Electromagnetism
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Lecture – 30
Tutorial on Electrostatics

Hello. So, now, we are going to do the fourth Tutorial that is on Electrostatics.

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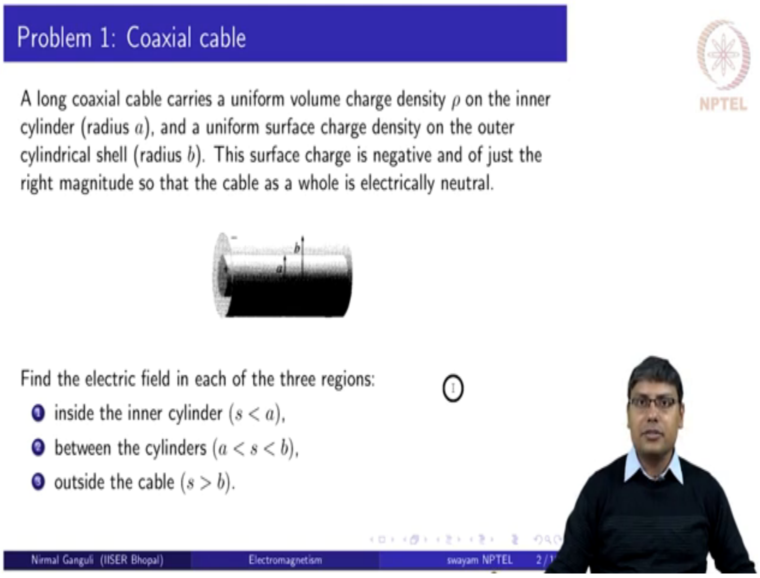

In the first problem in this tutorial is about a coaxial cable. You can see that there is one cable inside one cylinder and one cylindrical shell outside of it, that makes a coaxial cable. For the inner cylinder the radius of that cylinder is a and the outer shell its radius is b. And we have a uniform volume charge density rho on the inner cylinder that is of radius a; and we have a uniform surface charge density on the outer cylindrical shell.

So, that is only surface charge density that is we can call it sigma. But the surface charge density is negative rho is positive the volume charge inside the inner cylinder is positive. And the surface charge is negative and its of the right magnitude that makes the entire cable electrically neutral that is the amount of the charge; that means, the total amount of the charge in the entire cable is 0. You are supposed to find the electric field in each of the three regions; that means all space.

So, we can divide the problem into three parts inside the inner cylinder first when where s is less than small a; between the cylinders where a is less than s and less than b. And outside the cable where is where, s is less than b in the outer space. The coaxial cable is quite long so, we will assume it to be infinite. Now, you can pause this video and try solving the problem yourself, once you are done trying with it you can restart the video and see the solution ok.

After you have worked this out, please try to plot the magnitude of the electric field as a function of s; s is the distance from the axis of this cylinder. Now, you can pause the video again try to plot the electric field and then restart the video to check the solution.

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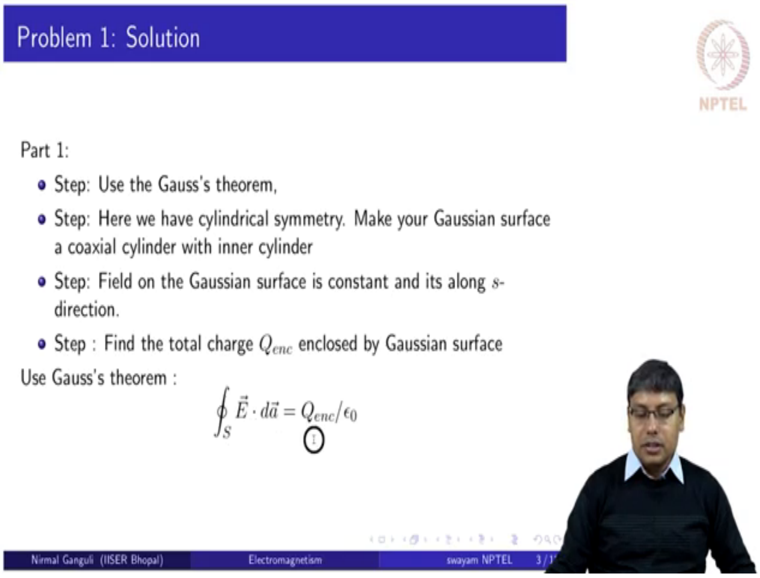

Let us go through the solution. In part 1, the steps to solve the problem is first you have to use the Gauss's theorem. Here we have cylindrical symmetry. So, make your Gaussian surface a coaxial cylinder with the inner cylinder. And in the first one we will have the inner Gaussian surface inside the inner cylinder.

Filled on the Gaussian surface is constant and its along s direction the radial direction. And finally, find the total charge enclosed by that Gaussian surface that will help you find the electric field. So, we have this Gauss theorems enclosed surface integral of E dot da would give you the total charge enclosed qe nc over epsilon naught.

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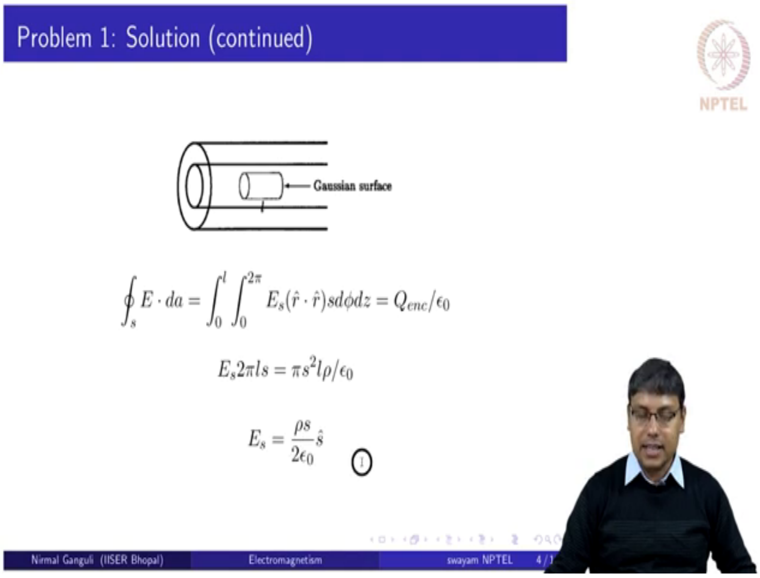

How do we do that? So, here we have shown a Gaussian surface inside the inner cylinder and the length of this Gaussian surface cylinder is l. So, we will have a closed integral over E dot da given as integration from 0 to l over z that is the length of this Gaussian surface. And integration over 0 to 2 pi over phi that is the azimuthal angle that we have E s times r cap dot r cap; s d phi d z and that will give us total charge enclosed over epsilon naught.

If we work this out, we will find the s component of electric field multiplied with twice pi l s that gives us pi s square l times rho over epsilon naught. So, E s can be given as rho times s over twice epsilon naught and its along the s cap direction.

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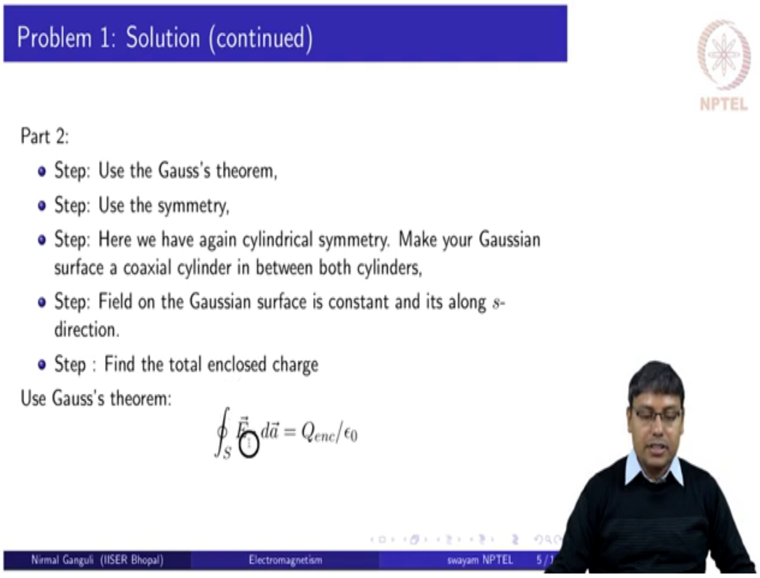

And in the second part, we are supposed to find the electric field in between the cylinders. So, there we have to consider again the Gauss Gaussian the Gauss theorem, we have to consider the symmetry and we need again a cylindrical Gaussian surface. And this time this Gaussian surface will enclose the smaller cylinder but it will not enclose the larger cylinder; it will lie somewhere in between the cylinders; that means, it will be inside the coaxial cable.

And the field on the Gaussian surface is constant and it will again be along the s direction for sure. Now you are supposed to find the total enclosed charge and then you will be able to find the electric field there. So, the Gauss theorem is closed surface integral over E dot da is given as total charge enclosed over epsilon naught.

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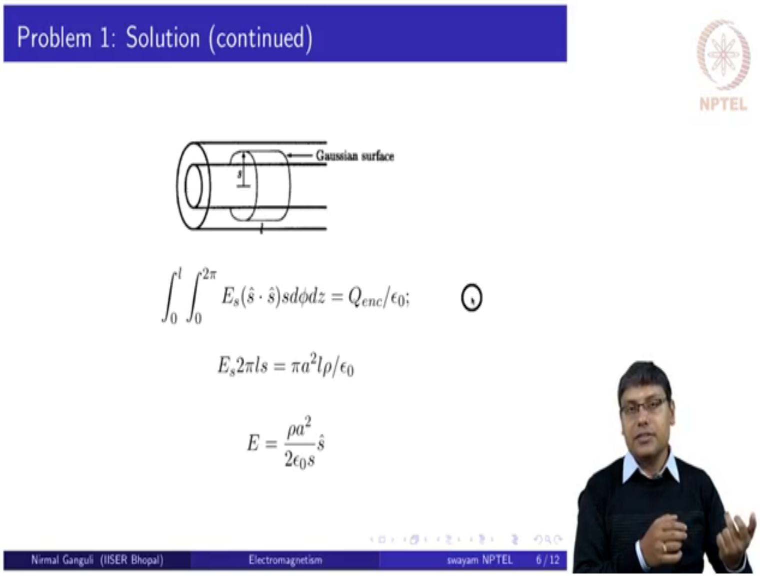

And the Gaussian surface will be drawn this way as shown in this picture here again we considered that it has a radius s and length l. Then we perform the surface integral over the Gaussian surface as we have seen earlier. Of course, we are we have not explicitly shown the surface integral with the flat surfaces because on the flat surface the direction of the surface is perpendicular to the electric field. Therefore, the dot product of E and da that will take it to 0 that part will not contribute to the left hand side of the equation. So, with this we can again find that the E s times twice pi ls is this time pi a squared naught s squared.

a squared a is the radius of the inner cylinder, times l rho over epsilon naught. So, the electric field this time can be written as rho times a squared over twice epsilon naught s; s cap. So, if we compare with the earlier situation, where the electric field was proportional to the radial distance. Here once we have come out of the charge distribution its proportional to 1 over the radial distance s; because a is a constant.

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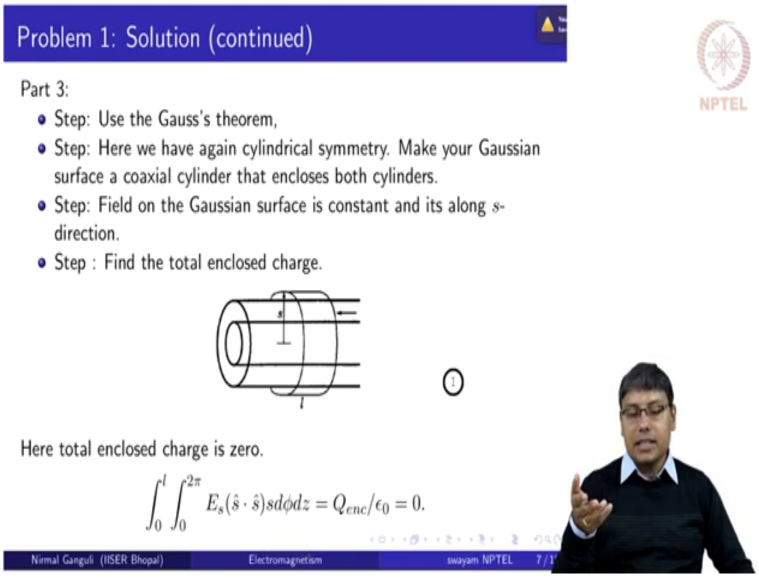

And in the part 3, we are supposed to find the electric field outside everything outside the entire coaxial cable. Again the method would remain the same, you are supposed to apply Gauss theorem you are supposed to consider the cylindrical symmetry and make a cylindrical Gaussian surface coaxial with the cylinder and that enclose both cylinders.

And then the field on the Gaussian surface will be constant and it would be along s direction therefore, you can integrate easily over E dot da and you are supposed to find the total enclosed charge. So, this time the Gaussian surface as we have discussed would lie outside the entire coaxial cable. And because its outside the entire coaxial cable the inner cylinder carries a volume charge the outer cylindrical shell carries a surface charge and they exactly cancel each other.

Therefore, the total charge enclosed by this Gaussian surface is 0 and if that is 0 then the then we can easily say that the electric field outside would also go to 0.

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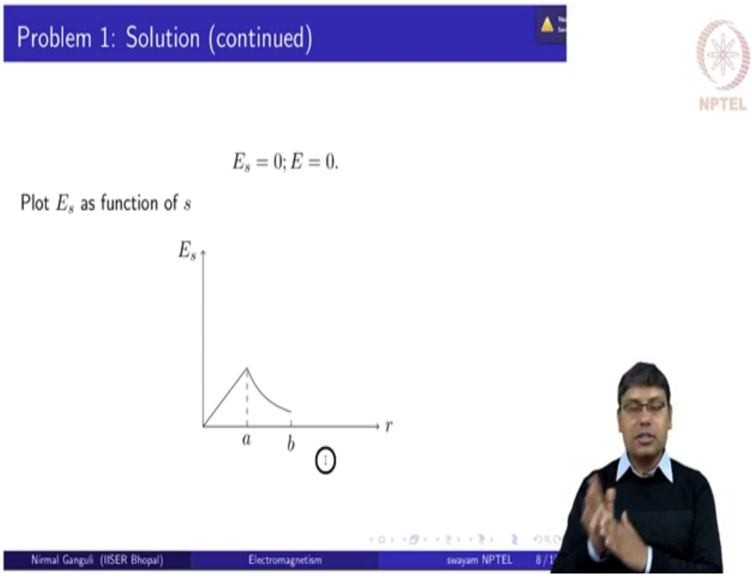

Now, comes the last part of the problem that is plotting E s as a function of s the plot could look something like this. Initially, the value of E s will linearly increase with increasing s and then it will decay as 1 over s up to r equals b; beyond r equals b we are coming outside of the coaxial cable and outside of the coaxial cable there is no electric field at all.

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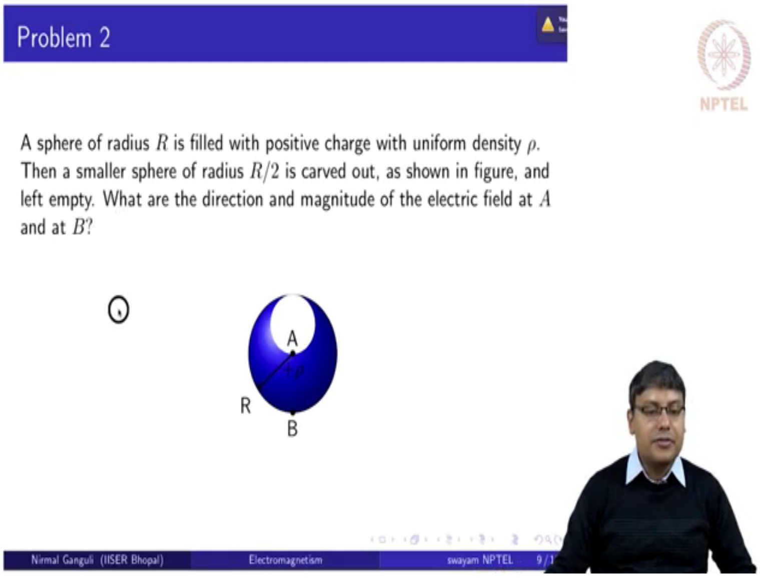

Let us consider another problem. We have a sphere of radius R that is marked as a blue sphere here and its filled with a positive charge with uniform charge density rho; its plus rho here we have marked. Then a smaller sphere of radius R over 2 is carved out, as shown in the figure and it is left empty. So, here we have a smaller sphere taken out of this bigger sphere.

So, the charge is also taken out there is no charge in this region white region now. So, you are supposed to find what are the direction and magnitude of the electric field at points A and B? The point A is here and the point B is here, you are supposed to find the direction and magnitude of the electric field at points A and B. Now, you can pause the video and try solving this problem. Once you are done with trying the problem, you can restart the video and look at the solution ok.

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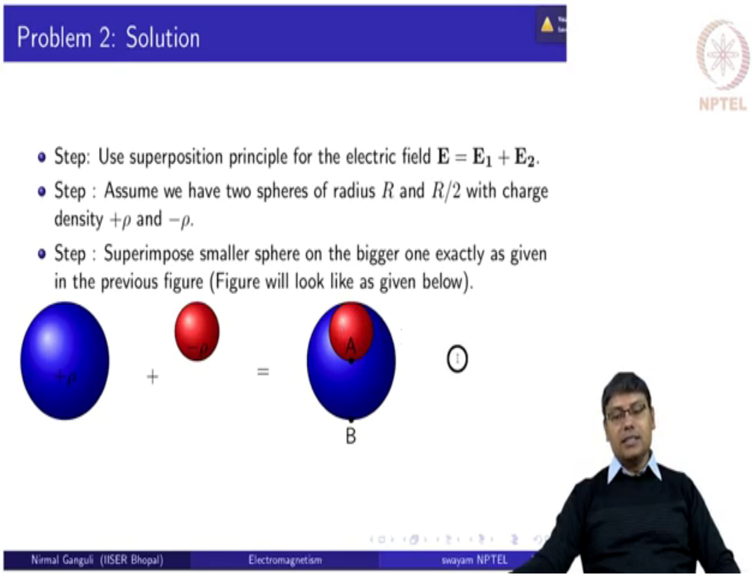

So, what are the steps that we will use to solve this problem? We use the superposition principle for the electric field. So, the total electric field is the electric field due to one part and the electric field due to second part E 1 plus E 2 gives us the total electric field. And we will assume that we have two spheres one of radius R and the other one of radius R over 2 with charge densities plus rho and minus rho. And if we superimpose the smaller sphere on the bigger sphere, we exactly simulate the situation that we are given.

So, this kind of a situation here is the bigger sphere with plus rho charge density and here is the smaller sphere in red with minus rho charge density. Now, if we superimpose them like shown here, the charge density in this smaller spherical region will become 0 plus rho and minus rho will cancel each other exactly.

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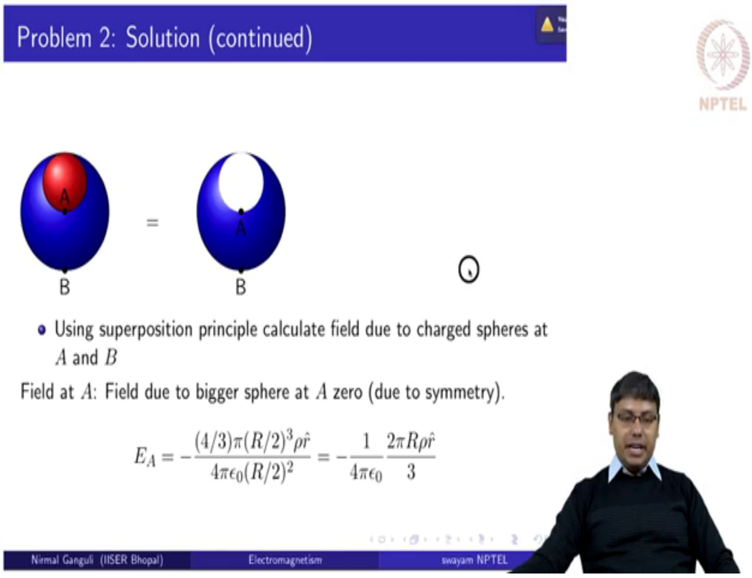

So, after doing this so; that means, this situation is equivalent to this situation that is that we are given with. So, we can find the electric field corresponding to this situation. By applying this principle of superposition on this situation and applying principles of superposition on this principle this situation is much easier using principle of superposition now you can calculate the electric fields at point A and point B.

How do we do that? Field at point A would be due to the bigger sphere at A and with for due to the bigger sphere, if we consider this point here A; that is at the center of the bigger sphere. So, there will not be any electric field due to the biggest sphere here. And if we consider the contribution from the smaller sphere with negative charge we can find out the electric field due to the negative charge of the smaller sphere as minus 1 over 4 pi epsilon naught, twice pi capital R rho over 3 along r cap direction, that will be the magnitude of electric field at point A.

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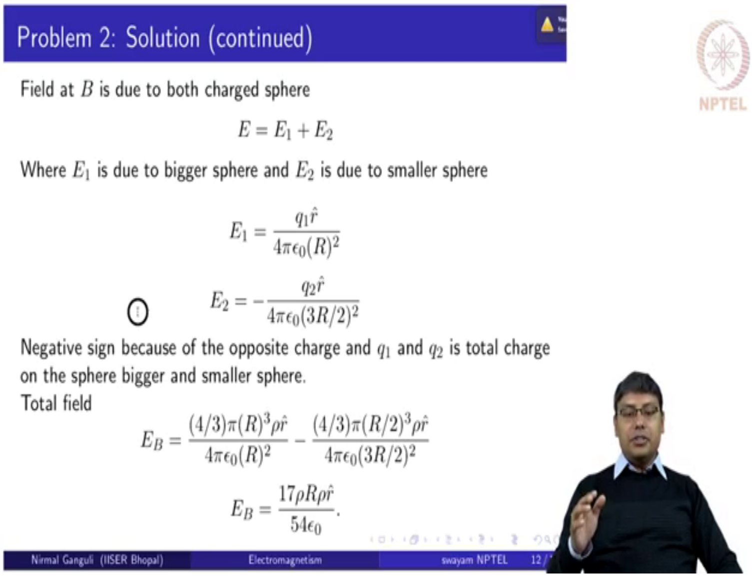

And point B will have the contribution coming from both spheres. So, there we can have E equals written as E 1 plus E 2 and E 1 is due to the bigger sphere E 2 is due to the smaller sphere. So, E 1 can be given as q 1 r cap over 4 pi epsilon naught times R squared. And E 2 that can be given as minus q 2 r cap over 4 pi epsilon naught times 3 R over 2 whole squared. The negative sign is because we have opposite charge in the form of q 1 and q 2 q 1 is positive charge and q 2 is negative charge.

So, the total charge on the bigger sphere and the smaller sphere we have to calculate in terms of the volume charge density that is given. And by doing that we can find the expression for E at point B as 17 rho R r cap over 54 epsilon naught.