Electromagnetism
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Lecture – 29
Tutorial on Dirac delta function and electrostatics

Hello, in the third tutorial we will discuss Dirac Delta Function and we will also try to evaluate electric field in some simple configuration using different methods. So, you are already familiar with Dirac delta functions.

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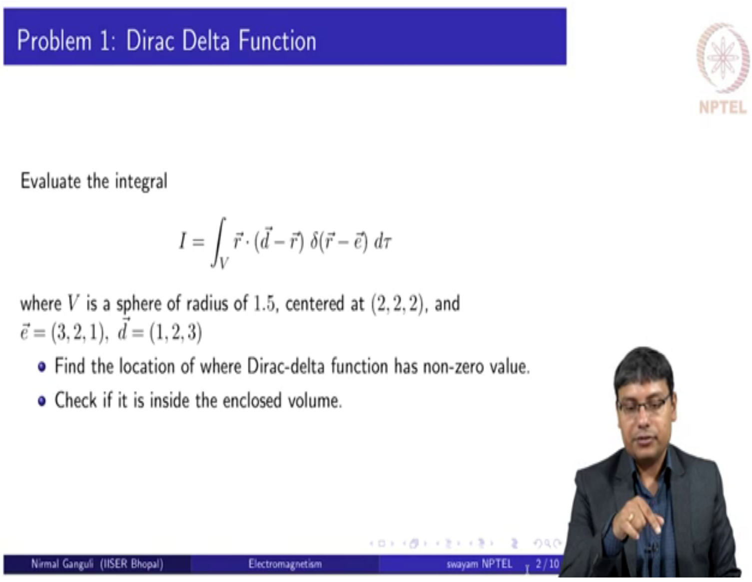

Let us evaluate this integral I equals a volume integral over r vector that is position vector dot d a constant vector minus r the position vector times the Dirac delta function r with argument r minus e integrated over the volume. So, the volume element is multiplied that is d tau.

And the V the volume that we have is that is a sphere of radius 1.5 units centered at the point 2, 2, 2. And we have 2 constant vectors here; e is given as 3, 2, 1; that is 3 x cap plus 2 y cap plus z cap. And d is given as 1 2 3 that is x cap plus 2 y cap plus 3 z cap. You are supposed to find this integral. Please pause the video and try to evaluate the integral after trying please replay restart the video so, that you can see how it can be solved ok.

So, the steps that you need to follow in order to find this integral is to find the location where the Dirac delta function has nonzero value; that means the argument of the Dirac delta function goes to 0. And check if that point is enclosed within that volume within the given volume here. If that point lies outside the volume then this integral will be anyway 0; because except the point where the argument of delta function goes to 0, everywhere else the Dirac delta function will return 0. Therefore the first thing is to check whether the point where the argument goes to 0 is inside the volume that we are interested in.

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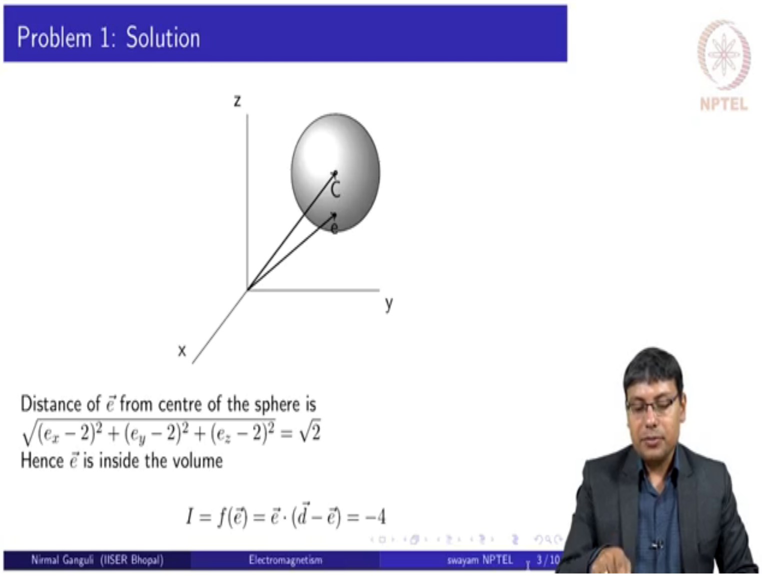

So, here is the situation we have drawn a sphere in x y z Cartesian coordinate system with the center at 2 2 2 and the radius is half. And if we now locate the e vector e vector we find somewhere inside that sphere. So, when r equals e according to this Dirac delta function, the argument of the delta function will go to 0; that means, the delta function will give us some nonzero value at that point.

So, we will have r equals e in order to have some nonzero value of the delta function. And for the other parts we can also put wherever we find r we can put the value of e there we can perform the integral that way. So, let us find the distance from the center of the sphere. So, this distance of e from the center of the sphere we have the expression of e given. So, e x minus the coordinate of the center x coordinate of the center that is 2 square plus e y minus 2 square plus e z minus 2 square if we take a square root that square root of 2. And if that is square root of 2 so, since the radius of this sphere is half and square root of 2 is less than half e will be inside the volume and we have pictorially shown that e lies inside the volume.

So, I would be a function. So, integration was a function of r times the Dirac delta volume integral of that. So, when r equals e the direct integration with Dirac delta will give us the function of e; that means e dot d minus e that will be the result of this integration which is minus 4. So, minus 4 would be the answer of this integration that we are supposed to perform ok.

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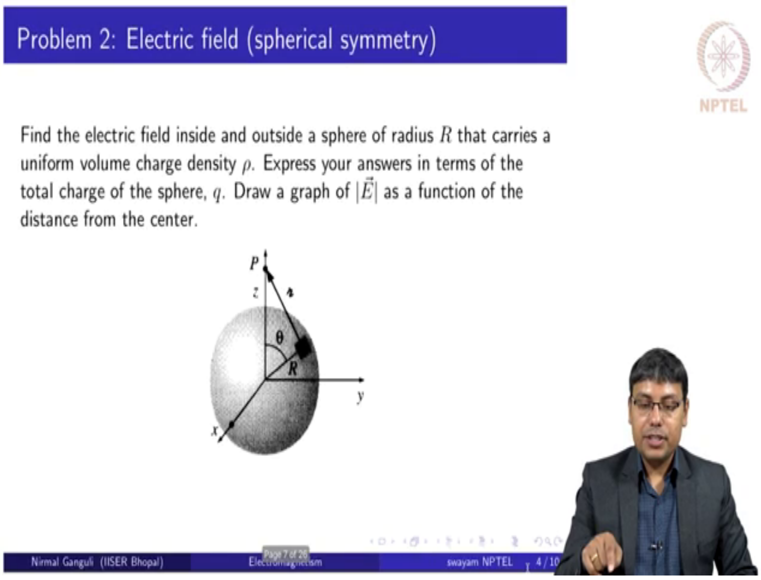

After solving this problem with Dirac delta, let us move on to finding the electric field in the presence of spherical symmetry. So, you are supposed to find the electric field inside and outside of a sphere of radius capital r and this spheres carries a uniform volume charge density rho. Express your answer in terms of the total charge of the sphere let us assume that the total charge of the sphere is q and after expressing your answer in terms of q draw a graph of the magnitude of e as a function of the distance from the center ok. You can pause the video now and try to solve this problem, once you are done or you are stuck you can restart the video and move on.

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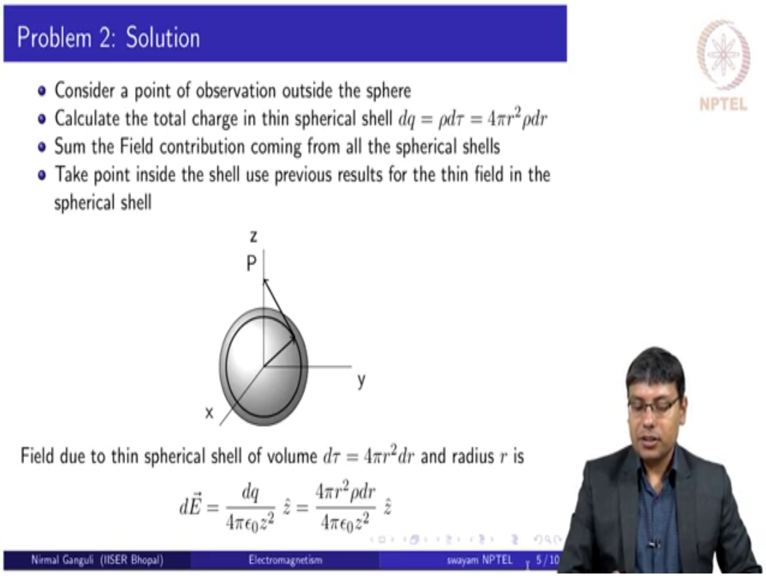

Let us consider a point of observation outside the sphere to begin with. And we can calculate the total charge in thin spherical shell. So, if we consider a spherical shell, the total charge on that shell we can call it dq and that will be rho times the volume of that thin shell that is rho d tau that is 4 pi r square times rho d r. And then we can sum the field contribution coming from all such spherical shells and that will give us the field. And then we can take the point of observation inside the shell somewhere and use the previous result for the thin film a thin field in the spherical shell.

What are we talking about? So, here as is shown here we can consider a P point of observation inside the sphere we can consider a spherical shell the dark shell that is marked here in the sphere. And the there is dq charge on it if uniform charge distribution is there rho that will be rho times d tau. So, d tau is given as 4 pi r squared d r for radius r. So, the differential so, the elemental electric field due to this shell would be dq over 4 pi epsilon naught z squared z cap if our point of observation is on z axis.

Now, we can orient the Cartesian coordinate system in such a way that our point of observation falls on z axis. So, thinking that our point of observation is on z axis is no loss of generality its completely general. With that we can write down that the elemental electric field is d e equals 4 pi r square times rho dr over 4 pi epsilon naught z squared z cap.

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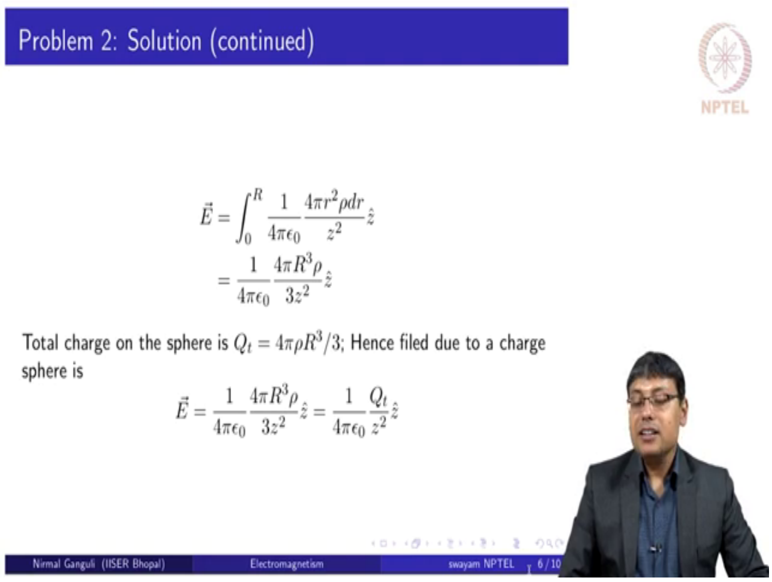

We can now integrate over all such shells to find the total electric field. So, integration over 0 to r 1 over 4 pi epsilon naught times 4 pi r squared rho dr over z squared along that cap direction this integral will give us the total electric field which turns out to be. If you perform the integration carefully 1 over 4 pi epsilon naught times 4 pi r cubed times rho over 3 z squared and this is along the z direction.

Now, the total charge on the sphere that is Q t Q total that is 4 pi rho R cubed times 3; that is the volume of the sphere times the volume charge density. So, the field due to the charge the density in terms of the total charge can be given as 1 over 4 pi epsilon naught q over z squared z cap.

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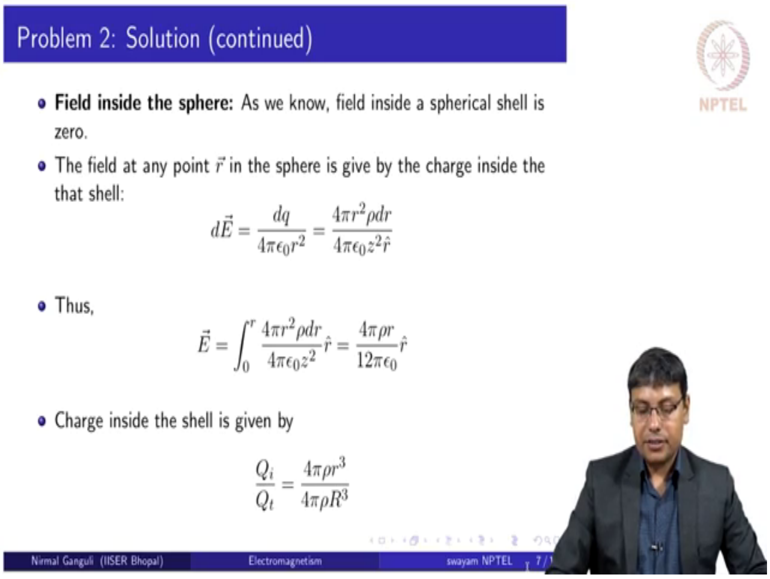

If we now want to find the field inside the sphere, as we know the field inside a spherical shell is zero. So, if we consider a point r somewhere in the sphere then the electric field due to the spherical charge distribution at that point will come from whatever charge is inside the sphere with that much radius.

That means, in this case we will have to consider. So, the differential amount of field element d E will remain the same that is 4 pi r squared rho dr over 4 pi epsilon naught z squared r cap. But, our limit of integration will change from 0 to small r instead of 0 to capital R. So, we cannot integrate over the entire sphere rather we have to integrate over integrate up to our point of observation not beyond that. So, the integration over r is from 0 to small r over the same quantity and that gives us 4 pi rho smaller over 12 pi epsilon naught r cap.

And now, if we express this in terms of the total charge, then the charge inside the part of our point of observation from the shell there if that is Q i that. So, Q i over Q t can be given as 4 pi epsilon sorry 4 pi rho r cubed over 4 pi rho capital R cubed.

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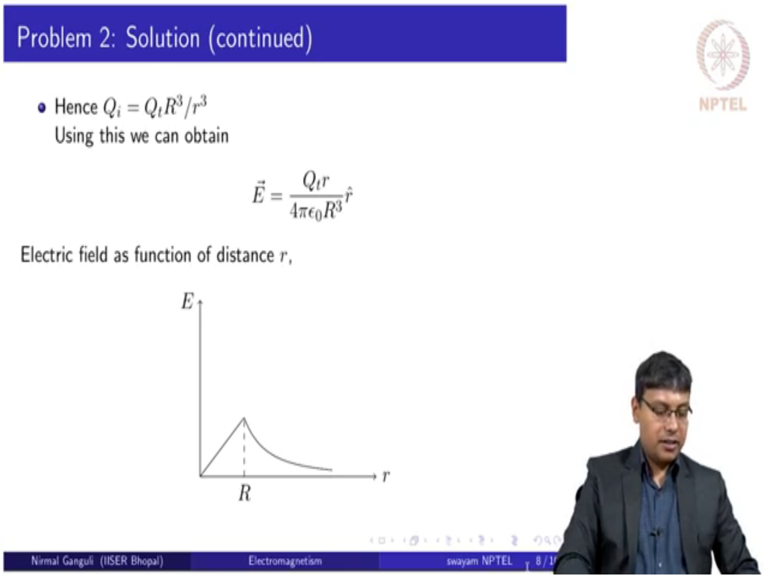

Qi would be given as Q t times capital R cubed over small r cubed using this and then we can now obtain the electric field as the total charge Q t times small r over 4 pi epsilon naught capital R cubed r cap. And now we are supposed to plot the electric field as a function of small r; if we do that inside we can see that the electric field is linear in r small r.

So, the electric field increases as we increase r up to capital R and it takes the maximum at capital R what about outside? From the expression here outside the electric field goes as 1 over z squared 1 over distance squared. So, if it is 1 over distance squared it will have a decay like this 1 over r squared kind of a decay that we have plotted here beyond capital R.

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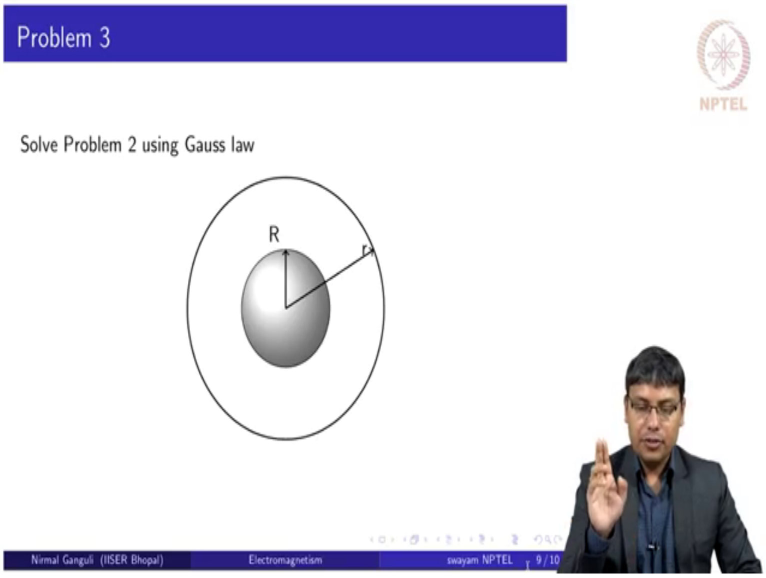

Let us come to problem 3 for this tutorial. In the problem 3 you are asked to solve the previous problem that is problem to find the electric field due to the spherical charge distribution inside and outside the sphere that is everywhere in space, but this time using Gausss law not using Coulombs law and performing so, difficult integrals.

How? So, you can pause the video here and then restart the video once you are done or once you are stuck ok. How do we do this? We can we have this sphere of radius capital R with uniform charge distribution and outside that sphere we have considered a large Gaussian surface its spherical. Because the electric field is uniform its along r cap direction and products perpendicular to this Gaussian surface sorry its not uniform, uniform means along 1 direction and r cap is changing.

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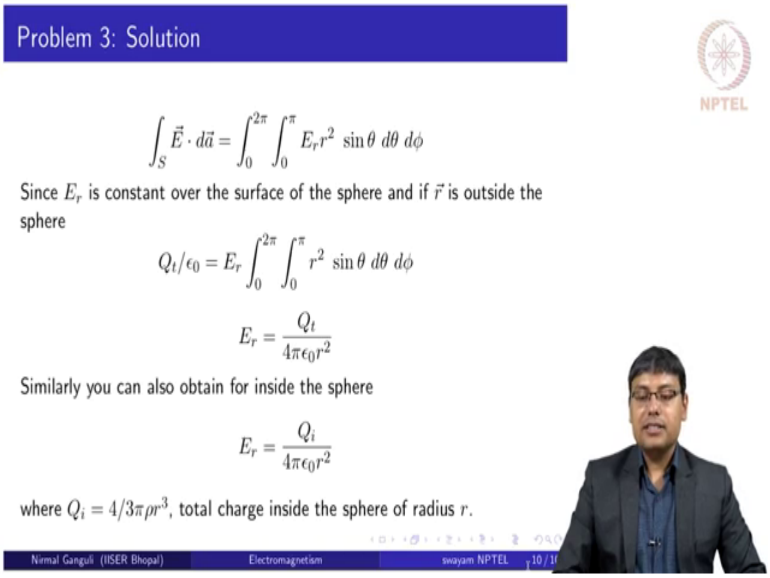

So, for this Gaussian surface we can write E dot da is given as integration over phi ranging from 0 to 2 pi integration over theta ranging from 0 to pi r component of the electric field times r squared sine theta d theta d phi. If we perform this then we will get the r component of the electric field. So, E r is constant over the surface and outside the sphere that now we can write total charge Q t over epsilon naught as E r times the integral over the surface element r squared sin theta d theta d phi over the sphere surface.

Where theta ranges from 0 to pi and phi ranges from 0 to 2 pi. So, E r would be given as Q t the total charge over 4 pi epsilon naught r squared. Similarly, you can also obtain the electric field inside the sphere that will be Q i that is the charge enclosed up to our point of observation not beyond that over 4 pi epsilon naught r squared. And we know that Q i would be 4 over 3 pi rho r cubed small r cubed. So, the total charge inside the sphere up to radius r and using whatever we did earlier the ratio between Q t and q r we can express this electric field also in terms of the total charge. Please do that and this is where we end the third tutorial.