Electromagnetism
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Lecture – 34
Boundary conditions on electric field and potential

Ok, with that we analyze the Boundary Conditions that we have due to Electric Fields.

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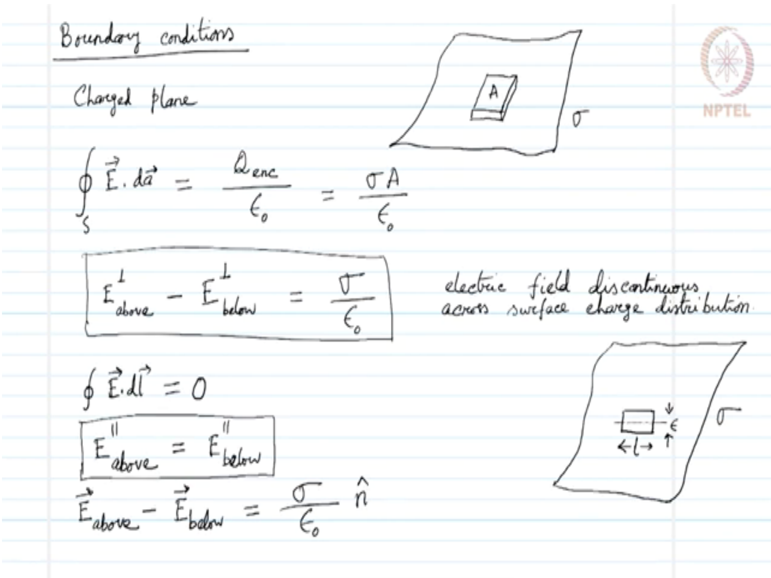

Boundary conditions are very important. So, we will consider a surface and charge distributed on that surface and if we go across that surface what happens to the electric field that is what we will try to analyze in this context.

Let us consider a charged plane I am trying to draw one here and we assume a charge density sigma on this plane and in order to find the electric field we consider a Gaussian pillbox here. It also extends below and its width is almost I mean the height of this pillbox is almost negligible, it is tiny. With that and if we have area of this pillbox as A, we can write that the closed surface integral over E dot da following Gauss law can be given as the total charge enclosed over epsilon naught which is sigma times A over epsilon naught.

So, from this we can only evaluate the perpendicular component of the electric field. So, here we have not considered unlike earlier a flat plane this plane can have some curve curvature and we have not considered something of infinite extent, it is a finite extent. With this situation we can write down that the perpendicular component of the electric field above this plane minus the perpendicular component of the electric field below the plane, this would be given as sigma over epsilon naught looking at this equation.

So, what we find here? We clearly see that if there is a surface charge distribution with uniform density sigma the perpendicular component of the electric field above and the perpendicular component of the electric field 0 below that is the difference is not 0; that means, they are not the same. The electric field is not continuous across this surface charge distribution ok.

In that situation, we have to find out the tangential component of the electric field also. If we are supposed to find out the tangential component of the electric field, we have to consider similar charge distribution with density sigma and this time let us consider a loop across this charge distribution this surface. So, this loop cuts the surface here. The surface cuts the loop here.

Let us consider the length to be L and epsilon be the width of this loop which is very small. So, if we consider a cyclic integral over E dot dl on this loop that will be 0 because curl of electric field is 0. So, any cyclic integral of electric field any cyclic line integral of electric field is going to be 0. And, if that has to be the case, then we must have the parallel component of the electric field above equals the parallel component of the electric field below. So, we must have E parallel equals E parallel below.

Now, summarizing this expression and this expression if we write this in full vector form, then we can have E vector above minus E vector below that equals sigma over epsilon naught n cap, where n cap is the normal to this surface on which we have the charge distribution.

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$$V_{above} - V_{below} = -\int_a^b \vec{E} \cdot \vec{dl} \qquad \lim_{(a \to b) \to 0} -\int_a^b \vec{E} \cdot \vec{dl} = 0$$

$$V_{above} = V_{below}$$

$$\vec{\nabla} V_{above} - \vec{\nabla} V_{below} = -\frac{\sigma}{\epsilon_0} \hat{n}$$

$$\frac{\partial}{\partial n} V_{above} - \frac{\partial}{\partial n} V_{below} = -\frac{\sigma}{\epsilon_0}$$

$$\frac{\partial V}{\partial n} = (\vec{\nabla} V) \cdot \hat{n}$$

How about the potential? Is that continuous or discontinuous? This would be given as minus integral over point a to point b; a is above b is below let us consider it this way E dot dl. Now, if we consider that the length of this path tends to 0, then the right hand side goes to 0. This when this length of this path tends to 0, then this integral minus a to b minus integral over a to b E dot dl that goes to 0; that means, the potential above the plane equals the potential below the plane.

There is no discontinuity in the potential and potential being a scalar has the only one component. So, it is continuous and it has to be continuous although the electric perpendicular component of the electric field is discontinuous because the first order derivative of the potential gives us the electric field. If the potential is discontinuous then it would not be differentiable and we would not have the concept of electric field. So, the potential must be continuous. So, we can write that the gradient of this potential above minus gradient of the potential below that is not continuous and this quantity can be expressed as minus sigma over epsilon naught n cap which is just representing the discontinuity in electric field in terms of potential that is all.

And, we can write it actually a little more conveniently and del n that is normal derivative of the potential above minus the normal derivative of the potential below is given as minus sigma over epsilon naught we can write this, where the normal derivative can be represented as gradient of the potential dot product with the normal direction.