Electromagnetism
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Lecture - 03
Vector triple products

So, after learning about dot product and cross product of two vectors, we will now be learning the triple products. That means involvement of three vectors.

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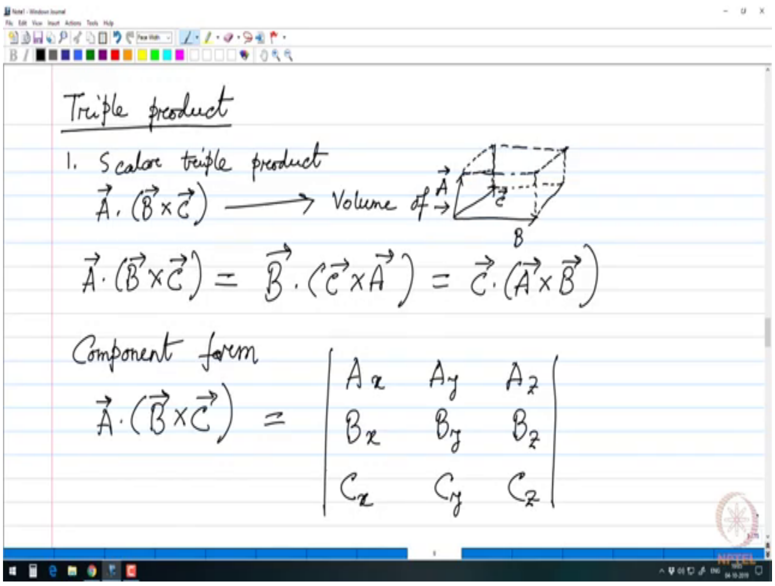

You can have two different type of triple products, as a result of the product one type becomes a scalar that we call scalar triple product. It is given as three vectors A, B and C: it is A dot B cross C. This B cross C is a vector and when you make dot product of that with A that makes the entire product a scalar one.

And if we try now to draw these vectors we can have say this is vector B, vector C is like this and this is vector A not in the plane of B and C. So, the vector B and vector C gives us this kind of parallelogram and now if we take a vector A into account we will have drawing a line parallel to B here, drawing a line parallel to C here, making a similar parallelepiped here we will get, sorry, making a similar parallelogram above this we will get a parallelepiped. The drawing is not that good, but this is the idea. And if you look at the product the scalar triple product, this represents the volume of this parallelepiped. This scalar triple product is the volume of this 3D structure here.

And scalar triple product obeys the rule. That means, A dot B cross C equals B dot C cross A equals C dot A cross B in cyclic order. And if we break this cyclic order then we get a minus sign. How do we write this product in component form? In the component form it can be written as A dot B cross C equals A x A y A z, B x B y B z, C x C y C z at determinant of this.

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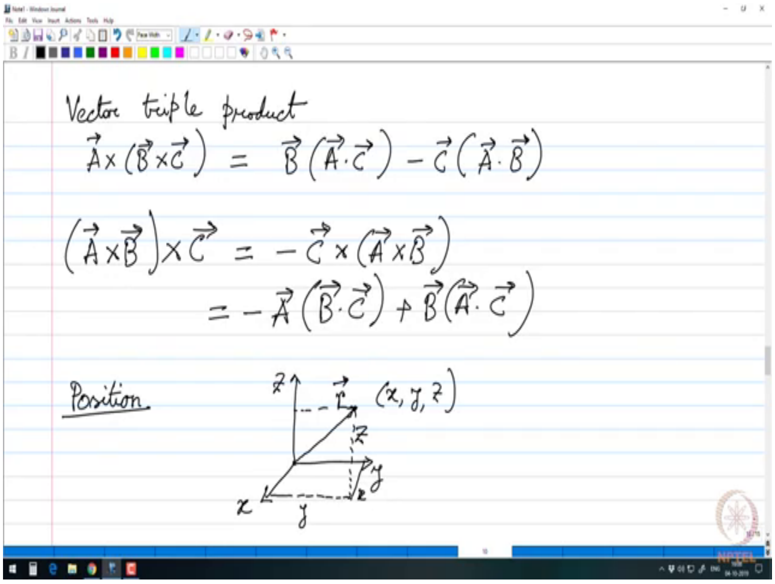

Now, let us come to vector triple product. Vector triple product is represented by A cross B cross C. And that equals to B times A dot C minus C times A dot B. We also note that if we have A cross B cross C that is minus of C cross A cross B. And that becomes minus A B dot C plus B A dot C. And this quantity is completely different from A cross B cross C that we have started with.

Now, let us consider some vectors vector quantities that we are habituated with that represent some interesting physical quantities, like position. So, in a Cartesian coordinate system the position of a particle let us say is here that can be represented by a position vector like this. This is usually represented as r, and it has three components: x, y, and z and those that is exactly the coordinate of this particle.

So, this is the x component, this is y component, and this is the z component of the position vector: this part x, y, z.

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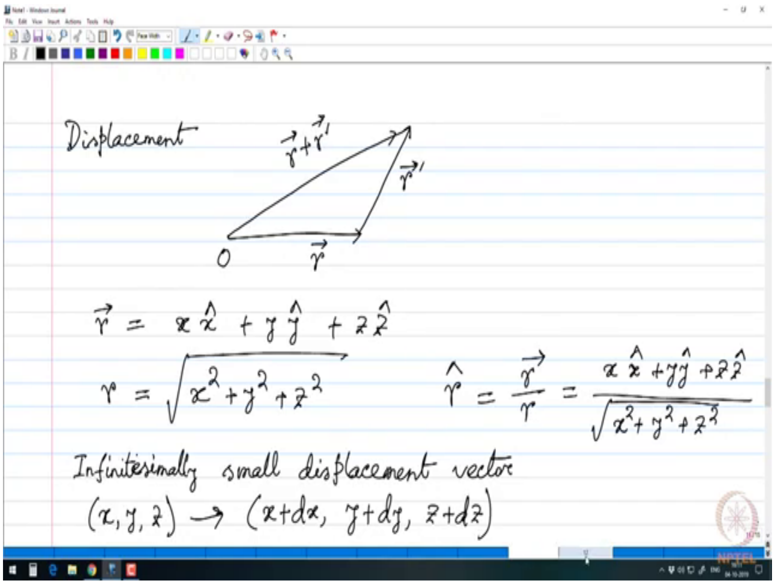

We can also consider displacement vector. If from the origin O we have a displacement along this direction of amount r and along this direction of a vector r prime, then we know that the resultant displacement is given as this vector that is r plus r prime.

So, the position vector of a particle in a coordinate system can be written as x x cap plus y y cap plus z z cap in a Cartesian coordinate system. The magnitude of this position vector r is given as x squared plus y square plus z square, square root of this quantity. And the unit vector along this position vector is given as r vector over the magnitude of this. That means, x x cap plus y y cap plus z z cap over x squared plus y squared plus z square square root of this.

Now, if we consider an infinitesimally small displacement vector from the coordinate x, y, z to the coordinate x plus dx y plus dy z plus dz.

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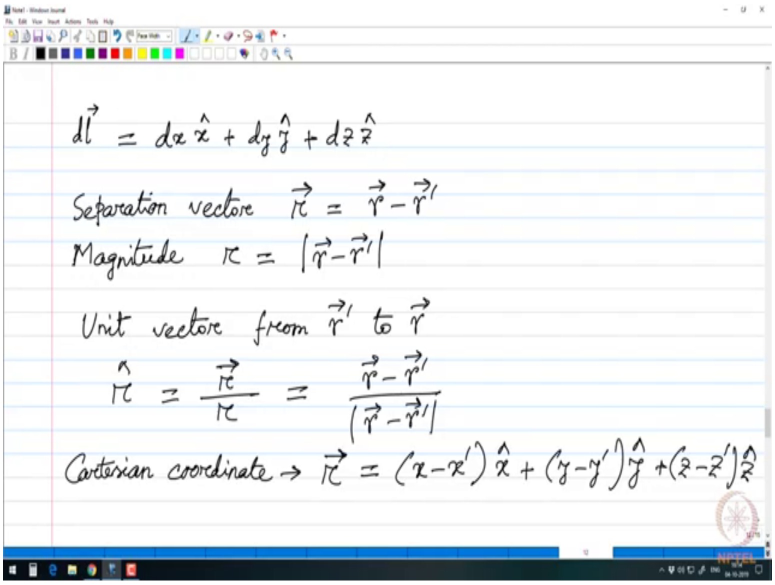

Then, this small displacement can be represented by an infinitesimal displacement element dl that is dx x cap plus dy y cap plus dz z cap.

The separation between two points, that is the separation vectors vector can be given as; the difference between position vectors r minus r prime and the magnitude of this separation vector is the absolute value of r minus r prime vector. And a unit vector in the direction from r prime to r can be given as; curly r cap that is curly r vector over the magnitude of curly r that is r minus r prime vector over r minus r prime absolute value.

In Cartesian coordinate system this curly r vector; that is the separation vector can be represented as x minus x prime x cap plus y minus y prime y cap plus z minus z prime z cap.

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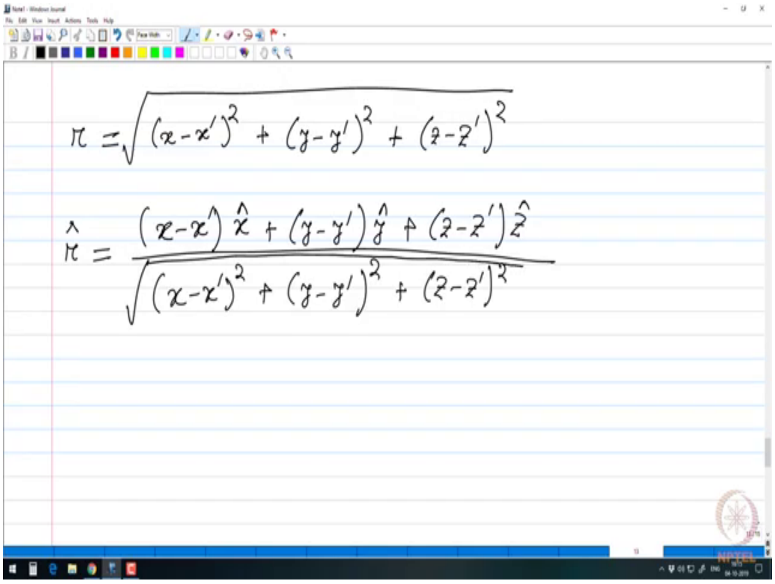

So, the magnitude of the separation vector is given as: x minus x prime squared plus y minus y prime squared plus z minus z prime squared and square root of this entire thing. Similarly, by whatever we have learned earlier the unit vector along this direction; that is unit vector along the direction of the separation can be given as: x minus x prime x cap plus y minus y prime y cap plus z minus z prime z cap over x minus x prime square plus y minus y prime square plus z minus z prime square and square root of this.