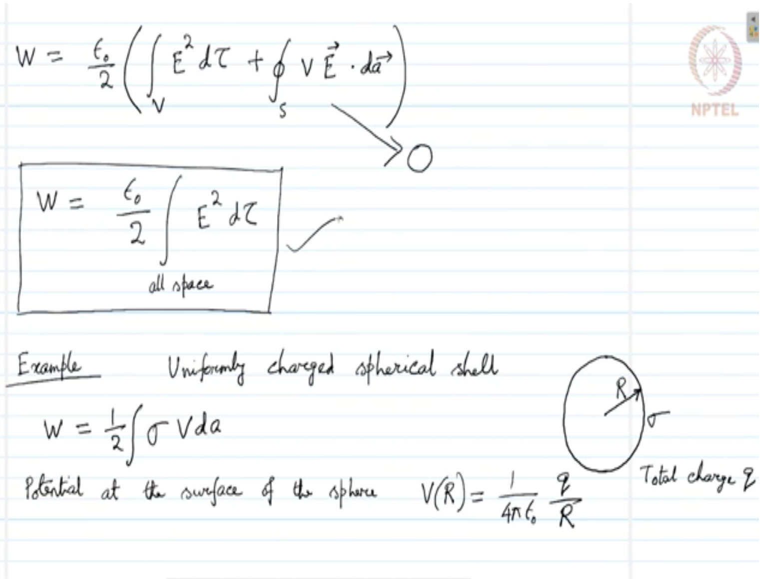

And, if we have that then we can write the work done as epsilon naught over 2 volume integral of E squared d tau that comes from the fact that gradient of the potential is the electric field plus a closed surface integral over V E dot da.

Now, which volume are we considering in this case? We have a in the second term we have a surface integral that surface integral encloses a volume and that volume confines the charges; the charge distribution that we are talking about. Now, if we consider a larger volume than what confines the charges that is also fine we will still have this expression valid. If we consider for example, an infinite sphere a sphere with radius infinity, still this expression is valid and if we do consider that then at infinity due to this charge distribution under consideration the electric field and the potential both go to 0.

And, if we have that situation then this integral gives us no contribution this integral goes to 0 and that means, the work done to assemble this continuous charge distribution that is volume charge density rho would be given as integration; this is a volume integration over all space now if we consider it over all space only then we can set the right hand side term 0 not otherwise.

So, we have to consider this integral over all space now E squared d tau. So, here we have an expression for the work done to assemble that charge distribution in terms of the electric field that it has resulted in. We do not have the charge distribution explicitly in our expression, but it is in terms of the energy and this is very useful. After deriving this important expression which we will analyze further let us consider an example to understand what this actually means.

So, we will try to find the energy of a uniformly charged spherical shell of total charge q and radius R, we consider a uniformly charged spherical shell as drawn in here and because it is a spherical shell it will only have a surface charge distribution this much is the radius R and the work done for this distribution can be. So, well the total charge is q let us write that down also here as well ok.

So, the work done to assemble this charge distribution can be expressed as half integral over sigma V da. Now, the potential at the surface of this sphere, let us try to calculate that surface of the sphere. We have seen an example earlier from which we can write down that V at capital R that is on the surface of the sphere is constant and it can be given as 1 over 4 pi epsilon naught q over capital R.

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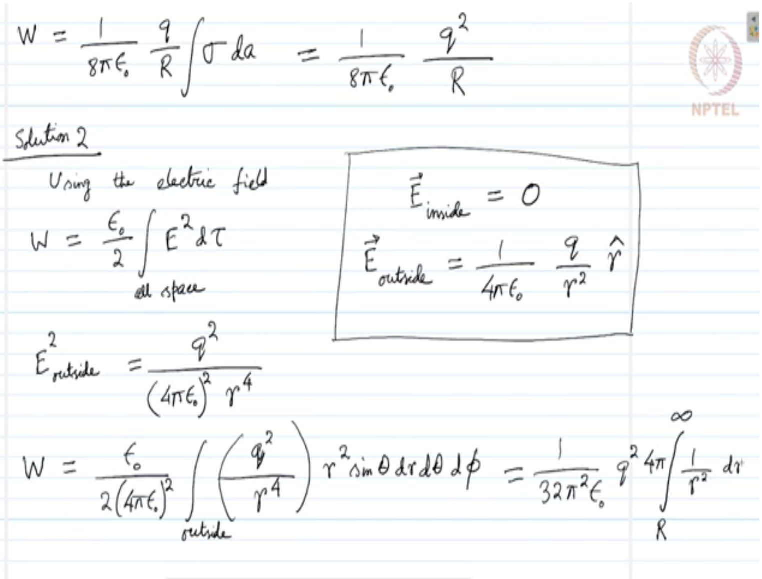

And, if that is the case then we can write down the work done as 1 over 8 pi epsilon naught q over capital R that is a constant and then integration over sigma da. We can do this turns out to be 1 over 8 pi epsilon naught q squared because integration over sigma da will give us q once again over R and this is the energy due to the distribution of uniform charge on the spherical shell.

Now, let us consider another way of solving this problem. We have talked about the electric field expression here this expression. If we now find the electric field corresponding to the spherical shell and apply this expression to find out the energy let us see what we find. Using the electric field we can write down the work done equals epsilon naught square epsilon naught over 2 E squared d tau integration over all space. So, we have to find the electric field everywhere.

We have considered this example of spherical shell with uniform charge distribution earlier; from Gauss law we found that the electric field inside the sphere is 0 because no charge is enclosed within the Gaussian surface if we consider a Gaussian surface inside the spherical shell.

Outside the spherical shell it can be given as 1 over 4 pi epsilon naught q over r squared r cap. We have seen that uniform distribution of charge over a spherical shell behaves like a point charge at the center of the sphere. We have proved this using the Gauss law and so, we are going to use these expressions for our calculation now.

So, E squared outside is given as q squared over 4 pi epsilon naught squared r power 4. Therefore, the work done can be written as epsilon naught over 2 times 4 pi epsilon naught squared integration only outside because inside the electric field is 0. So, integral will also give a 0, q squared over r power 4 and the volume element in spherical coordinate system r square sin theta dr d theta d phi.

If we perform this integral we will get one over 32 pi squared epsilon naught q square times 4 pi; only the r integral is left that will run from capital R upto infinity for outside the sphere, it is 1 over r squared dr.

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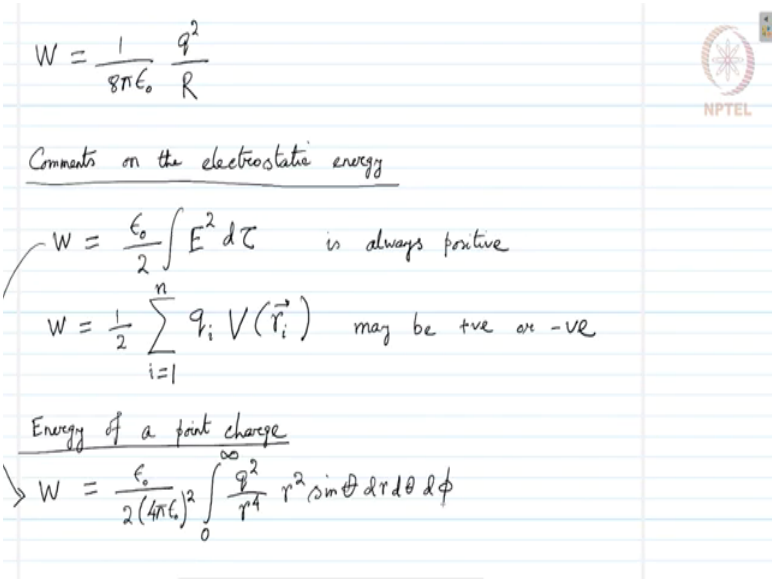

Now, once we perform the r integral also we get this work done as 1 over 8 pi epsilon naught q squared over R. And, these two with these two methods we get the same expression for the electric for the work done to assemble this surface charge density. So, it is fine. Let us make a few comments on the electrostatic energy.

Let us consider the work done to assemble a charge density from the electric field point of view from the expression with electric field we write this work done as E integral over E squared over d tau with a pre-factor of epsilon naught over 2. Now, this quantity is always positive. Why because even though even if electric field is negative E squared is going to be positive and the rest of the quantities cannot be negative. So, the work done will always be positive from this expression.

From the other expression that we have; we have work done equals half sum over i equals 1 to n q i and the potential due to this charge distribution this is for the assembly of discrete charges q i. And, this one this expression clearly suggests that the work done may be positive or negative is that an inconsistency which of these two expression is correct? Let us examine that.

If we consider the energy of a point charge that can be expressed as in terms of the field expression with the help of this one this work done can be written as epsilon naught over 2 times 4 pi epsilon naught squared integration from 0 to infinity q squared over r power 4 r squared sin theta dr d theta d phi.

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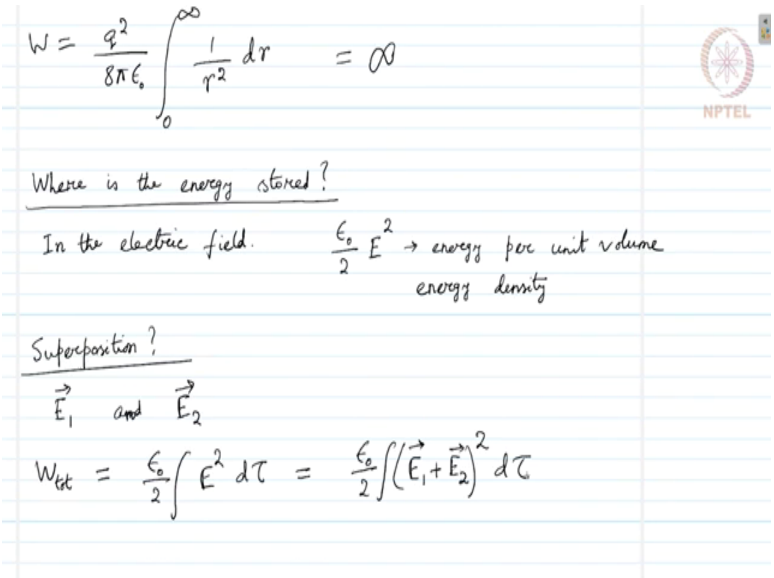

And, once we evaluate this integral it gives us q squared over 8 pi epsilon naught integral from 0 to infinity 1 over r squared dr, we have already performed the theta and phi integral and have taken them outside because we did not have any function of theta and phi. So, if we perform this integral now from 0 to infinity what is going to be the result? We can find that this integral is going to give us infinity.

So, according to this expression of electric field for a point charge in order to bring a point charge and place it at one point looking at the electric field expression for the work done we find that the work done amount is infinity. However, while assembling point charges we considered that when we bring one point charge from infinity to any position here in this case the origin of our coordinate system we did not make any work because there was no electric field before there was any charge there was no electric field.

And, in that condition we have noted that one expression gave us no in no work done because we have considered the entire amount of point charge bringing together bringing that entire amount of point charge together and for the continuous charge distribution or the electric field expression gives us bringing in infinitesimal charges at a time and arranging it and distributing it over a volume or surface or whatever.

So, these two approaches the field approach and the assembly of point charge approach these two approaches consider two different processes two different circumstances and we cannot compare one with the other and say that one is correct and the other is wrong. It depends on the situation exactly what process you follow to make that assembly of charges that will tell you whether the field approach is what is suitable for your work or the assembly of point charges approach that depends on the context and cannot one cannot be said correct over the other ok.

Then comes the question, where is the energy stored? We have considered the work done so far. So, the work done means some energy and in a conservative force field this energy will be stored somewhere where is it in the charge distribution itself or in the electric field. The answer is the work or the energy is stored in the electric field. Therefore, epsilon naught over 2 E squared this is the energy per unit volume also called the energy density.

And, then comes the question how about the principle of superposition? If we consider two electric fields E 1 and E 2 that is due to some charge distribution and the total electric field because of this principle of superposition is E 1 plus E 2. How about the total energy? Let us see. The total energy for these two electric fields can be given as epsilon naught over 2 integration over E squared d tau which means epsilon naught over 2 integration over E 1 plus E 2 squared d tau.

Now, E 1 plus E 2 whole squared is different from E 1 squared plus E 2 squared. So, what are we doing in terms of energy? Is the energy conserved or what is happening here?

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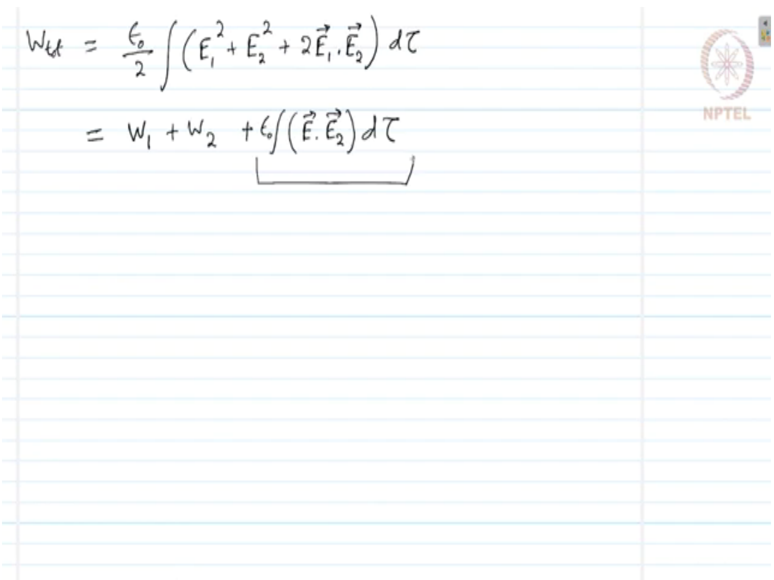

That means the total work is epsilon naught over 2 integration of E 1 squared plus E 2 squared plus 2 E 1 dot E 2 integrated over d tau which is work for the first electric field plus work for the second electric field plus epsilon naught times E 1 dot E 2 d tau. That means, if we have two electric fields superimposing we will have more energy than the sum of the individual energies.

Why so because we will clearly understand that here we are bringing in the point bringing in the charges be it point charge or infinitesimal continuous charge whatever it is in some electric field that existed from earlier. So, we have to do we have to perform more work and that more work is given by this term here. So, the energy of the electrostatic fields there the principle of superposition is not valid.