Electromagnetism

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Lecture – 27
Application of Gauss law with cylindrical symmetry

Now, let us consider the Application of Gauss Law on interesting geometries.

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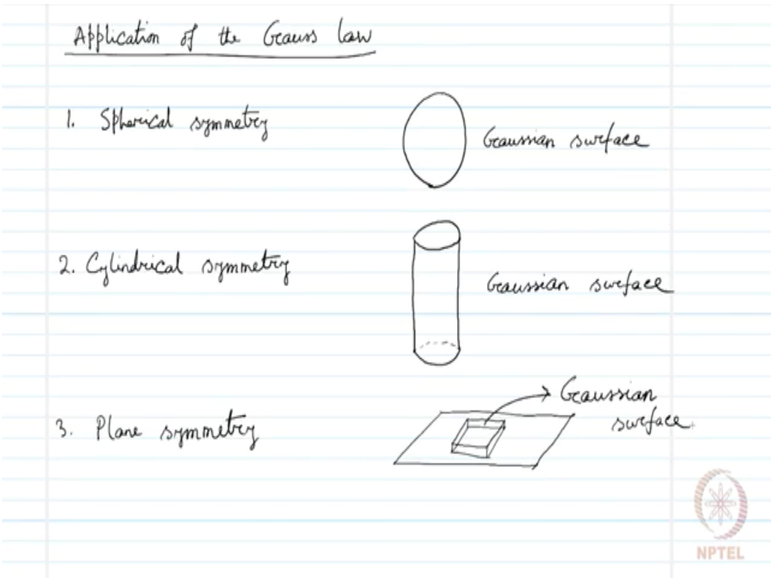

If we want to apply Gauss law, then, so Gauss law is valid everywhere. But when we apply Gauss law, we have to find out a surface on which the electric field is symmetric. That is necessary otherwise looking at few examples we will find that Gauss law although valid everywhere will not become of much use.

So, if we have spherical symmetry of the electric field, then we will have to consider a spherical Gaussian surface. If we have a cylindrical symmetry, then we will have to consider a cylindrical Gaussian surface and this also applies when we consider a line charge distribution because line with line charge distribution, we will have a cylindrical symmetry in the electric field. And if we have plane symmetry, then let us consider this kind of a plane, then electric field will point upward or downward and we can have this kind of a box acting as the Gaussian surface upward as well as downward.

So, this becomes the Gaussian surface then and if we do not have any of these symmetries then it becomes very difficult to apply Gauss law and find out the electric field. Although, Gauss law is valid everywhere no matter whether there is symmetry in the electric field or not. Let us consider a simple example to begin with.

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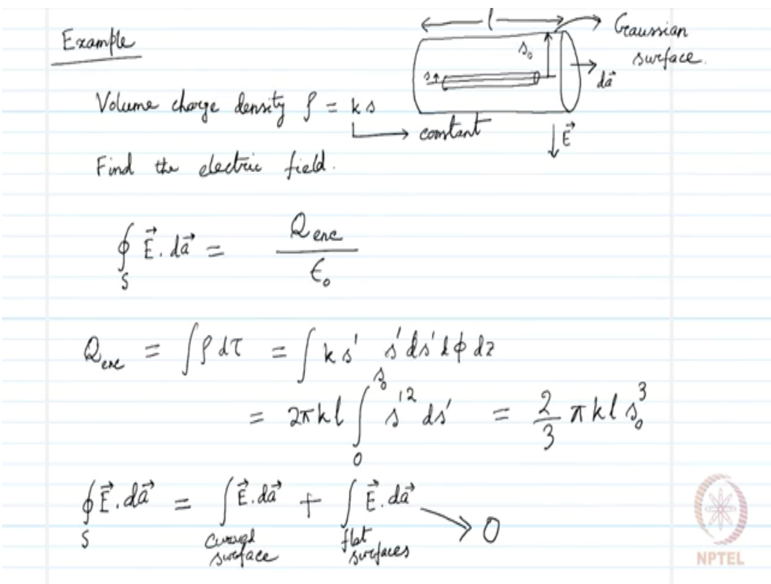

We considered a long cylinder like this. So, I have drawn it short. It is actually much longer than this and it carries a volume charge density. So, here is the axis of that cylinder. The distance from the axis is given as s and the volume charge density that it carries is a constant times the distance of the point from the axis.

So, we have a volume charge confined in this cylinder, very long cylinder and so, we will for practical purposes assume that the length of the cylinder is infinity, although I have drawn a short 1 for simplicity and the volume charge density on this cylinder is given as rho equals a constant k times s; the distance from the axis. k is a constant.

Now, we are asked to find the electric field for this kind of and this kind of a charge distribution. How do we go about solving this problem? We have to first consider the direction of this electric field. If the charge distribution is cylindrically symmetric, its only a function of s and not theta or z. That means, its cylindrically symmetric the charge distribution is itself cylindrically symmetric. Then, we will certainly have the electric field also cylindrically symmetric.

That means, we will have electric field at a distance at any distance from the axis of the cylinder at a given distance if that field will be constant over that cylindrical surface. So, electric field will also have cylindrical symmetry, it will only be a function of s and nothing else and that electric field will point along s direction, the radial direction along the direction of distance from the axis in no other direction. That means, if we consider a Gaussian surface like this, a bigger cylinder that is that has the same axis as the smaller one.

So, this one is our Gaussian surface and the length of this Gaussian surface cylinder is l and the length of the cylinder carrying charge is infinity for practical purpose. If we have this kind of a situation, then we can write surface integral in over a closed surface E dot da that equals total charge enclosed over epsilon naught. Now, what is the total charge enclosed? If we have l length for the cylinder forming this Gaussian surface, then whatever is within this surface that is the charge enclosed.

So, let us try to find out Q enc first; enclose total charge. That is nothing but the charge density times the volume element integrated over that is integration k s prime; prime because we have primed coordinate for source region and this one multiplied with the volume element is prime d s prime d phi d z and that means, because of cylindrical symmetry, we can take 2 pi outside. k is a constant, we can take it outside. The integration over z will give us l. We can take that outside and we are left with integration from 0 to s, this range s prime squared d s prime.

With this performing this integral, we can find the value to be 2 over 3 pi k l s cubed. So, this is the value of the charge enclosed provided the radius of this smaller cylinder inside that is the charge carrying cylinder is s. Now, if we consider the integral over the electric field over a closed surface, this integral will be over the Gaussian surface. So, we can divide this integral into 2 parts; integration over E dot da that is on the curved surface of the cylinder plus integral over E dot da that is on the flat surface of the cylinder, flat surfaces.

But looking at the symmetry of the problem, we realize that the electric field is along the radius of this cylinder of the. So, it is along the s direction, radially outward. If we have that kind of an electric field then over flat surface E dot da is going to be 0. Because d a on the flat surface would point in this direction which will be perpendicular to the electric field. The electric field is pointing in this direction or upward direction that is radially outward direction. This is the direction of da for flat surface. This is the direction of electric field anywhere. Therefore, this is going to give us 0 and we have to evaluate only this integral, the left hand side integral.

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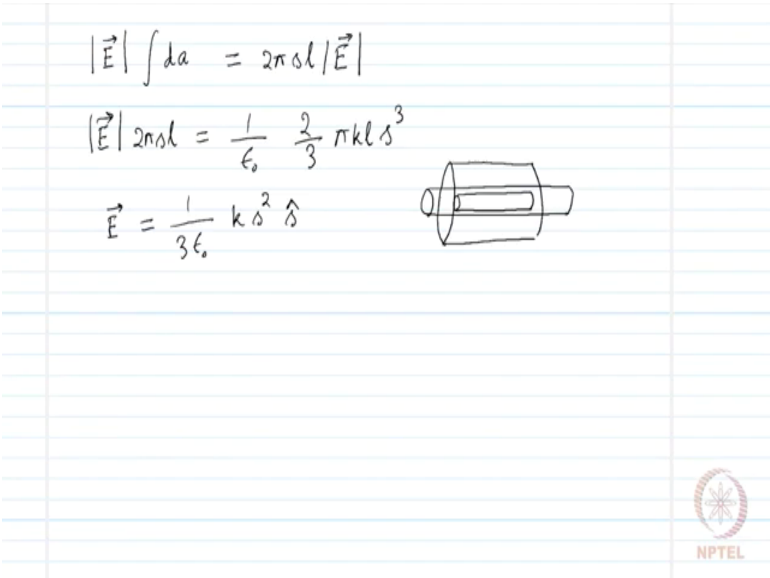

And if we do that, we will find for a constant magnitude of electric field for a constant value of s that is on the Gaussian surface, we will have this integral as given as this which is nothing but 2 pi s l times the magnitude of the electric field. So, we can equate the 2 quantities applying Gauss law and find the magnitude of the electric field times 2 pi s l equals 1 over epsilon naught multiplied with the total charge 2 over 3 pi k l s cubed and that makes electric field equals 1 over 3 epsilon naught k s squared s cap.

Now, this happens, if we consider the Gaussian surface to be within this material not outside the material. So, this result would be valid, if we have this kind of a material that extends to infinity cylindrical material that carries the charge and we have a Gaussian surface like this; inside the material. If we consider a Gaussian surface outside the material like this and we try to calculate the electric field outside, then our integral over charge, the charge density that should be confined to some value of s say s naught here and we will find s naught cubed here.

So, we can consider the Gaussian surface inside or outside the cylinder now. So, we have to perform this integral over the electric field over the curved surface.

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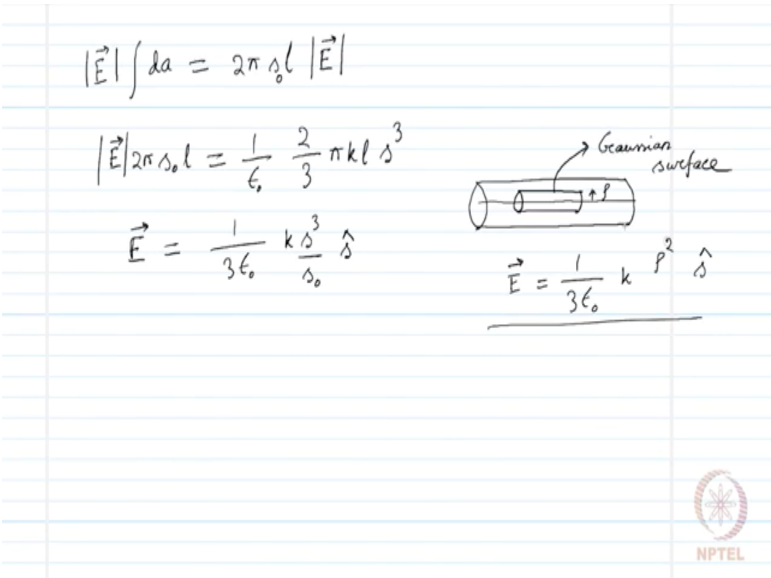

And over the curved surface, the magnitude of the electric field remains constant. So, we take it outside the integral E times da sorry because we have a dot product. It is a scalar that becomes 2 pi s l times the electric field magnitude. And if we consider this Gaussian surface at a radius say s naught, then here we will put s naught and with that applying Gauss law, we can write the electric field times to electric field magnitude times 2 pi s naught l equals 1 over epsilon naught 2 over 3 pi k l s cubed. s is the radius of the cylinder with smaller radius; that means, the charge carrying cylinder.

And hence, we can find the expression for electric field electric field vector that would be given as 1 over 3 epsilon naught. We will have pi is getting cancelled. We will be left with l’s will also cancel. We will be left with k s cubed over s naught along s direction and if we consider the opposite situation, where we have we want to find out the electric field inside the cylinder like this and the Gaussian surface is inside the cylinder; this one is the Gaussian surface. Then, we will have to perform the integral over the charge on the radius of this Gaussian surface say rho.

Then, the situation will change like with this the electric field will become looking at the other expression, we can write 1 over 3 epsilon naught k rho cubed over rho along s cap direction. That means, we can write it down as rho squared. So, this will be the expression for the electric field in this kind of a situation, where the outside cylinder is the charge carrying cylinder and inside cylinder is the Gaussian surface. So, we are finding the electric field inside the charge carrying cylinder itself.