Electromagnetism
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Lecture - 32
Scalar potential

So, we have the expression for curl of the electric field and that is 0, for any arbitrary charge distribution in general as long as we consider electrostatics.

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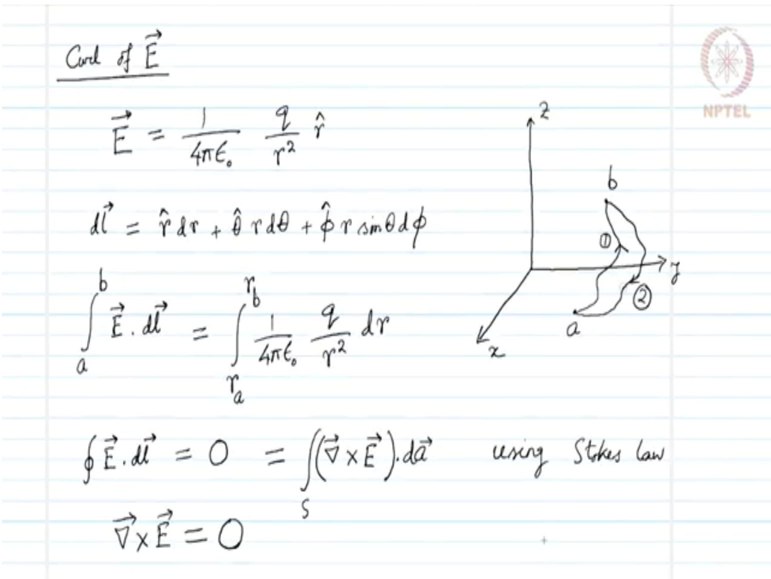

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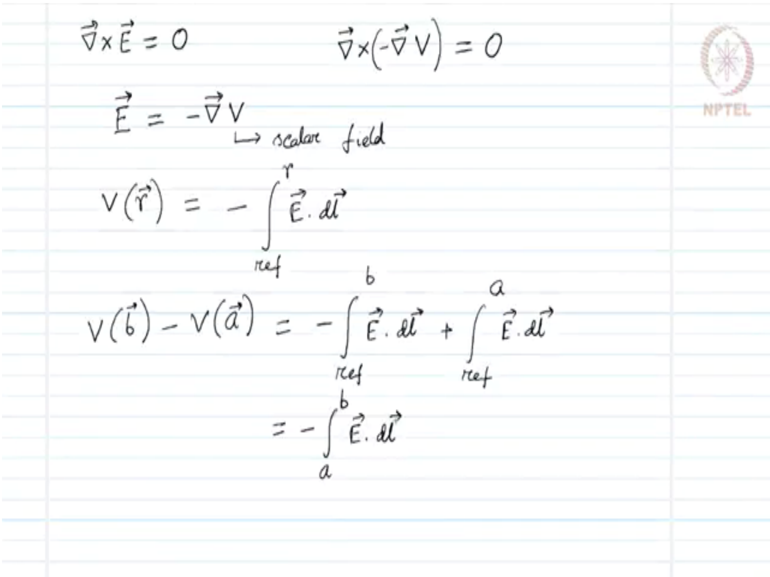

And if that is the case, let us develop something new from this. We have curl of the electric field going to 0 and in vector calculus what have we find? If we have the gradient of a scalar field then the curl of that gradient goes to 0; that means, we can write the curl of the gradient of a scalar field equals to 0. Minus sign is just a convention. I will explain why we take a minus sign here; that means we can express electric field as the gradient of a scalar field V.

That is very interesting. We had a vector E that had three components in three dimensional space. Now we have found a scalar field V, that has only one component because it is a scalar and we can express the vector field completely by the scalar field by writing it as the gradient of the electric, gradient of the scalar field; that is very interesting. And why do we have this negative sign? Because we want to have physical meaning associated with this scalar field V, we call it the electrostatic potential or the electric potential. So, we can define this as from any integration over from any arbitrary reference to r E dot dl.

This is how we can find this scale the scalar quantity and if we have this kind of a situation then the difference of this potential from point b; position vector b V as a function of position vector V minus V as a function of position vector a. The difference in potential that will be given as minus integration from reference to point b E dot dl plus integration over reference to point a E dot dl which is minus integral over point a to point b E dot dl.

That means, the difference in potential suggests the integral the line integral from point a to point b over the electric field, because it is not path dependent, it equates to just a scalar and it is always valid because the scalar we have just the function of two points nothing else.

Now, with this expression if we have this negative sign we can have positive potential due to positive charge due to positive electric field that is why we have this negative sign in here. We will find out in some examples how that works. What would this potential physically mean? If we consider central force like electric field or gravitational electric force electrostatic force or gravitational force, we can get the work done by the potential difference.

So, here the difference in potential from point b to point a gives us the work done for taking a unit charge from point a to point b. If we have a q amount of charge, we can multiply that with the potential difference and get the amount of work done. So, the potential difference is the amount of work done in taking one charge one unit charge from point a to point b, that is how we can physically interpret the scalar potential in the context of electric field.

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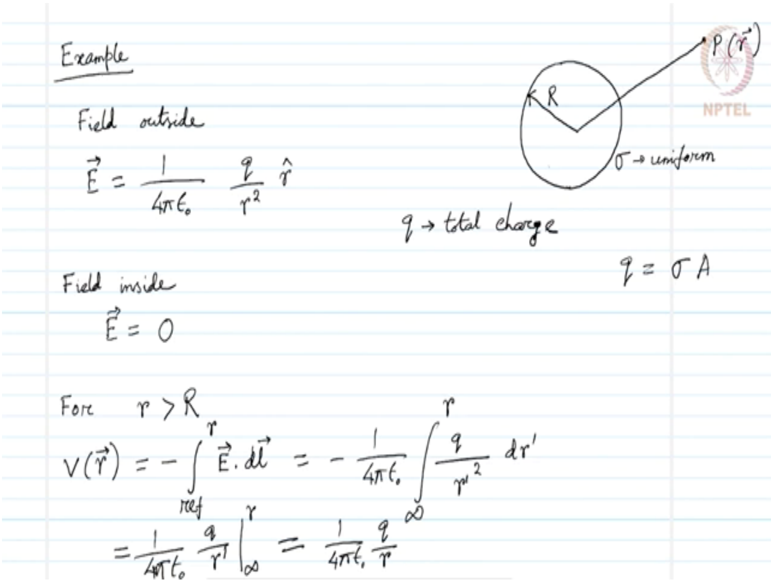

Now, let us consider an example. Let us consider a sphere of radius R capital R and let us consider an arbitrary point P of observation at position vector small r. We want to find the potential inside and outside the spherical shell of radius r that carries uniform surface charge. So, this is a spherical shell carrying a surface charge density sigma that is uniform. And we need to set the reference point at infinity because at infinite distance from the sphere we will assume that the electric field and electric potential both go to 0.

The field outside this sphere with surface charge distribution sigma can be given as E equals 1 over 4 pi epsilon naught q over r squared r cap, where q is the total charge on the sphere. So, what does q have to do with sigma? q will be equal to sigma times the area of the sphere, it can be given like this because sigma and sigma is uniform charge density, just multiplying it with the area would give us the total charge. With that this is the field outside. We must find and this was found using Gauss law, applying Gauss law you can easily find this expression for the field.

How about the field inside the sphere? We can see that if we consider a Gaussian surface inside the sphere that will enclose no charge. And if no charge is enclosed then the electric field inside the sphere inside the spherical shell will certainly be 0 from Gauss law. And with that we are supposed to find out the potential outside as well as inside.

For potential outside for points outside the sphere, we will have r greater than capital R. And in that case we can find, we can write the expression for the potential as minus integration over the range reference to r E dot dl and this is given as minus 1 over 4 pi epsilon naught integration, the reference is infinity from there we come to r q over r prime squared d r prime. And this if we evaluate we find 1 over 4 pi epsilon naught q over r prime infinity r, these are the limits that gives us 1 over 4 pi epsilon naught q over r.

So, we see that for a positive charge distribution we get a positive electric field outside the sphere and corresponding to that is a positive potential. So, in order to make positive potential for a positive charge distribution we have set the convention that the negative gradient of the scalar field is the electric that represents the electric field, the negative gradient of the potential represents the electric field.

So, this is the expression for electric field for small r greater than capital R ok. So, we have found the expression for electric field for small r greater than capital R that is, outside the sphere. Now let us try finding the expression for the electric potential inside the sphere where small r is less than capital R.

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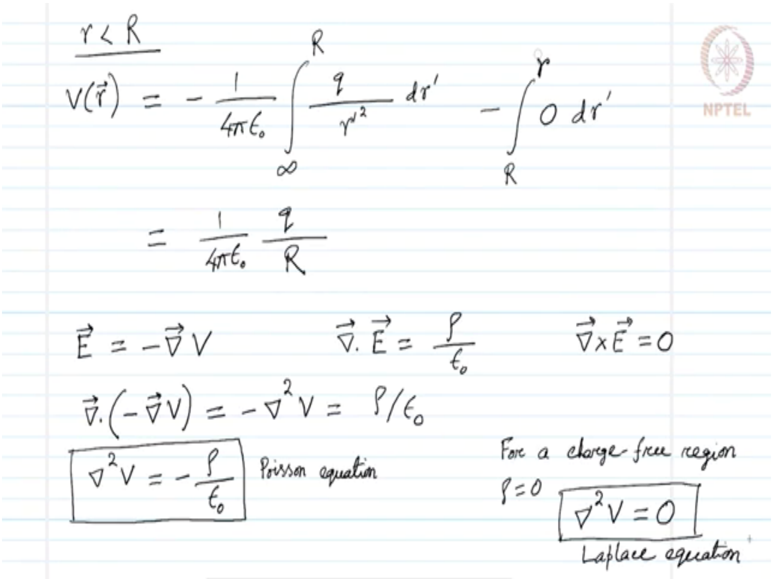

And for that we need to perform two integrals: for r less than capital R we will have the potential given by minus 1 over 4 pi epsilon naught integration infinity to capital R on the surface of the sphere and outside we have an electric field. So, we are writing this expression for the electric field minus from capital R to small sorry capital R to small r we have no electric field, so this integration over is over 0.

And that means, it will give us 1 over 4 pi epsilon naught. The total charge on the sphere over capital R. This will be the expression for the potential electric potential inside this sphere; that means, inside this sphere where there is no electric field we have a finite electric potential, but this potential is constant. It is not a function of the position of the point of observation inside the sphere that is interesting. Now let us consider something interesting. We have electric field given as negative gradient of the scalar potential and we know that the divergence of this electric field is rho over epsilon naught.

If we have that then we can and we also know that the curl of this electric field is 0. With this we can write that the divergence of the negative gradient of the scalar potential is minus of Laplacian of scalar potential and that is rho over epsilon naught from the differential form of the Gauss law. Therefore, we can write Laplacian of the scalar potential is negative of the volume charge density over epsilon naught. This expression is the second order differential equation and is known as the Poisson equation.

Now, if we consider a charge free region, we will have rho equals 0 and Poisson equation will turn into del square V equals 0. This equation we call Laplace equation and we will see how important these two equations are in the context of electrostatics.